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Instructor's Manual

to accompany

Applied Strength of Materials

Fifth Edition

Robert L. Mott

PEARSON
Prentice Hall

Upper Saddle River, New Jersey
Columbus, Ohio

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ISBN-13: 978-0-13-174340-3
ISBN-10: 0-13-174340-6

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Options for Course Organization

INTRODUCTION

Course organization is one of the most important responsibilities for an instructor. Knowledge of the specific objectives of the program or programs of which the course is a part is critical, particularly with regard to the prerequisite knowledge and skills students are expected to have when they begin the course and the outcomes expected as they relate to career paths of the students and abilities required for successful completion of following courses.

With these overarching considerations in mind, this document attempts to provide options for how to structure a course in Strength of Materials using this textbook. Variables considered include specific prerequisites for mathematics, statics, centroids and moments of inertia, physics mechanics, and materials science. Comments are then presented about each of the 13 chapters of the book.

Users of previous editions of this book will notice significant changes in the arrangement of topic coverage in this edition, in response to feedback from colleagues and users, both instructors and students.

- **Mathematics:** Students are expected, as a minimum, to have good abilities in college algebra and trigonometry. Additional skills in calculus are beneficial but not necessary. Comprehension of virtually all topics in the book and completion of almost all problems for student solution require only algebra and trigonometry. The principles of strength of materials in each chapter are developed first with logical observations of the behavior of materials when subjected to particular forces, moments, and torques with specific support conditions. Typically, those observations are presented in the introduction to each chapter in the form of ***The Big Picture*** in which students are asked to observe structures and various devices with which they are familiar and to engage in simple activities from which they can discover underlying principles. Then the primary formulas governing the mathematical representation of those behaviors are stated along with the definition of variables and statements of limitations on the use of the formulas. For most concepts, a separate section is included that presents a more complete development of the formulas, often using differential and integral calculus. This is beneficial for students who have completed such mathematics courses and

for instructors who prefer this approach. However, it is not essential to include coverage of these sections and they are marked [Optional] in the following chapter overviews.

- **Statics:** It is considered essential that students have fundamental understanding of *forces, moments, vectors,* and *static equilibrium* to learn adequately the principles of strength of materials and the problem solution techniques presented in this book. An extensive review of the principles of **Statics** is included in Appendix A-27 for students needing reinforcement. Also, the study of **Physics Mechanics** is beneficial and is typically included as a prerequisite to **Statics**.
- **Centroids and Moments of Inertia of Areas:** Many courses in Statics include these topics. However, there is some advantage in delaying this coverage until these concepts are needed for application to beam analysis within the study of strength of materials. This provides just-in-time coverage that flows naturally as presented in Chapters 5, 6, and 7 in this book. When a particular course requires prerequisite knowledge of **Centroids and Moments of Inertia of Areas**, Chapter 6 can be skipped. Having the material in the book should be useful for students to review as needed.
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POSSIBLE COURSE ORGANIZATIONS

The order of presentation of topics in this book is, in the opinion of the author, logical and would lead to a rather linear progression through the chapters in the order given. The primary options for course organization involve consideration of which topics are essential to the objectives of the specific course. Options are presented here in a chapter-by-chapter basis.

Chapter 1 – Basic Concepts of Strength of Materials

- Sections 1-1 through 1-12 should be covered completely in order to present a foundation for the study of later chapters, to present basic expectations for student performance, and to give students an overview of many of the Appendix tables related to the properties of areas and standard shapes used for structural and mechanical applications. [See Appendixes A-1 through A-13.]
- Sections 1-6 through 1-11 give the basic concepts of stress and strain for direct tension, direct compression, and direct shear.
- The emphasis is on analysis and the understanding of the ability of materials to resist external forces applied to them. This is necessary for progression into Chapter 2 on Design Properties of Materials where some additional material properties are discussed. These basic concepts are expanded upon in Chapter 3.
- Mention should also be made of Appendix A-26 Conversion Factors and Appendix A-27 Review of the Fundamentals of Statics.
- Coverage of Section 1-13 Experimental and Computational Stress Analysis is optional and may depend on the connection of this course with companion laboratory courses.

Chapter 2 – Design Properties of Materials

Refer to the discussion of **Materials Science** given above in regard to prerequisite study. Most students will benefit from at least a quick review of all parts of this chapter and the related Appendixes. Those without prerequisite knowledge of materials will need more intensive study. Some considerations for coverage are discussed next.

- Students in mechanical, manufacturing, civil and construction programs all require sound knowledge of metals and plastics.
- Most would also benefit from coverage of wood, concrete, and composites.
- Section 2-12 [Optional] on Composites may be delayed until Chapter 7 is covered and linked with Section 7-12 on the design of beams to be made from composite materials.
- The section on Materials Selection gives approaches to relating the expected performance of a structure or product to the behavior of appropriate materials. The method featured here leads to consideration of a wide variety of materials and refers to other references giving more extensive treatment of the materials selection processes. Of particular note is the reference for Dr. Michael Ashby's book, *Materials Selection in Mechanical Design.*

Chapter 3 – Direct Stress, Deformation, and Design

This chapter builds on the basic introductory treatment of direct stresses from Chapter 1 and adds significant competencies in design of load-carrying members. Design stresses are defined and related to the yield strength or ultimate strength of the materials and to the manner of loading; steady, repeated, and impact or shock. Coverage can be grouped as follows:

- The Big Picture, Activity, and Chapter Objectives

- Sections 3-2 through 3-6: Design of members under direct normal stresses, including the definition of design stress, design factor (factor of safety), and design approaches.
- Sections 3-7 through 3-11: Deformation, thermal stresses, members made from more than one material, and stress concentration factors for direct axial stresses
- Sections 3-12 and 3-13 on bearing stress, including design bearing stresses
- Section 3-14 – Design Shear Stress

Users of previous editions of this book will note that, in response to feedback from colleagues and external reviewers, a significant re-ordering of topics has been done in this new 5th edition. For example, Bearing Stresses were formerly presented in Chapter 1 and deformations and related topics were covered in a separate chapter. It was recommended that both stress and strain (with deformations) be included in one chapter for each type of stress.

Chapter 4 – Torsional Shear Stress and Torsional Deformation

Coverage of this chapter can be groups as follows:

- Big Picture, Activity, and Objectives
- Section 4-2 on Torque, Power, and Rotational Speed: These topics should be review for most students but it has been found that careful study is required before applying them to stress analysis.
- Section 4-3 presents the fundamental torsional shear stress formula and demonstrates its application to the analysis of stresses.
- Sections 4-4 and 4-5 [Optional] use calculus to derive the torsional shear stress formula and the equations for polar moment for solid circular bars.
- Section 4-6 extends the coverage to hollow circular sections. While some calculus is used to develop equations for polar moment of inertia, the final equations are all that is required for problem solving.
- Section 4-7 presents an approach to design of circular members under torsion, extending the design stress concepts from Chapter 3 to include torsional shear strength of materials.
- Section 4-8: This section provides interesting and useful comparison of the behavior of hollow circular sections and emphasizes their efficiency as compared with solid sections.
- Section 4-9: The study of stress concentrations in torsionally loaded members is essential to proper design and analysis of shafts.
- Section 4-10: The twisting of circular bars is discussed with the application of the equation for torsional deformation.
- Section 4-11 [Optional] Torsion in noncircular sections is less frequently encountered in practice. However, it is important for students to understand that such shapes behave quite differently from circular sections.

Chapters 5 through 9: All these chapters deal with beams; members carrying loads perpendicular to their axes. Students should be advised to scan all five chapters to see the progression of topics and to observe how each chapter relates to the others.

Chapter 5 – Shearing Forces and Bending Moments in Beams

- The Big Picture, Activity, and Sections 5-1 through 5-9 are essential. On rare occasions, some programs include some of these topics in the Statics course.
- Section 5-10 [Optional] Free-Body Diagrams of Parts of Structures: Mastery of this topic gives students a better fundamental understanding of the behavior of load carrying members by visualizing the internal forces, moments, and stresses created by various external loads.
- Section 5-11 [Optional] Mathematical Analysis of Beam Diagrams: Here students apply calculus to derive equations for shearing force and bending moments from given beam loading and support conditions. This skill is required for later study of Section 9-7 Successive Integration Method for deflection of beams, which is, itself, optional.
- Section 5-12 [Optional] Continuous Beams – Theorem of Three Moments: Students should, at least, understand that the behavior of beams with three or more supports is quite different from those with only two simple supports as covered in other sections of this chapter. Extensive study of this topic, however, would be most beneficial for the civil and construction fields where such beams are frequently applied in bridges and buildings.
- ***Note: This is one place where the Beam Calculator program supplied with this book can be used effectively for analyzing complex loading patterns after students have mastered the manual process of creating shearing force and bending moment diagrams. The 'Shear' and 'Moment' selections produce complete diagrams immediately after the beam loading and support conditions are defined.***

Chapter 6 – Centroids and Moments of Inertia of Areas

This entire chapter may be skipped for those programs in which the coverage of this topic is included in a prerequisite course in Statics. However, review of the procedures for computing the location of centroids and the computation of moments of inertia of areas is typically required. This can be done by moving directly to Sections 6-5, 6-6, and 6-8 where sections commonly encountered in strength of materials are considered, especially those including standard structural shapes such as W-beams, channels, and angles.

For those programs that do not include this topic in prior courses, coverage of Sections 6-1 through 6-6 and 6-8 should be covered as a minimum. These skills are essential to the understanding of concepts in Chapters 7 – 11. Coverage of the other sections of this chapter are optional as discussed next.

- Section 6-7 [Optional] uses calculus to derive the moment of inertia of an area, I.

- Section 6-9 [Optional] provides a useful method of analyzing shapes with all rectangular parts. The process can be implemented effectively in a spreadsheet.
- Section 6-10 [Optional] Radius of Gyration is an important property of an area and is most directly applicable to Chapter 11 on Columns. It may be desirable to delay the coverage of this topic to combine it with the study of columns.
- Section 6-11 [Optional] Section Modulus is an important property of an area and is most directly applicable to Chapter 7 on Stress Due to Bending. It may be desirable to delay the coverage of this topic to combine it with the study of beams.

Chapter 7 – Stress Due to Bending

- Sections 7-1 through 7-4 present the foundation material for the analysis of beams.
- Section 7-5 [Optional] uses calculus to derive the flexure formula. It can be skipped or discussed lightly for those programs where detailed use of the calculus is not expected.
- Sections 7-6 through 7-8 cover the transitions from analysis to design of beams.
- Section 7-9 covers stress concentrations in bending situations.
- Section 7-10 is critical, at least from the standpoint that students must understand that the flexure formula applies only to symmetrical sections or when the load path passes through the flexural center (shear center) of the section. Otherwise twisting combines with the bending stress, reducing the capacity of the beam.
- Section 7-11 on Preferred Shapes for Beam Cross Sections is designed to help the novice student understand better why certain shapes are preferred for beams.
- Section 7-12 [Optional] on beams made from composites presents mostly conceptual information about the advantages of composites in bending cases and how the shape can be optimized to make best use of the special properties of composites. This section refers back to Section 2-12 and it may be desirable to cover those two sections together at this point.
- ***Note: This is one place where the Beam Calculator program supplied with this book can be used effectively for analyzing bending stress produced by complex loading patterns after students have mastered the manual process making such calculations on more simple beams. The 'Stress' selection produces the complete diagram of bending stress distribution immediately after the beam loading and support conditions are defined. Students should compare this result with the bending moment diagram.***

Chapter 8 – Shearing Stresses in Beams

- Sections 8-1 through 8-4 present the fundamental concepts and the general shear formula.
- Section 8-5 [Optional] uses calculus to derive the general shear formula. It can be skipped or discussed lightly for those programs where detailed use of the calculus is not expected.
- Section 8-6 shows the special shear formulas applicable to rectangular, circular, hollow, and thin-webbed sections (e.g. W-beams). These formulas are frequently used.
- Section 8-7 transitions the coverage of shear in beams from analysis to design.

- Section 8-8 on shear flow [Optional] is applicable to beam sections made from component shapes that are fastened, glued, or otherwise assembled where connections are subjected to shear.

Chapter 9 – Deflection of Beams

There appears to be a wide divergence of opinion about what types of beam deflection approaches to cover in a basic course in strength of materials. This book attempts to show all popular approaches and let individual instructors and program faculty members decide which is best for their programs.

Note: This is the place where the Beam Calculator program supplied with this book is most applicable. The complete deflection curve is produced immediately after the beam loading and support conditions are defined by selecting the 'Deflection' button. Comparison of the Deflection curve with the Shear, Moment, and Stress diagrams is advised.

That said, here are some factors to consider in course planning:

- Sections 9-1 through 9-4 present the basic concepts and the widely used formulas for beam deflection, using the extensive list of formulas from Appendixes A-23, A-24, and A-25.
- Section 9-5 gives students some experience in comparing the performance of several ways of supporting a given load with regard to the stresses and deflections that result. This should help the novice student gain a better 'feel' for what approaches are preferred in different applications.
- Section 9-6 extends the material in Section 9-4 to the permit use of beam deflection formulas to a much broader array of applications.
- Section 9-7 on the Successive Integration Method [Optional] provides a more analytical approach to deflection analysis. It requires the use of differential and integral calculus and should be combined with Section 5-11 Mathematical Analysis of Beam Diagrams. Mastery of these concepts would be expected for students who intend to continue their study of applied mechanics in later courses or graduate study. However, their application to typical design and analysis cases, especially those with multiple loads, is typically very cumbersome and it has become normal procedure to use commercially-available beam analysis software for such problems. ***The Beam Calculator program supplied with this book is a basic example.***
- Section 9-8 – Moment-Area Method [Optional] is preferred by some designers for applications that do not lend themselves to the use of formulas, superposition, or the successive integration approach. A notable example is the analysis of beams with varying cross sections as illustrated in this section.

Chapter 10 – Combined Stresses

The extent of coverage of the several topics in this chapter is best done by the individual instructor and/or program faculty members.

- Sections 10-1 through 10-6 give good introductory coverage of the issues presented when two or more types of stresses occur at a given point. They also tie material from previous chapters together to help students understand the distribution of stresses and the interactions involved. Combined normal stresses and combined normal and shear stresses are discussed.
- Sections 10-7 through 10-11 cover stress transformations, equations for stresses in any direction, principal stresses (maximum normal stress, maximum shear stress), and Mohr's circle.
- Section 10-12 covers the use of strain-gage rosettes to determine principal stresses and ties well with the preceding sections. It is also related to Section 1-13 – Experimental and Computational Stress Analysis, and is useful for connecting this course with companion laboratory courses.

Chapter 11 – Columns

- This chapter is a succinct, but comprehensive coverage of column analysis.
- Included are basic concepts, Euler formula for long columns, J. B. Johnson formula for short columns, and non-centrally loaded columns (crooked and eccentrically loaded).
- A Column Analysis Spreadsheet is shown that facilitates the calculations.

Chapter 12 – Pressure Vessels

- Basic concepts for thin-walled spheres and cylinders are recommended as a minimum, using Sections 12-1 through 12-4.
- Sections 12-5 through 12-7 [Optional] present extended coverage of thick-walled pressure vessels.
- Sections 12-8 and 12-9 [Optional] present additional considerations for column design.
- Section 12-9 [Optional] discusses the advantages of applying composite materials to pressure vessels. Reference to Section 2-12 should be made for basic properties of composites.

Chapter 13 – Connections

- This chapter covers bolted and riveted joints and welded connections.

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Software Included with the Book

INTRODUCTION

Two types of software on a CD-ROM are included with this book:

1. A set of 12 interactive video lessons that students can use to:
    a. Review material from the text for a given topic
    b. Observe the solution of a representative problem
    c. Complete a quiz at the end of each module to test understanding
2. A versatile beam calculator program that allows:
    a. The creation of a beam and its loading and support patterns
    b. Analysis of:
        i. Shearing force distribution
        ii. Bending moment distribution
        iii. Deflection of the beam at all points in the beam
        iv. Stress due to bending at all points in the beam

The software was created by Professor Jack Zecher of Indiana University - Purdue University – Indianapolis (IUPUI) in Indianapolis, Indiana.

ADVICE ON THE USE OF THE SOFTWARE

As with any software, students are advised to read pertinent text material and master the fundamental principles of the subject and the methods of problem solution prior to using the software.

INTERACTIVE VIDEO LESSONS

The following lessons with quizzes are included in this software:

1. ***NORMAL STRESS*** – Reviews the direct normal stress equation, σ = *Force/Area* for both tension and compression. Illustrates the calculation of direct normal stress on a member with multiple cross section sizes. Relevant to Chapters 1 – 3.
2. ***DIRECT SHEAR*** – Reviews the direct shear stress equation, τ = *Force/Area in shear*, for both single shear and double shear. Relevant to Chapters 1 – 3.
3. ***PUNCHING SHEAR*** – Reviews shearing stress that occurs in a cutting or punching situation using the direct shear stress equation, τ = *Force/Area in shear*, with emphasis on identifying the correct area in shear. Relevant to Chapters 1 and 3.
4. ***POISSON'S RATIO*** – Reviews the definition of strain and the fact that strains in both longitudinal and transverse directions are created when a load-carrying member is subjected to direct normal stress. Reviews the definition of Poisson's ratio. Relevant to Chapters 2 and 3.
5. ***STRESS CONCENTRATION*** – Reviews the concept of increased stresses occurring near sections of load-carrying members with abrupt changes in cross section. Illustrates the stress concentration factor for a member loaded in tension. Includes color graphic illustrations of stress lines around a hole and the plot of results of a finite element analysis. Relevant to Chapter 3.
6. ***AXIAL DEFORMATION*** – Reviews the deformation of members loaded in direct tension or compression using the formula, $\delta = FL/EA$. Relevant to Chapter 3.
7. ***THERMAL STRESSES*** – Reviews the property of coefficient of thermal expansion, α. Demonstrates the calculation of thermal expansion using the formula, $\delta = \alpha L(\Delta t)$ for a given change of temperature, Δt. Also demonstrates the stress created when members are restrained as temperatures change. Relevant to Chapter 3.
8. ***STATICALLY INDETERMINATE*** – Reviews the principles of axial deformation and considers the case when two or more members, possibly made from different materials, are loaded together. Relevant to Chapter 3.
9. ***TORSIONAL STRESS AND DEFORMATION*** – Reviews both the torsional shear stress equation, $\tau = Tc/J$ and the torsional deformation equation, $\theta = TL/GJ$. Illustrates calculations for a stepped shaft loaded by two torques and shows a torque diagram. Relevant to Chapter 4.
10. ***BENDING STRESS*** – Reviews the bending stress equation, $\sigma = Mc/I$, along with shearing force and bending moment diagrams. A finite element analysis animation is included illustrating how bending stresses are produced as a section of a T-beam deforms. Relevant to Chapters 5 – 7.
11. ***SHEAR IN BEAMS*** – Reviews shearing forces and stresses produced in beams along with bending. Illustrates the application of the beam shearing stress formula, $\tau = VQ/It$, using a rectangular beam made from glued laminations. Relevant to Chapter 8.

12. ***COMBINED NORMAL STRESSES*** – Reviews the case when a member is subjected to simultaneous bending and direct normal stresses. Includes a finite element model of such a member. Relevant to Chapter 10.

Notes on the quizzes: After viewing the video of any module, the student may access an interactive quiz in which a situation similar to the example shown in the video is presented with data. The student must complete the analysis on paper and enter the result. The program determines whether the entered result is correct or not and reports back. Students are permitted to enter values twice before the correct solution is shown.

BEAM CALCULATOR

This versatile software permits students to perform analyses of beams with complex loading patterns and with many combinations of support conditions. Its use, after students have mastered the principles of beam analysis by hand calculations, facilitates the evaluation of multiple alternative designs for a beam to explore relationships among variables such as:

- Types of support and their placement relative to the applied loads
- Magnitude of the loads and their placement relative to the supports
- Beam materials and cross section properties such as modulus of elasticity, moment of inertia, and shape

Many more and more complex examples can be analyzed in a given amount of time, extending learning beyond the typical problems that are assigned for practice by hand calculations.

The software uses a finite element analysis-based process that divides the beam into 50 segments. Calculations of results are made for each of the 50 points and at any applied load or support. ***If the user desires that the results for any other point be given, a concentrated load of zero value may be placed at that point.***

Features of the software include:

1. **Units** - Units of length are first selected by the user in either English (feet or inches) or Metric (meters or millimeters).
2. **Beam Properties** – Beam properties are entered by the user for:
    a. Beam length
    b. Modulus of elasticity, E, for the material of the beam
    c. Moment of inertia, I, for the cross section shape and dimensions of the beam
    d. Distance from the neutral axis of the cross section to the top of the beam
    e. Distance from the neutral axis of the cross section to the bottom of the beam
3. **Supports** – The type or types of supports and their placement are defined by the user. Up to 20 supports may be used in any combination of:
    a. Roller support providing only vertical support
    b. Pinned support providing vertical or horizontal support

    i. Note: Theoretically one roller support and one pin support should be provided for a simply supported beam to ensure equilibrium. However, this program permits only vertical concentrated or distributed loads and couples for which only vertical reactions are computed.

  c. Fixed support providing vertical and moment resistance, such as the support for a cantilever
  d. Before the analysis can proceed, the beam design must have a minimum of either:
     i. Two pinned supports
     ii. One pinned and one roller support
     iii. One fixed support
  e. The user may modify any support type or location before analysis is performed. This feature facilitates correction of entered data or the exploration of several alternative designs.

4. **Loads** – The user defines any combination of up to 20 loads by giving their placement and magnitudes. The load types available are:
   a. **Concentrated**
   b. **Distributed** – Either uniformly or uniformly varying distributed loads can be used. The user enters the placement and magnitude (force per unit length) at the start and at the end of the loading.
   c. **Couple** – This is a concentrated moment applied at any point along the beam. A counterclockwise couple is considered positive.
5. **Analyze** – After the beam is defined completely, the user selects the 'Analyze' button. If an incomplete or an excessive set of data are provided, the analysis will not be completed. The following analyses are completed:
   a. **Shear** – A complete shearing force diagram is shown under the beam design
   b. **Moment** – A complete bending moment diagram is shown under the beam design
   c. **Deflection** – A complete diagram of the shape of the deflected beam is shown
   d. **Stress** – The distribution of bending stress across the entire length of the beam is shown
   e. **Notes**:
      i. Values at any point on any diagram can be displayed by placing the cursor at the desired point.
      ii. The ESC (escape) key must be used to stop the interaction with the currently displayed diagram before switching from one type of output to another.

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- Students in mechanical, manufacturing, civil and construction programs all require sound knowledge of metals and plastics.
- Most would also benefit from coverage of wood, concrete, and composites.
- Section 2-12 [Optional] on Composites may be delayed until Chapter 7 is covered and linked with Section 7-12 on the design of beams to be made from composite materials.
- The section on Materials Selection gives approaches to relating the expected performance of a structure or product to the behavior of appropriate materials. The method featured here leads to consideration of a wide variety of materials and refers to other references giving more extensive treatment of the materials selection processes. Of particular note is the reference for Dr. Michael Ashby's book, *Materials Selection in Mechanical Design.*

Chapter 3 – Direct Stress, Deformation, and Design

This chapter builds on the basic introductory treatment of direct stresses from Chapter 1 and adds significant competencies in design of load-carrying members. Design stresses are defined and related to the yield strength or ultimate strength of the materials and to the manner of loading; steady, repeated, and impact or shock. Coverage can be grouped as follows:

- The Big Picture, Activity, and Chapter Objectives

- Sections 3-2 through 3-6: Design of members under direct normal stresses, including the definition of design stress, design factor (factor of safety), and design approaches.
- Sections 3-7 through 3-11: Deformation, thermal stresses, members made from more than one material, and stress concentration factors for direct axial stresses
- Sections 3-12 and 3-13 on bearing stress, including design bearing stresses
- Section 3-14 – Design Shear Stress

Users of previous editions of this book will note that, in response to feedback from colleagues and external reviewers, a significant re-ordering of topics has been done in this new 5th edition. For example, Bearing Stresses were formerly presented in Chapter 1 and deformations and related topics were covered in a separate chapter. It was recommended that both stress and strain (with deformations) be included in one chapter for each type of stress.

Chapter 4 – Torsional Shear Stress and Torsional Deformation

Coverage of this chapter can be groups as follows:

- Big Picture, Activity, and Objectives
- Section 4-2 on Torque, Power, and Rotational Speed: These topics should be review for most students but it has been found that careful study is required before applying them to stress analysis.
- Section 4-3 presents the fundamental torsional shear stress formula and demonstrates its application to the analysis of stresses.
- Sections 4-4 and 4-5 [Optional] use calculus to derive the torsional shear stress formula and the equations for polar moment for solid circular bars.
- Section 4-6 extends the coverage to hollow circular sections. While some calculus is used to develop equations for polar moment of inertia, the final equations are all that is required for problem solving.
- Section 4-7 presents an approach to design of circular members under torsion, extending the design stress concepts from Chapter 3 to include torsional shear strength of materials.
- Section 4-8: This section provides interesting and useful comparison of the behavior of hollow circular sections and emphasizes their efficiency as compared with solid sections.
- Section 4-9: The study of stress concentrations in torsionally loaded members is essential to proper design and analysis of shafts.
- Section 4-10: The twisting of circular bars is discussed with the application of the equation for torsional deformation.
- Section 4-11 [Optional] Torsion in noncircular sections is less frequently encountered in practice. However, it is important for students to understand that such shapes behave quite differently from circular sections.

Chapters 5 through 9: All these chapters deal with beams; members carrying loads perpendicular to their axes. Students should be advised to scan all five chapters to see the progression of topics and to observe how each chapter relates to the others.

Chapter 5 – Shearing Forces and Bending Moments in Beams

- The Big Picture, Activity, and Sections 5-1 through 5-9 are essential. On rare occasions, some programs include some of these topics in the Statics course.
- Section 5-10 [Optional] Free-Body Diagrams of Parts of Structures: Mastery of this topic gives students a better fundamental understanding of the behavior of load carrying members by visualizing the internal forces, moments, and stresses created by various external loads.
- Section 5-11 [Optional] Mathematical Analysis of Beam Diagrams: Here students apply calculus to derive equations for shearing force and bending moments from given beam loading and support conditions. This skill is required for later study of Section 9-7 Successive Integration Method for deflection of beams, which is, itself, optional.
- Section 5-12 [Optional] Continuous Beams – Theorem of Three Moments: Students should, at least, understand that the behavior of beams with three or more supports is quite different from those with only two simple supports as covered in other sections of this chapter. Extensive study of this topic, however, would be most beneficial for the civil and construction fields where such beams are frequently applied in bridges and buildings.
- ***Note: This is one place where the Beam Calculator program supplied with this book can be used effectively for analyzing complex loading patterns after students have mastered the manual process of creating shearing force and bending moment diagrams. The 'Shear' and 'Moment' selections produce complete diagrams immediately after the beam loading and support conditions are defined.***

Chapter 6 – Centroids and Moments of Inertia of Areas

This entire chapter may be skipped for those programs in which the coverage of this topic is included in a prerequisite course in Statics. However, review of the procedures for computing the location of centroids and the computation of moments of inertia of areas is typically required. This can be done by moving directly to Sections 6-5, 6-6, and 6-8 where sections commonly encountered in strength of materials are considered, especially those including standard structural shapes such as W-beams, channels, and angles.

For those programs that do not include this topic in prior courses, coverage of Sections 6-1 through 6-6 and 6-8 should be covered as a minimum. These skills are essential to the understanding of concepts in Chapters 7 – 11. Coverage of the other sections of this chapter are optional as discussed next.

- Section 6-7 [Optional] uses calculus to derive the moment of inertia of an area, *I*.

- Section 6-9 [Optional] provides a useful method of analyzing shapes with all rectangular parts. The process can be implemented effectively in a spreadsheet.
- Section 6-10 [Optional] Radius of Gyration is an important property of an area and is most directly applicable to Chapter 11 on Columns. It may be desirable to delay the coverage of this topic to combine it with the study of columns.
- Section 6-11 [Optional] Section Modulus is an important property of an area and is most directly applicable to Chapter 7 on Stress Due to Bending. It may be desirable to delay the coverage of this topic to combine it with the study of beams.

Chapter 7 – Stress Due to Bending

- Sections 7-1 through 7-4 present the foundation material for the analysis of beams.
- Section 7-5 [Optional] uses calculus to derive the flexure formula. It can be skipped or discussed lightly for those programs where detailed use of the calculus is not expected.
- Sections 7-6 through 7-8 cover the transitions from analysis to design of beams.
- Section 7-9 covers stress concentrations in bending situations.
- Section 7-10 is critical, at least from the standpoint that students must understand that the flexure formula applies only to symmetrical sections or when the load path passes through the flexural center (shear center) of the section. Otherwise twisting combines with the bending stress, reducing the capacity of the beam.
- Section 7-11 on Preferred Shapes for Beam Cross Sections is designed to help the novice student understand better why certain shapes are preferred for beams.
- Section 7-12 [Optional] on beams made from composites presents mostly conceptual information about the advantages of composites in bending cases and how the shape can be optimized to make best use of the special properties of composites. This section refers back to Section 2-12 and it may be desirable to cover those two sections together at this point.
- ***Note: This is one place where the Beam Calculator program supplied with this book can be used effectively for analyzing bending stress produced by complex loading patterns after students have mastered the manual process making such calculations on more simple beams. The 'Stress' selection produces the complete diagram of bending stress distribution immediately after the beam loading and support conditions are defined. Students should compare this result with the bending moment diagram.***

Chapter 8 – Shearing Stresses in Beams

- Sections 8-1 through 8-4 present the fundamental concepts and the general shear formula.
- Section 8-5 [Optional] uses calculus to derive the general shear formula. It can be skipped or discussed lightly for those programs where detailed use of the calculus is not expected.
- Section 8-6 shows the special shear formulas applicable to rectangular, circular, hollow, and thin-webbed sections (e.g. W-beams). These formulas are frequently used.
- Section 8-7 transitions the coverage of shear in beams from analysis to design.

- Section 8-8 on shear flow [Optional] is applicable to beam sections made from component shapes that are fastened, glued, or otherwise assembled where connections are subjected to shear.

Chapter 9 – Deflection of Beams

There appears to be a wide divergence of opinion about what types of beam deflection approaches to cover in a basic course in strength of materials. This book attempts to show all popular approaches and let individual instructors and program faculty members decide which is best for their programs.

Note: This is the place where the Beam Calculator program supplied with this book is most applicable. The complete deflection curve is produced immediately after the beam loading and support conditions are defined by selecting the 'Deflection' button. Comparison of the Deflection curve with the Shear, Moment, and Stress diagrams is advised.

That said, here are some factors to consider in course planning:

- Sections 9-1 through 9-4 present the basic concepts and the widely used formulas for beam deflection, using the extensive list of formulas from Appendixes A-23, A-24, and A-25.
- Section 9-5 gives students some experience in comparing the performance of several ways of supporting a given load with regard to the stresses and deflections that result. This should help the novice student gain a better 'feel' for what approaches are preferred in different applications.
- Section 9-6 extends the material in Section 9-4 to the permit use of beam deflection formulas to a much broader array of applications.
- Section 9-7 on the Successive Integration Method [Optional] provides a more analytical approach to deflection analysis. It requires the use of differential and integral calculus and should be combined with Section 5-11 Mathematical Analysis of Beam Diagrams. Mastery of these concepts would be expected for students who intend to continue their study of applied mechanics in later courses or graduate study. However, their application to typical design and analysis cases, especially those with multiple loads, is typically very cumbersome and it has become normal procedure to use commercially-available beam analysis software for such problems. ***The Beam Calculator program supplied with this book is a basic example.***
- Section 9-8 – Moment-Area Method [Optional] is preferred by some designers for applications that do not lend themselves to the use of formulas, superposition, or the successive integration approach. A notable example is the analysis of beams with varying cross sections as illustrated in this section.

Chapter 10 – Combined Stresses

The extent of coverage of the several topics in this chapter is best done by the individual instructor and/or program faculty members.

- Sections 10-1 through 10-6 give good introductory coverage of the issues presented when two or more types of stresses occur at a given point. They also tie material from previous chapters together to help students understand the distribution of stresses and the interactions involved. Combined normal stresses and combined normal and shear stresses are discussed.
- Sections 10-7 through 10-11 cover stress transformations, equations for stresses in any direction, principal stresses (maximum normal stress, maximum shear stress), and Mohr's circle.
- Section 10-12 covers the use of strain-gage rosettes to determine principal stresses and ties well with the preceding sections. It is also related to Section 1-13 – Experimental and Computational Stress Analysis, and is useful for connecting this course with companion laboratory courses.

Chapter 11 – Columns

- This chapter is a succinct, but comprehensive coverage of column analysis.
- Included are basic concepts, Euler formula for long columns, J. B. Johnson formula for short columns, and non-centrally loaded columns (crooked and eccentrically loaded).
- A Column Analysis Spreadsheet is shown that facilitates the calculations.

Chapter 12 – Pressure Vessels

- Basic concepts for thin-walled spheres and cylinders are recommended as a minimum, using Sections 12-1 through 12-4.
- Sections 12-5 through 12-7 [Optional] present extended coverage of thick-walled pressure vessels.
- Sections 12-8 and 12-9 [Optional] present additional considerations for column design.
- Section 12-9 [Optional] discusses the advantages of applying composite materials to pressure vessels. Reference to Section 2-12 should be made for basic properties of composites.

Chapter 13 – Connections

- This chapter covers bolted and riveted joints and welded connections.

APPLIED STRENGTH OF MATERIALS
5^{TH} Ed.
by
Robert L. Mott

Software Included with the Book

INTRODUCTION

Two types of software on a CD-ROM are included with this book:

1. A set of 12 interactive video lessons that students can use to:
    a. Review material from the text for a given topic
    b. Observe the solution of a representative problem
    c. Complete a quiz at the end of each module to test understanding
2. A versatile beam calculator program that allows:
    a. The creation of a beam and its loading and support patterns
    b. Analysis of:
        i. Shearing force distribution
        ii. Bending moment distribution
        iii. Deflection of the beam at all points in the beam
        iv. Stress due to bending at all points in the beam

The software was created by Professor Jack Zecher of Indiana University - Purdue University – Indianapolis (IUPUI) in Indianapolis, Indiana.

ADVICE ON THE USE OF THE SOFTWARE

As with any software, students are advised to read pertinent text material and master the fundamental principles of the subject and the methods of problem solution prior to using the software.

INTERACTIVE VIDEO LESSONS

The following lessons with quizzes are included in this software:

1. ***NORMAL STRESS*** – Reviews the direct normal stress equation, σ = *Force/Area* for both tension and compression. Illustrates the calculation of direct normal stress on a member with multiple cross section sizes. Relevant to Chapters 1 – 3.
2. ***DIRECT SHEAR*** – Reviews the direct shear stress equation, τ = *Force/Area in shear*, for both single shear and double shear. Relevant to Chapters 1 – 3.
3. ***PUNCHING SHEAR*** – Reviews shearing stress that occurs in a cutting or punching situation using the direct shear stress equation, τ = *Force/Area in shear*, with emphasis on identifying the correct area in shear. Relevant to Chapters 1 and 3.
4. ***POISSON'S RATIO*** – Reviews the definition of strain and the fact that strains in both longitudinal and transverse directions are created when a load-carrying member is subjected to direct normal stress. Reviews the definition of Poisson's ratio. Relevant to Chapters 2 and 3.
5. ***STRESS CONCENTRATION*** – Reviews the concept of increased stresses occurring near sections of load-carrying members with abrupt changes in cross section. Illustrates the stress concentration factor for a member loaded in tension. Includes color graphic illustrations of stress lines around a hole and the plot of results of a finite element analysis. Relevant to Chapter 3.
6. ***AXIAL DEFORMATION*** – Reviews the deformation of members loaded in direct tension or compression using the formula, $\delta = FL/EA$. Relevant to Chapter 3.
7. ***THERMAL STRESSES*** – Reviews the property of coefficient of thermal expansion, α. Demonstrates the calculation of thermal expansion using the formula, $\delta = \alpha L(\Delta t)$ for a given change of temperature, Δt. Also demonstrates the stress created when members are restrained as temperatures change. Relevant to Chapter 3.
8. ***STATICALLY INDETERMINATE*** – Reviews the principles of axial deformation and considers the case when two or more members, possibly made from different materials, are loaded together. Relevant to Chapter 3.
9. ***TORSIONAL STRESS AND DEFORMATION*** – Reviews both the torsional shear stress equation, $\tau = Tc/J$ and the torsional deformation equation, $\theta = TL/GJ$. Illustrates calculations for a stepped shaft loaded by two torques and shows a torque diagram. Relevant to Chapter 4.
10. ***BENDING STRESS*** – Reviews the bending stress equation, $\sigma = Mc/I$, along with shearing force and bending moment diagrams. A finite element analysis animation is included illustrating how bending stresses are produced as a section of a T-beam deforms. Relevant to Chapters 5 – 7.
11. ***SHEAR IN BEAMS*** – Reviews shearing forces and stresses produced in beams along with bending. Illustrates the application of the beam shearing stress formula, $\tau = VQ/It$, using a rectangular beam made from glued laminations. Relevant to Chapter 8.

12. ***COMBINED NORMAL STRESSES*** – Reviews the case when a member is subjected to simultaneous bending and direct normal stresses. Includes a finite element model of such a member. Relevant to Chapter 10.

Notes on the quizzes: After viewing the video of any module, the student may access an interactive quiz in which a situation similar to the example shown in the video is presented with data. The student must complete the analysis on paper and enter the result. The program determines whether the entered result is correct or not and reports back. Students are permitted to enter values twice before the correct solution is shown.

BEAM CALCULATOR

This versatile software permits students to perform analyses of beams with complex loading patterns and with many combinations of support conditions. Its use, after students have mastered the principles of beam analysis by hand calculations, facilitates the evaluation of multiple alternative designs for a beam to explore relationships among variables such as:

- Types of support and their placement relative to the applied loads
- Magnitude of the loads and their placement relative to the supports
- Beam materials and cross section properties such as modulus of elasticity, moment of inertia, and shape

Many more and more complex examples can be analyzed in a given amount of time, extending learning beyond the typical problems that are assigned for practice by hand calculations.

The software uses a finite element analysis-based process that divides the beam into 50 segments. Calculations of results are made for each of the 50 points and at any applied load or support. ***If the user desires that the results for any other point be given, a concentrated load of zero value may be placed at that point.***

Features of the software include:

1. **Units** - Units of length are first selected by the user in either English (feet or inches) or Metric (meters or millimeters).
2. **Beam Properties** – Beam properties are entered by the user for:
   a. Beam length
   b. Modulus of elasticity, E, for the material of the beam
   c. Moment of inertia, I, for the cross section shape and dimensions of the beam
   d. Distance from the neutral axis of the cross section to the top of the beam
   e. Distance from the neutral axis of the cross section to the bottom of the beam
3. **Supports** – The type or types of supports and their placement are defined by the user. Up to 20 supports may be used in any combination of:
   a. Roller support providing only vertical support
   b. Pinned support providing vertical or horizontal support

i. Note: Theoretically one roller support and one pin support should be provided for a simply supported beam to ensure equilibrium. However, this program permits only vertical concentrated or distributed loads and couples for which only vertical reactions are computed.

c. Fixed support providing vertical and moment resistance, such as the support for a cantilever

d. Before the analysis can proceed, the beam design must have a minimum of either:
   i. Two pinned supports
   ii. One pinned and one roller support
   iii. One fixed support

e. The user may modify any support type or location before analysis is performed. This feature facilitates correction of entered data or the exploration of several alternative designs.

4. **Loads** – The user defines any combination of up to 20 loads by giving their placement and magnitudes. The load types available are:
   a. **Concentrated**
   b. **Distributed** – Either uniformly or uniformly varying distributed loads can be used. The user enters the placement and magnitude (force per unit length) at the start and at the end of the loading.
   c. **Couple** – This is a concentrated moment applied at any point along the beam. A counterclockwise couple is considered positive.

5. **Analyze** – After the beam is defined completely, the user selects the 'Analyze' button. If an incomplete or an excessive set of data are provided, the analysis will not be completed. The following analyses are completed:
   a. **Shear** – A complete shearing force diagram is shown under the beam design
   b. **Moment** – A complete bending moment diagram is shown under the beam design
   c. **Deflection** – A complete diagram of the shape of the deflected beam is shown
   d. **Stress** – The distribution of bending stress across the entire length of the beam is shown
   e. **Notes**:
      i. Values at any point on any diagram can be displayed by placing the cursor at the desired point.
      ii. The ESC (escape) key must be used to stop the interaction with the currently displayed diagram before switching from one type of output to another.

CHAPTER 1 Basic Concepts in Strength of Materials

1-1 TO **1-15** ANSWERS IN TEXT.

1-16 $W = m \cdot g = 1800\,kg \cdot 9.81\,m/s^2 = 17658\,kg\,m/s^2 = 17.7 \times 10^3\,N$
$\underline{W = 17.7\,kN}$

1-17 TOTAL WT. $= mg = 4000\,kg \cdot 9.81\,m/s^2 = 39.24\,kN$
EACH FRONT WHEEL: $F_F = (\frac{1}{2})(0.40)(39.24\,kN) = \underline{7.85\,kN}$
EACH REAR WHEEL: $F_R = (\frac{1}{2})(0.60)(39.24\,kN) = \underline{11.77\,kN}$

1-18 LOADING = TOTAL FORCE/AREA
TOTAL FORCE = $6800\,kg \cdot 9.81\,m/s^2 = 66.7\,kN$
AREA = $(5.0\,m)(3.5\,m) = 17.5\,m^2$
LOADING = $66.7\,kN / 17.5\,m^2 = 3.81\,kN/m^2 = \underline{3.81\,kPa}$

1-19 FORCE = WT = $m \cdot g = 25\,kg \cdot 9.81\,m/s^2 = 245\,N$
k = SPRING SCALE = $4500\,N/m = F/\Delta L$
$\Delta L = \frac{F}{K} = \frac{245\,N}{4500\,N/m} = 0.0545\,m = 54.5 \times 10^{-3}\,m = \underline{54.5\,mm}$

1-22 $W = 17.7\,kN = 17700\,N \times 0.2248\,LB/N = \underline{3980\,LB}$

1-23 $F_F = 7.85\,kN = 7850\,N \times 0.2248\,LB/N = \underline{1765\,LB}$
$F_R = 11.77\,kN = 11770\,N \times 0.2248\,LB/N = \underline{2646\,LB}$

1-24 LOADING = $3.81\,kPa = 3.81 \times 10^3 \frac{N}{m^2} \times \frac{0.2248\,LB}{N} \times \frac{1\,m^2}{(3.28\,ft)^2} = \underline{79.6\,\frac{LB}{FT^2}}$

1-25 $F = 245\,N \cdot 0.2248\,LB/N = \underline{55.1\,LB}$
$k = 4500\,\frac{N}{m} \times \frac{0.2248\,LB}{N} \times \frac{1\,m}{39.37\,IN} = \underline{25.7\,LB/IN}$
$\Delta L = \frac{F}{k} = \frac{55.1\,LB}{25.7\,LB/IN} = \underline{2.14\,IN}$

1-26 $m = \frac{w}{g} = \frac{2750\ LB}{32.2\ FT/s^2} = 85.4\ \frac{LB\text{-}s^2}{FT} = \underline{85.4\ SLUGS}$

1-27 $m = \frac{w}{g} = \frac{12800\ LB}{32.2\ FT/s^2} = 398\ \frac{LB\text{-}s^2}{FT} = \underline{398\ SLUGS}$

1-29 $p = 1200\ psi \times 6.895\ kPa/psi = \underline{8274\ kPa}$

1-30 $\sigma = 21600\ psi \times 6.895\ kPa/psi = 149{,}000\ kPa = \underline{149\ MPa}$

1-31 $S_u = 14000\ psi \times 6.895\ kPa/psi = 96{,}500\ kPa = \underline{96.5\ MPa}$

$S_u = 76000\ psi \times 6.895\ kPa/psi = 524000\ kPa = \underline{524\ MPa}$

1-32 $n = 1750\ \frac{REV}{MIN} \times \frac{2\pi\ RAD}{REV} \times \frac{1\ MIN}{60\ s} = \underline{183\ RAD/s}$

1-33 $A = 14.1\ in^2 \times \frac{(25.4\ mm)^2}{IN^2} = \underline{9097\ mm^2}$

1-34 $y = 0.08\ IN \times 25.4\ mm/IN = \underline{2.03\ mm}$

1-35 DIMENSIONS: $18\ IN \times 25.4\ mm/IN = 457\ mm$

$12\ IN \times 25.4\ mm/IN = 305\ mm$

AREA $= (18\ IN)^2 = \underline{324\ IN^2}$

AREA $= (457\ mm)^2 = \underline{209 \times 10^5\ mm^2}$

VOLUME $= V =$ AREA $\times$ HEIGHT

$V = 324\ IN^2 \times 12\ IN = \underline{3888\ IN^3}$

$V = (1.5\ FT)^2 \times 1.0\ FT = \underline{2.25\ FT^3}$

$V = (209 \times 10^3\ mm^2) \times 305\ mm = \underline{6.37 \times 10^7\ mm^3}$

$V = (0.457\ m)^2 \times 0.305\ m = 0.0637\ m^3 = \underline{6.37 \times 10^{-2}\ m^3}$

1-36 $A = \pi D^2/4 = \pi(0.505\ IN)^2/4 = \underline{0.200\ IN^2}$

$A = 0.200\ IN^2 \times \frac{(25.4\ mm)^2}{IN^2} = \underline{129\ mm^2}$

1-37 $\sigma = \frac{P}{A} = \frac{3200\ N}{\pi D^2/4} = \frac{3200\ N}{\pi(10\ mm)^2/4} = 40.7\ N/mm^2 = \underline{40.7\ MPa}$

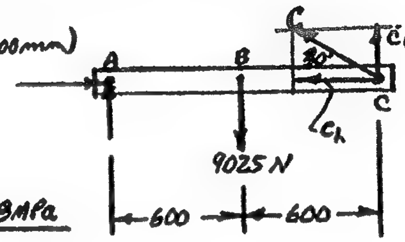
1-38 $\sigma = \frac{P}{A} = \frac{20 \times 10^3\ N}{(10)(30)\ mm^2} = 66.7\ N/mm^2 = \underline{66.7\ MPa}$

1-39 $\sigma = \frac{P}{A} = \frac{860\ LB}{(0.40\ IN)^2} = \underline{5375\ psi}$

1-40 $\sigma = \frac{P}{A} = \frac{1850\ LB}{\pi(0.375\ IN)^2/4} = \underline{16750\ psi.}$

1-41 LOAD ON SHELF $= W = mg = 1840\,kg \cdot 9.81\,m/s^2 = 18050\,N$

$W/2 = 9025\,N$ ON EACH SIDE



$\Sigma M_A = 0 = (9025\,N)(600\,mm) - C_y(1200\,mm)$

$C_y = 4512\,N$

$C = C_v / \sin 30° = 9025\,N$

$\sigma = \frac{P}{A} = \frac{C}{A} = \frac{9025\,N}{\pi(12\,mm)^2/4} = \underline{79.8\,MPa}$

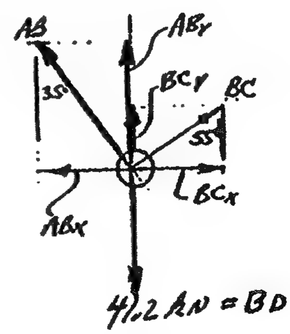
1-42 $\sigma = \frac{P}{A} = \frac{70\,000\,LB}{\pi(8\,in)^2/4} = \underline{1393\,psi}$

1-43 $\sigma = \frac{P}{A} = \frac{29\,500\,LB/3}{(3.5\,IN)^2} = \underline{803\,psi}$

1-44 $\sigma = \frac{P}{A} = \frac{3500\,N}{(8.0\,mm)^2} = \underline{54.7\,MPa}$

1-45

$w = mg = 4200\,kg \cdot 9.81\,m/s^2 = 41.2\,kN$



$AB_x = AB \sin 35°$

$AB_y = AB \cos 35°$

$BC_x = BC \sin 55°$

$BC_y = BC \cos 55°$

$\Sigma F_x = 0 = AB_x - BC_x$

$0 = AB \sin 35° - BC \sin 55°$

$AB = BC \cdot \frac{\sin 55°}{\sin 35°} = 1.428\,BC$

$\Sigma F_V = 0 = AB_y + BC_y - 41.2\,kN = AB \cos 35° + BC \cos 55° - 41.2\,kN$

$0 = (1.428\,BC)\cos 35° + BC \cos 55° - 41.2\,kN$

$41.2\,kN = BC[1.170 + 0.574] = 1.743\,BC$

$BC = 41.2\,kN / 1.743 = 23.63\,kN$

$AB = 1.428\,BC = 33.75\,kN$

STRESS IN ROD AB: $\sigma_{AB} = \frac{AB}{A} = \frac{33.75 \times 10^3\,N}{\pi(20\,mm)^2/4} = \underline{107.4\,MPa}$

STRESS IN ROD BC: $\sigma_{BC} = \frac{BC}{A} = \frac{23.63 \times 10^3\,N}{\pi(20\,mm)^2/4} = \underline{75.2\,MPa}$

STRESS IN ROD BD: $\sigma_{BD} = \frac{BD}{A} = \frac{41.2 \times 10^3\,N}{\pi(20\,mm)^2/4} = \underline{131.1\,MPa}$

1-46 $F = 0.01097\ m R n^2 = (0.01097)(0.40)(0.60)(3000)^2\ N$

$F = 23695\ N$

$A = \pi(16\ mm)^2/4 = 201\ mm^2$

$\sigma = \frac{F}{A} = \frac{23695 N}{201\ mm^2} = \underline{118\ MPa}$

1-47 $A = (30\ mm)^2 = 900\ mm^2$

FOR AB: $F_{AB} = (110 - 40 + 80)\ kN = 150\ kN$

$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{150 \times 10^3 N}{900\ mm^2} = \underline{167\ MPa}$ TENSION

FOR BC: $F_{BC} = 110 - 40 = 70\ kN$

$\sigma_{BC} = \frac{F_{BC}}{A} = \frac{70 \times 10^3 N}{900\ mm^2} = \underline{77.8\ MPa}$ TENSION

FOR CD: $F_{CD} = 110\ kN$

$\sigma_{CD} = \frac{F_{CD}}{A} = \frac{110 \times 10^3 N}{900\ mm^2} = \underline{122\ MPa}$ TENSION

1-48 AREAS: A-C; $A_1 = \pi(25)^2/4 = 491\ mm^2$

C-D; $A_2 = \pi(16)^2/4 = 201\ mm^2$

FOR AB: $F_{AB} = -9.65 - 12.32 + 4.45 = -17.52\ kN$

$\sigma_{AB} = \frac{F_{AB}}{A_1} = \frac{-17.52 \times 10^3 N}{491\ mm^2} = \underline{-35.7\ MPa}$ COMPR.

FOR BC: $F_{BC} = -9.65 - 12.32 = -21.97\ kN$

$\sigma_{BC} = \frac{F_{BC}}{A_1} = \frac{-21.97 \times 10^3 N}{491\ mm^2} = \underline{-44.7\ MPa}$ COMPR.

FOR CD: $F_{CD} = -9.65\ kN$

$\sigma_{CD} = \frac{F_{CD}}{A_2} = \frac{-9.65 \times 10^3 N}{201\ mm^2} = \underline{-48.0\ MPa}$ COMPR.

1-49 $A = \pi[(1.90)^2 - (1.61)^2]/4 = 0.799\ in^2$ (1½ IN PIPE-APP. A-12)

FOR BC: $\sigma_{BC} = \frac{F_{BC}}{A} = \frac{2500\ LB}{0.799\ in^2} = \underline{3129\ PSI}$ TENSION

FOR AB: $F_{AB} = 2500 + 2(8000 \cos 30°) = 16356\ LB$

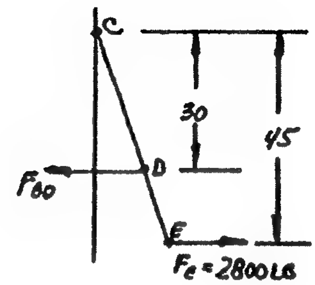
$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{16356\ LB}{0.799\ in^2} = \underline{20471\ PSI}$ TENSION

1-50

$\Sigma M_C = 0 = 2800(45) - F_{BD}(30)$

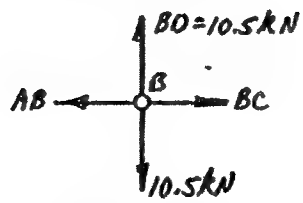
$F_{BD} = 4200$ LB

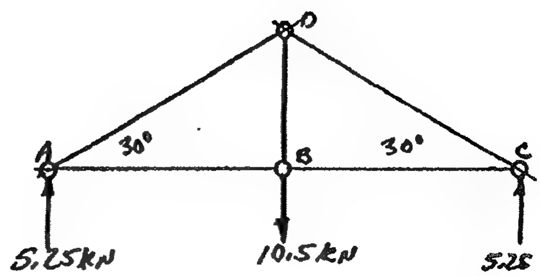
$\sigma_{BD} = \dfrac{F_{BD}}{A} = \dfrac{4200 \text{ LB}}{(2.0)(0.65)\text{ IN}^2} = \underline{3231 \text{ psi}}$ TENSION



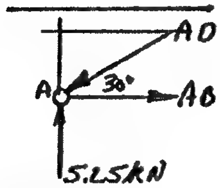
1-51

JOINT B





JOINT A



$AD \sin 30° = 5.25$ kN

$AD = 10.5$ kN $= CD$

$AB = AD \cos 30° = 9.09$ kN $= BC$

STRESSES:

AB, BC: $\sigma_{AB} = \sigma_{BC} = \dfrac{9.09 \times 10^3 \text{ N}}{(12)(30)\text{ mm}^2} = \underline{25.3 \text{ MPa}}$ TENSION

BD: $\sigma_{BD} = \dfrac{10.5 \times 10^3 \text{ N}}{(2)(10)(30)\text{ mm}^2} = \underline{17.5 \text{ MPa}}$ TENSION

AD, CD: $A = (30)^2 - (20)^2 = 500 \text{ mm}^2$

$\sigma_{AD} = \sigma_{CD} = \dfrac{-10.5 \times 10^3 \text{ N}}{500 \text{ mm}^2} = \underline{-21.0 \text{ MPa}}$ COMPRESSION

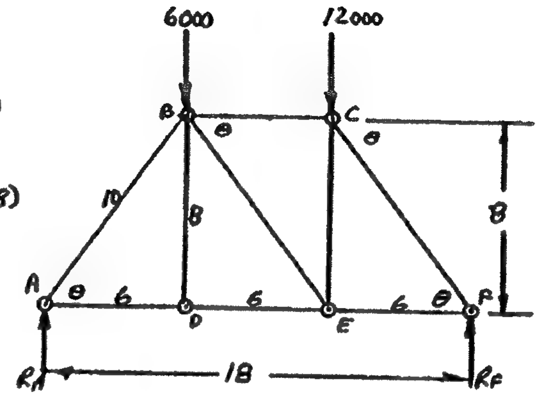
1-52

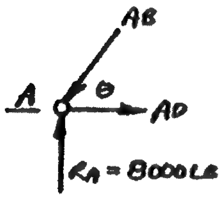
$\Sigma M_A = 0 = 6000(6) + 12000(12) - R_F(18)$

$R_F = 10000 \text{ LB}$

$\Sigma M_F = 0 = 12000(6) + 6000(12) - R_A(18)$

$R_A = 8000 \text{ LB}$

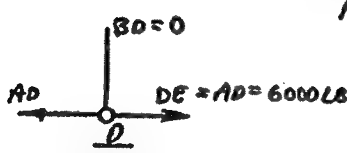


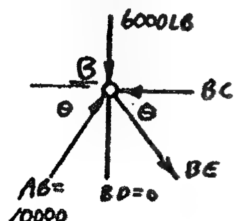


$R_A = AB \sin\theta = AB(0.8)$

$AB = R_A/0.8 = 8000/0.8 = 10000 \text{ LB COMP.}$

$AD = AB\cos\theta = 10000(0.6) = 6000 \text{ LB TENS.}$



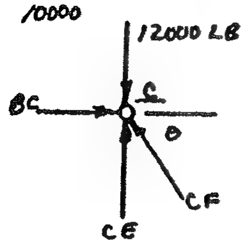


$BE\sin\theta + 6000 - AB\sin\theta = 0$

$BE = \dfrac{AB\sin\theta - 6000}{\sin\theta} = \dfrac{10000(0.8) - 6000}{0.8} = 2500 \text{ LB TENS.}$

$BC = AB\cos\theta + BE\cos\theta = 10000(0.6) + 2500(0.6) =$

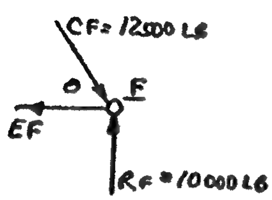
$BC = 7500 \text{ LB COMP.}$



$BC = CF\cos\theta$

$CF = BC/\cos\theta = 7500/0.6 = 12500 \text{ LB COMP.}$

$CE = 12000 - CF\sin\theta = 12000 - 12500(0.8) = 2000 \text{ LB } \underline{C}$



$EF = CF\cos\theta = 12500(0.6) = 7500 \text{ LB TENS.}$

AREAS OF MEMBERS: (APP. A5, A6)

AD, DE, EF – $2(0.484) = 0.968 \text{ IN}^2$

BD, BE, CE – 0.484 IN^2

AB, BC, CF – $2(1.21) = 2.42 \text{ IN}^2$

NOTE: COMPRESSION MEMBERS MUST BE CHECKED FOR COLUMN BUCKLING

STRESSES:

$\sigma_{AD} = \sigma_{DE} = 6000/0.968 = +6198 \text{ psi}$

$\sigma_{EF} = 7500/0.968 = +7748 \text{ psi}$

$\sigma_{BD} = 0$

$\sigma_{BE} = 2500/0.484 = +5165 \text{ psi}$

$\sigma_{CE} = -2000/0.484 = -4132 \text{ psi}$

$\sigma_{AB} = -10000/2.42 = -4132 \text{ psi}$

$\sigma_{BC} = -7500/2.42 = -3099 \text{ psi}$

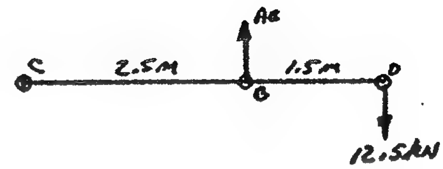
$\sigma_{CF} = -12500/2.42 = -5165 \text{ psi}$

1-53

$\Sigma M_C = 0 = (2.5)(4.0) - AB(2.5)$

$AB = 20\text{ kN}$

$\sigma = \dfrac{20\times10^3\text{ N}}{(20)^2\text{ mm}^2} = \underline{50\text{ MPa}}$



1-54 $A = \pi(0.505)^2/4 = 0.200\text{ in}^2$

$\sigma = F/A = 12600\text{ LB}/0.200\text{ IN}^2 = \underline{63000\text{ psi}}$

1-55 $A = (2.65)(1.40) + 2[(1.40)(0.5)(t)] = 4.41\text{ in}^2$

$\sigma = F/A = (52000\text{ LB}/4.41\text{ IN}^2) = \underline{11791\text{ psi}}$

1-56 $A = (80)(40) - (60)(15) + \pi(40)^2/4 = 3557\text{ mm}^2$

$\sigma = F/A = 640\times10^3\text{ N}/3557\text{ mm}^2 = \underline{180\text{ MPa}}$

1-57 DIRECT SHEAR - SINGLE SHEAR

$A_S = [\pi(12.0)^2/4]\text{ mm}^2 = 113\text{ mm}^2$

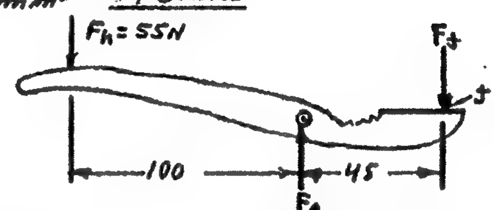
$\tau = F/A_s = 16.5\times10^3\text{ N}/113\text{ mm}^2 = \underline{146\text{ MPa}}$

1-58 $\Sigma F_t = 0 = 55(145) - F_p(45)$

$F_p = 177\text{ N}$

$A_S = \pi(3.0)^2/4 = 7.07\text{ mm}^2$

$\tau = F_P/A_s = 177\text{ N}/7.07\text{ mm}^2 = \underline{25.1\text{ MPa}}$



PIN IS IN SINGLE SHEAR

1-59 FROM PROB 1-46: $F = 23695\text{ N}$

$A_S = 2[\pi(10)^2/4] = 157\text{ mm}^2$ DOUBLE SHEAR

$\tau = F/A_s = 23695\text{ N}/157\text{ mm}^2 = \underline{151\text{ MPa}}$

1-60 $A_s = (3.0)(3.5) = 10.5\ \text{in}^2$

$\tau = F/A_s = (1800\ \text{lb})/(10.5\ \text{in}^2) = \underline{171\ \text{psi}}$

1-61 $A_s = [2(35) + \pi(8)](5.0) = 475.7\ \text{mm}^2$

$\tau = F/A_s = 38.6 \times 10^3\ \text{N} / 475.7\ \text{mm}^2 = \underline{81.1\ \text{MPa}}$

1-62 $L = \sqrt{.4^2 + .6^2} = 0.721\ \text{in.}$

$A_s = [2(1.60) + \pi(0.8)/2 + 2(0.721)]\,0.194$

$A_s = 1.144\ \text{in}^2$

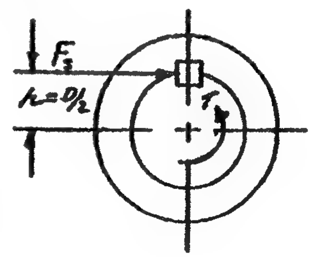
$\tau = F/A_s = 45000\ \text{lb}/1.144\ \text{in}^2 = \underline{39324\ \text{psi}}$

1-63 $T = F_s \cdot R$

$F_s = T/R = \dfrac{95\ \text{N}\cdot\text{m}}{35\ \text{mm}/2} \cdot \dfrac{10^3\ \text{mm}}{\text{m}} = 5429\ \text{N}$

$A_s = b \cdot L = (10)(22) = 220\ \text{mm}^2$

$\tau = F_s/A_s = 5429\ \text{N}/220\ \text{mm}^2 = \underline{24.7\ \text{MPa}}$



1-64 $F_s = T/R = 8000\ \text{lb}\cdot\text{in}/1.0\ \text{in} = 8000\ \text{lb}$

$A_s = b \cdot L = (0.50)(2.25) = 1.125\ \text{in}^2$

$\tau = F_s/A_s = 8000\ \text{lb}/1.125\ \text{in}^2 = \underline{7111\ \text{psi}}$

1-65 PIN DOUBLE SHEAR; $A_s = 2[\pi(0.5)^2/4] = 0.393\ \text{in}^2$

$\tau = F/A_s = 20\,000\ \text{lb}/0.393\ \text{in}^2 = \underline{50\,930\ \text{psi}}$

COLLAR SHEAR COLLAR FROM CONNECTOR BODY

$A_s = \pi d t = \pi(0.875)(0.1875) = 0.5154\ \text{in}^2$

$\tau = F/A_s = 20000\ \text{lb}/0.5154\ \text{in}^2 = \underline{38\,800\ \text{psi}}$

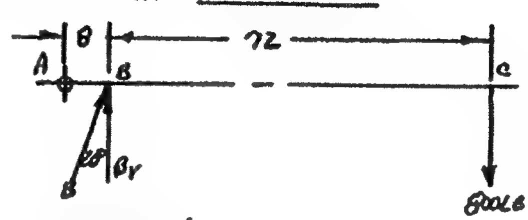
1-66 $\Sigma M_A = 0 = 800(80) - B_y(8)$

$B_y = 8000\ \text{lb}$

$B = B_y/\cos 20° = 8513\ \text{lb}$

$A_s = 2(\pi(0.375)^2/4) = 0.221\ \text{in}^2$

$\tau = B/A_s = 8513\ \text{lb}/0.221\ \text{in}^2 = \underline{38\,540\ \text{psi}}$



1-67 $A_s = (40)(12) = 480\ \text{mm}^2$

$\tau = F/A_s = 88 \times 10^3\ \text{N}/480\ \text{mm}^2 = \underline{183\ \text{MPa}}$

1-68 $A_s = (40)(120) = 4800\ \text{mm}^2$

$\tau = F/A_s = 88.2 \times 10^3\ \text{N} / 4800\ \text{mm}^2 = \underline{18.4\ \text{MPa}}$

1-69 $A_s = \pi D t = \pi(12)(8) = 301.6\ mm^2$

$\tau = F/A_s = 22.3 \times 10^3 N / 301.6\ mm^2 = \underline{73.9\ MPa}$

1-70 $A_s = 2[\pi(12)^2/4] = 226.2\ mm^2$ TWO RIVETS – SINGLE SHEAR

$\tau = F/A_s = 10.2 \times 10^3 N / 226.2\ mm^2 = \underline{45.1\ MPa}$

1-71 $A_s = 4[\pi(12)^2/4] = 452.4\ mm^2$ TWO RIVETS – DOUBLE SHEAR

$\tau = F/A_s = 10.2 \times 10^3 N / 452.4\ mm^2 = \underline{22.55\ MPa}$

CHAPTER 2 Design Properties of Materials

ONLY THOSE PROBLEMS REQUIRING NUMERICAL DATA ARE SHOWN.

2-14 S_u = 90ksi (621 MPa); S_y = 60 ksi (414 MPa); 25% ELONG.
BECAUSE % ELONGATION > 5%, IT IS DUCTILE. (APP. A-14)

2-15 1020 HR: 36 % ELONGATION – GREATER DUCTILITY
1040 HR: 25 % ELONGATION (APP. A-14)

2-16 AISI 1141 OQT 700: HIGH SULFUR ALLOY STEEL WITH 0.41% CARBON, QUENCHED IN OIL, TEMPERED AT 700°F. (APP. A-14)

2-17 YES. S_y = 172 ksi @ OQT 700, S_y = 129 ksi @ OQT 900
BY INTERPOLATION $S_y \approx 150$ ksi @ OQT 800. (APP. A-14)

2-18 $E = 30 \times 10^6$ psi (207 GPa) FOR ALL CARBON AND ALLOY STEELS. (APP. A-14)

2-19 WT = DENSITY × VOLUME = $(0.283\ LB/in^3)(1.0)(4.0)(14.5)in^3 = \underline{16.4\ LB}$
(APP. A-14) VALUE OF LB_m = VALUE OF LB FORCE (WT.)

2-20 VOLUME = AREA × LENGTH = $\frac{\pi}{4}(50)^2 \times 250 = 4.909 \times 10^5\ mm^3$
STEEL BAR

$$MASS = \frac{7680\ kg}{m^3} \times \frac{4.909 \times 10^5\ mm^3}{1} \times \frac{1\ m^3}{(10^3\ mm)^3} = 3.77\ kg$$

(APP. A-14)

$$WT = m \cdot g = 3.77\ kg \cdot 9.81\ m/s^2 = 36.98\ kg \cdot m/s^2 = \underline{36.98\ N}$$

2-21 MAGNESIUM WOULD BECAUSE IT HAS A LOWER E.
E_{MAG} = 45 GPa ; E_{Ti} = 114 GPa ; Ti IS STIFFER. (APP. A-15)

2-23 ALLOY OF ALUMINUM WITH SILICON AND MAGNESIUM.
HEAT TREATED TO T6 TEMPER.

2-24 (APP. A-18)

	S_u	S_y	E	DENSITY
6061-O	18 ksi	8 ksi	10×10^6 psi	0.10 LB/in³
6061-T4	35 ksi	21 ksi	"	"
6061-T6	45 ksi	40 ksi	"	"

2-29 S_{ut} = 40 ksi ; S_{uc} = 140 ksi (APP. A-17)

2-31 BENDING σ_d = 1450 psi ; TENSION σ_d = 850 psi ; COMP. 1000 psi PARALLEL TO GRAIN, 385 psi PERPENDICULAR TO GRAIN ; SHEAR τ_d = 95 psi (APP. A-19)

2-32 2000 TO 7000 psi (SECTION 2-10)

2-44 Graphite fibers.

2-45 S-glass, quartz fibers, tungsten fibers coated with silicon carbide.

2-51

Material	*Specific strength* (in)	*Ratio to AISI 1020*
Graphite/Epoxy (High Strength)	4.86×10^6	25.0
Aramid/Epoxy Composite	4.00×10^6	20.6
Boron/Epoxy Composite	3.60×10^6	18.5
Graphite/Epoxy (Ultra-hi mod)	2.76×10^6	14.2
Glass/Epoxy Composite	1.87×10^6	9.63
Titanium Ti-6Al-4V	1.00×10^6	5.15
AISI 5160 OQT 700 Steel	0.929×10^6	4.78
Aluminum 7075-T6	0.822×10^6	4.23
Aluminum 6061-T6	0.459×10^6	2.36
AISI 1020 HR Steel	0.194×10^6	1.00

2-52

Material	*Specific modulus* (in)	*Ratio to AISI 1020*
Graphite/Epoxy (Ultra-hi mod)	8.28×10^8	7.81
Boron/Epoxy Composite	4.00×10^8	3.77
Graphite/Epoxy (High Strength)	3.45×10^8	3.25
Aramid/Epoxy Composite	2.20×10^8	2.07
AISI 1020 HR Steel	1.06×10^8	1.00
AISI 5160 OQT 700 Steel	1.06×10^8	1.00
Titanium Ti-6Al-4V	1.03×10^8	0.97
Aluminum 6061-T6	1.02×10^8	0.96
Aluminum 7075-T6	0.99×10^8	0.93
Glass/Epoxy Composite	0.66×10^8	0.62

2-60 $V_m = 1 - V_f = 1.0 - 0.60 = 0.40$

2-61 See Equation (2-10).

2-62 See Equations (2-11), (2-12), (2-13), (2-14).

2-63 Given: $V_f = 0.50$; Fibers are high strength carbon-PAN; Matrix is Epoxy
See Table 2-15 for data. $V_m = 1 - V_f = 1.0 - 0.50 = 0.50$
Use Equation (2-10): $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail: $\varepsilon_f = s_{uf} / E_f = (820 \times 10^3 \text{ psi})/(40 \times 10^6 \text{ psi})$

$\varepsilon_f = 0.0205$

Stress in matrix at this strain: $\sigma_m' = E_m \varepsilon = (0.56 \times 10^6 \text{ psi})(0.0205) = 11\,480 \text{ psi}$
Then: $s_{uc} = (820 \times 10^3 \text{ psi})(0.50) + (11\,480 \text{ psi})(0.50) = \underline{415 \times 10^3 \text{ psi}}$
Modulus of elasticity: $E_c = E_f V_f + E_m V_m = (40 \times 10^6)(0.5) + (0.56 \times 10^6)(0.50)$
$\underline{E_c = 20.3 \times 10^6 \text{ psi}}$
Specific weight: $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.065)(0.50) + (0.047)(0.50)$
$\underline{\gamma_c = 0.056 \text{ lb/in}^3}$

2-64 Given: $V_f = 0.50$; Fibers are high modulus carbon; Matrix is Epoxy
See Table 2-15 for data. $V_m = 1 - V_f = 1.0 - 0.50 = 0.50$
Use Equation (2-10): $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail: $\varepsilon_f = s_{uf} / E_f = (325 \times 10^3 \text{ psi})/(100 \times 10^6 \text{ psi})$

$\varepsilon_f = 0.00325$

Stress in matrix at this strain: $\sigma_m' = E_m \varepsilon = (0.56 \times 10^6 \text{ psi})(0.00325) = 1820 \text{ psi}$
Then: $s_{uc} = (325 \times 10^3 \text{ psi})(0.50) + (1820 \text{ psi})(0.50) = \underline{163 \times 10^3 \text{ psi}}$
Modulus of elasticity: $E_c = E_f V_f + E_m V_m = (100 \times 10^6)(0.5) + (0.56 \times 10^6)(0.50)$
$\underline{E_c = 50.3 \times 10^6 \text{ psi}}$
Specific weight: $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.078)(0.50) + (0.047)(0.50)$
$\underline{\gamma_c = 0.0625 \text{ lb/in}^3}$

2-65 Given: $V_f = 0.50$; Fibers are aramid; Matrix is Epoxy
See Table 2-15 for data. $V_m = 1 - V_f = 1.0 - 0.50 = 0.50$
Use Equation (2-10): $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail: $\varepsilon_f = s_{uf} / E_f = (500 \times 10^3 \text{ psi})/(19 \times 10^6 \text{ psi})$

$\varepsilon_f = 0.0263$

Stress in matrix at this strain: $\sigma_m' = E_m \varepsilon = (0.56 \times 10^6 \text{ psi})(0.0263) = 14\,740 \text{ psi}$
Then: $s_{uc} = (500 \times 10^3 \text{ psi})(0.50) + (14\,740 \text{ psi})(0.50) = \underline{257 \times 10^3 \text{ psi}}$
Modulus of elasticity: $E_c = E_f V_f + E_m V_m = (19 \times 10^6)(0.5) + (0.56 \times 10^6)(0.50)$
$\underline{E_c = 9.78 \times 10^6 \text{ psi}}$
Specific weight: $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.052)(0.50) + (0.047)(0.50)$
$\underline{\gamma_c = 0.0495 \text{ lb/in}^3}$

Solutions to Problems 2-66 to 2-67: Some data approximated from Figure P2-66.
Most accurate values are for Ultimate strength (b.)and % elongation (f).
Elastic limit (d.) estimated between proportional limit (c.) and yield strength (a.)
Modulus of elasticity (e.) computed from (Δ stress / Δ strain). Data are approximated
Materials found from Appendixes A-13 through A-17 matching s_u, s_y, % Elongation, and E

2-66
a. $s_y = 73$ ksi - Offset
b. $s_u = 83$ ksi
c. $s_p = 60$ ksi
d. $s_{el} = 67$ ksi
e. $E = 10.0 \times 10^6$ psi
f. 11% Elongation
g. Ductile
h. Aluminum
I. 7075-T6

2-67
a. $s_y = 173$ ksi Yield point
b. $s_u = 187$ ksi
c. $s_p = 162$ ksi
d. $s_{el} = 168$ ksi
e. $E = 29.0 \times 10^6$ psi
f. 15% Elongation
g. Ductile
h. Steel
I. AISI 4140 OQT 900

2-68
a. $s_y = 62$ ksi Offset
b. $s_u = 75$ ksi
c. $s_p = 50$ ksi
d. $s_{el} = 56$ ksi
e. $E = 16.7 \times 10^6$ psi
f. 15% Elongation
g. Ductile
h. Copper Alloy
I. C54400 Bronze-hard

2-69
a. $s_y = 49$ ksi - Yield point
b. $s_u = 65$ ksi
c. $s_p = 46$ ksi
d. $s_{el} = 48$ ksi
e. $E = 26.5 \times 10^6$ psi
f. 36% Elongation
g. Ductile
h. Steel
I. AISI 1020 CD

2-70
a. No s_y - Brittle
b. $s_u = 55$ ksi
c. $s_p = 50$ ksi
d. $s_{el} = 53$ ksi
e. $E = 20.0 \times 10^6$ psi
f. 0.5% Elongation
g. Brittle
h. Cast Iron
I. ASTM A48 Grade 60

2-71
a. $s_y = 53$ ksi - Offset
b. $s_u = 59$ ksi
c. $s_p = 31$ ksi
d. $s_{el} = 42$ ksi
e. $E = 12.0 \times 10^6$ psi
f. 5.0% Elongation
g. Borderline Brittle/Ductile
h. Zinc
I. Cast ZA-12

2-72
a. $s_y = 35$ ksi - Yield point
b. $s_u = 57$ ksi
c. $s_p = 30$ ksi
d. $s_{el} = 27$ ksi
e. $E = 26 \times 10^6$ psi
f. 21% Elongation
g. Ductile
h. Structural Steel
I. ASTM A36

2-73
a. $s_y = 19$ ksi - Offset
b. $s_u = 40$ ksi
c. $s_p = 14$ ksi
d. $s_{el} = 17$ ksi
e. $E = 6 \times 10^6$ psi
f. 5% Elongation
g. Borderline Brittle/Ductile
h. Magnesium
I. ASTM AZ 63A-T6

2-74
a. s_y = 155 ksi - Offset
b. s_u = 170 ksi
c. s_p = 142 ksi
d. s_{el} = 149 ksi
e. $E = 16.5 \times 10^6$ psi
f. 8% Elongation
g. Ductile
h. Titanium
I. 6Al-4V

2-75
a. s_y = 40 ksi - Offset
b. s_u = 45 ksi
c. s_p = 30 ksi
d. s_{el} = 35 ksi
e. $E = 10.0 \times 10^6$ psi
f. 17% Elongation
g. Ductile
h. Aluminum
I. 6061-T6

2-76
a. s_y = 80 ksi - Offset
b. s_u = 90 ksi
c. s_p = 62 ksi
d. s_{el} = 71 ksi
e. $E = 26 \times 10^6$ psi
f. 15% Elongation
g. Ductile
h. Stainless Steel
I. AISI 430 full hard

2-77
a. s_y = 80 ksi - Offset
b. s_u = 95 ksi
c. s_p = 55 ksi
d. s_{el} = 68 ksi
e. $E = 26 \times 10^6$ psi
f. 2.0% Elongation
g. Brittle, but does yield
h. Malleable Iron
I. ASTM A220 Grade 80002

3-1 $\sigma = P/A = \frac{8.50\times10^3 N}{\pi(10mm)^2/4} = 108\ MPa = \sigma_d = S_y/2$

REQ'D $S_y = 2\sigma_d = 2(108\ MPa) = \underline{216\ MPa}$

ALUMINUM $\underline{2014\text{-}T4}$ HAS $S_y = 290\ MPa$ (APPENDIX A-18)

3-2 $\sigma = P/A = \frac{20.0\times10^3 N}{(10)(30)\,mm^2} = 66.7\ MPa = \sigma_d = S_u/8$

REQD $\underline{S_u} = 8\sigma_d = 8(66.7\ MPa) = \underline{533\ MPa}$ PLUS GOOD DUCTILITY

$\underline{\text{AISI 1141 ANNEALED}}$ HAS $S_u = 600\ MPa$; 26% ELONGATION. (A-14)

3-3 $\sigma = P/A = 1720\ LB/(0.40\ IN)^2 = 10750\ PSI = \sigma_d = S_u/8$

REQD $\underline{S_u} = 8\sigma_d = 8(10750) = \underline{86000\ PSI}$ PLUS GOOD DUCTILITY

$\underline{\text{AISI 1040 WQT 1300}}$ HAS $S_u = 87\ ksi$; 32% ELONGATION (A-14)

3-4 $\sigma = P/A = \frac{1850\ LB}{\pi(0.375\ IN)^2/4} = 16750\ psi = \sigma_d = 0.60\,S_y$ (AISC)

REQD $\underline{S_y} = \sigma_d/0.60 = 16750\ psi/0.60 = \underline{27900\ psi}$

$\underline{\text{ASTM A36}}$ STRUCTURAL STEEL HAS $S_y = 36000\ PSI$ (A-16)

3-5 $\sigma = P/A = \frac{5200\ LB}{5.25\ IN^2} = 990\ psi$ $\underline{\text{TOO HIGH}}$; $\sigma_{ALLOW} = 575\ psi$ (A-19)

(A-4)

3-6 a) NO. 1 GRADE DOUGLAS FIR HAS $\sigma_{ALLOW} = 1050\ PSI$ (A-19)

b) TO USE NO. 2 GRADE SOUTHERN PINE: $\sigma_{ALLOW} = 575\ psi$

REQD AREA $= \frac{P}{\sigma} = \frac{5200\ LB}{575\ LB/IN^2} = 9.04\ IN^2$; USE 2X8 OR 4X4 OR $\underline{\text{TWO}}$ 2X4.

3-7 $\sigma = P/A$; $A = \frac{P}{\sigma} = \frac{6400\ LB}{12000\ LB/IN^2} = 0.533\ IN^2 = \pi D^2/4$

REQD $D = \sqrt{4A/\pi} = \sqrt{4(0.533\ IN^2)/\pi} = \underline{0.824\ IN}$

SPECIFY 7/8 IN (0.875 IN) OR 1.00 IN. DIA.

3-8 TOTAL MASS = 1150 + 6350 = 7500 kg; 1875 kg ON EACH STRAP.

$F = m\cdot g = 1875\ kg \cdot 9.81\ m/s^2 = 18394\ N$

REQD $A = \frac{P}{\sigma_d} = \frac{18394\ N}{70\ N/mm^2} = 263\ mm^2 = (w)(8\ mm)$

REQD $\underline{w} = 263\ mm^2/8\ mm = \underline{32.8\ mm}$

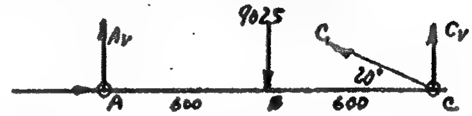
SPECIFY $w = 35.0\ mm$

3-9 FORCE ON SHELF $= 1840\,kg \cdot 9.81\,m/s^2 = 18050\,N$: 9025 N/SIDE

$C_V = A_y = 9025\,N/2 = 4513\,N$

$C = C_V/\sin 20^\circ = 13\,194\,N$

C = FORCE IN ROD



REQD $A = \dfrac{C}{\sigma_d} = \dfrac{13194\,N}{110\,N/mm^2} = 120\,mm^2 = \pi D^2/4$

REQD $D = \sqrt{4A/\pi} = \sqrt{4(120\,mm^2)/\pi} = \underline{12.4\,mm}$

SPECIFY $D = 14.0\,mm$

3-10 $\sigma = \dfrac{P}{A} = \dfrac{70\,000\,LB}{\pi(8.0\,IN)^2/4} = 1393\,PSI = \dfrac{1}{4} \times$ RATED STRENGTH (SEC. 2-10)

REQ'D RATED STRENGTH $= 4(1393) = 5570\,PSI$

SPECIFY 6000 PSI RATED STRENGTH

3-11 LOAD ON EACH BLOCK $= 29\,500\,LB/3 = 9833\,LB$

$\sigma = \dfrac{P}{A} = \dfrac{9833\,LB}{12.25\,IN^2} = 803\,PSI$

IF COMPRESSION IS PERPENDICULAR TO GRAIN, NO SUITABLE WOOD LISTED.

IF PARALLEL TO GRAIN: NO. 1 SOUTH. PINE $- \sigma_{ALL.} = 850\,PSI$ (A-19)

NO. 2 HEMLOCK $- \sigma_{ALL.} = 800\,PSI$

NO. 2 DOUGLAS FIR $- \sigma_{ALL} = 1000\,PSI$

3-12 $\sigma_{ALLOW.} = \dfrac{\text{RATED STRENGTH}}{4} = \dfrac{20.7\,MPa}{4} = 5.18\,MPa$ (SEC. 2-10)

REQD $A = \dfrac{P}{\sigma} = \dfrac{1.50 \times 10^6\,N}{5.18\,N/mm^2} = 2.90 \times 10^5\,mm^2 = \pi D^2/4$

REQD $D = \sqrt{4A/\pi} = 607\,mm$; SPECIFY 700 mm DIA.

3-13 $S_u = 483\,MPa = \sigma = P/A$; $P = \sigma \cdot A$ (A-18)

$A = \pi(12^2 - 10^2)/4 = 34.56\,mm^2$

$P = \sigma \cdot A = 483\,\dfrac{N}{mm^2} \cdot 34.56\,mm^2 = 16.7 \times 10^3\,N = \underline{16.7\,kN}$

3-14 $A = (40\,mm)^2 = 1600\,mm^2$; $\sigma_{ALLOW.} = 1.69\,MPa \perp$ GRAIN (A-19)

$\sigma_{ALLOW.} = 5.52\,MPa \parallel$ GRAIN (#2 HEMLOCK)

$P = \sigma \cdot A = (1.69\,N/mm^2)(1600\,mm^2) = \underline{2.70\,kN - \perp \text{GRAIN}}$

$P = \sigma \cdot A = (5.52\,N/mm^2)(1600\,mm^2) = \underline{8.83\,kN - \parallel \text{GRAIN}}$

3-15 $\sigma_d = 0.60\,S_y = 0.60(50\,ksi) = 30.0\,ksi = P/A$ (AISC) (A-16) $S_y = 50\,ksi$ IF $D < 0.75\,IN$

REQD $A = P/\sigma_d = \dfrac{4000\,LB}{30\,000\,LB/IN^2} = 0.133\,IN^2 = \pi D^2/4$

REQD $D = \sqrt{4A/\pi} = \sqrt{4(0.133\,IN^2)/\pi} = 0.412\,IN$

SPECIFY $\underline{D = 7/16\,IN.\ (0.4375\,IN.)}$ OR $\underline{0.500\,IN.}$ (OK - $D < 0.75\,IN.$)

3-16 $A = (2.65)(1.40) + 2[\frac{1}{2}(1.40)(0.5)] = 4.41\ IN^2$

$\sigma = P/A = \dfrac{52\,000\ LB}{4.41\ IN^2} = \underline{11\,791\ PSI} = \sigma_d = S_u/N$

$N = \dfrac{S_u}{\sigma} = \dfrac{80\,000\ PSI}{11\,791\ PSI} = \underline{6.78}$

3-17 FOR SHOCK LOADING - DUCTILE METALS $\sigma_d = S_u/12$

$\sigma_d = 1650\ MPa/12 = 137.5\ MPa$ (A-17)

REQD $A = \dfrac{P}{\sigma_d} = \dfrac{135 \times 10^3\ N}{137.5\ N/mm^2} = 982\ mm^2 = BH = B(2B) = 2B^2$

REQD $B = \sqrt{A/2} = \sqrt{982/2} = 22.2\ mm$; $H = 44.4\ mm$

SPECIFY $\underline{B = 25.0\ mm}$; $\underline{H = 50.0\ mm}$

3-18 $\sigma_d = S_u/8 = 8\,000\ PSI/8 = 1000\ PSI = P/A$ (A-20)

REQ'D $A = \dfrac{P}{\sigma_d} = \dfrac{110\ LB}{1000\ LB/IN^2} = 0.110\ IN^2 = (0.20)(w)$

REQ'D $w = A/0.20 = 0.110\ IN^2/0.20\ IN = 0.550\ IN$

SPECIFY $\underline{w = 0.600\ IN}$

3-19 $A = (80)(40) - (60)(15) + \pi(40)^2/4 = 3557\ mm^2$

$\sigma = \dfrac{P}{A} = \dfrac{640 \times 10^3\ N}{3557\ mm^2} = 180\ MPa = \sigma_d$

FOR DUCTILE METALS: $\sigma_d = S_y/2$

REQ'D. $S_y = 2(180) = 360\ MPa$

POSSIBLE METALS: AISI 1040 HR, $S_y = 414\ MPa$ (A-14)
AISI 4140 ANNEALED, $S_y = 414\ MPa$ (A-14)
C54400 BRONZE, $S_y = 393\ MPa$ (A-15)
ALUMINUM 2014-T6, $S_y = 414\ MPa$ (A-18)
AISI 1020 CD, $S = 441\ MPa$ (A-14)

3-20 SEE PROB. 1-48: $\sigma_{MAX} = -48.0\ MPa$ COMPRESSION

$\sigma_d = 0.6\ S_y = 0.6(248\ MPa) = 149\ MPa$ (AISC) (A-16)

<u>OK FOR COMPRESSIVE STRESS</u>.

COLUMN BUCKLING SHOULD BE CHECKED.

3-21 $\sigma = 50\ MPa$ IN MEMBER AB - <u>SEE PROB 1-53.</u>

$\sigma_d = S_u/8$; REQ'D $S_u = 8\sigma = 8(50) = 400\ MPa$

ALUMINUM 2014-T4, $S_u = 427\ MPa$; 20 % ELONG. (A-18)

3-22 $A = 0.944\ IN^2$; $\sigma_d = 0.60\ S_y = 0.60(36000\ PSI) = 21\,600\ PSI$

$P_{ALLOW.} = \sigma_d \cdot A = (21\,600\ LB/IN^2)(0.944\ IN^2) = \underline{20\,390\ LB}$

3-23 $\sigma = 800$ PSI; $E = 1.40\times10^6$ PSI; $L = 6.0$ FT $\times$ 12 IN/FT $= 72$ IN.

$$\delta = \frac{\sigma L}{E} = \frac{800\,\text{PSI}\,(72\,\text{IN})}{1.40\times10^6\,\text{PSI}} = \underline{0.041\ \text{IN}}$$

3-24 $A = (0.75)(12) = 9\ \text{mm}^2$; $\sigma = F/A = 90\,\text{N}/9.0\,\text{mm}^2 = 10.0\ \text{MPa}$

a) ABS; $S_u = 41$ MPa (OK) $\quad E = 2500\ \text{N/mm}^2$

$$\delta = \frac{FL}{EA} = \frac{(90\,\text{N})(375\,\text{mm})}{(2400\,\text{N/mm}^2)(9\,\text{mm}^2)} = \underline{1.57\ \text{mm}}$$

b) PHENOLIC; $S_u = 45$ MPa (OK); $E = 7580$ MPa $= 7580\ \text{N/mm}^2$

$$\delta = \frac{FL}{EA} = \frac{(90)(375)}{(7580)(9)} = \underline{0.495\ \text{mm}}$$

3-25 $D_o = 2.50$ IN; $D_i = 2.50 - 2(0.085) = 2.33$ IN; $E = 10.6\times10^6$ PSI

$A = \pi(D_o^2 - D_i^2)/4 = 0.645\ \text{IN}^2$

$$\delta = \frac{FL}{EA};\quad F = \frac{\delta EA}{L} = \frac{(0.005\,\text{IN})(10.6\times10^6\,\text{PSI})(0.645\,\text{IN}^2)}{14.5\,\text{IN}} = \underline{2357\ \text{LB}}$$

$$\sigma = \frac{F}{A} = \frac{2357\,\text{LB}}{0.645\,\text{IN}^2} = \underline{3655\ \text{PSI}}\quad \text{OK};\ S_y = 42000\ \text{PSI}$$

3-26 1) WEIGH IT. DENSITY OF ALUMINUM – 0.10 LB/IN³ (WT. DENSITY)
DENSITY OF MAGNESIUM – 0.066 LB/IN³

2) EXERT A KNOWN LOAD AND MEASURE DEFORMATION.
E FOR ALUMINUM – 10×10^6 PSI
E FOR MAGNESIUM – 6.5×10^6 PSI

3-27 a) AISI 1020 HR STEEL: STRENGTH: $\sigma_d = \frac{S_u}{8} = \frac{448}{8} = 56.0$ MPa

REQ'D. $A = \frac{F}{\sigma_d} = \frac{3500\,\text{N}}{56.0\,\text{N/mm}^2} = 62.5\ \text{mm}^2 = \pi D^2/4$

REQ'D. $D = \sqrt{4A/\pi} = \sqrt{4(62.5)/\pi} = 8.92$ mm

DEFORMATION: $E = 207$ GPa $= 207000$ MPa $= 207000\ \text{N/mm}^2$

REQ'D. $A = \frac{FL}{E\delta} = \frac{(3500\,\text{N})(630\,\text{mm})}{(207000\,\text{N/mm}^2)(0.12\,\text{mm})} = 88.8\ \text{mm}^2 = \frac{\pi D^2}{4}$

REQ'D $D = \sqrt{4A/\pi} = \underline{10.63\ \text{mm}}$ GOVERNS

b) AISI 4140 OQT 700 STEEL IS MUCH STRONGER THAN AISI 1020 HR.
THEREFORE: DEFORMATION GOVERNS. E IS SAME.
REQ'D. $D = 10.63$ mm

c) ALUM. 6061-T6: STRENGTH: $\sigma_d = \frac{S_u}{8} = \frac{310\,\text{MPa}}{8} = 38.75$ MPa

REQ'D. $A = F/\sigma_d = 3500\,\text{N}/38.75\,\text{N/mm}^2 = 90.32\ \text{mm}^2$

REQ'D. $D = \sqrt{4A/\pi} = 10.7$ mm

DEFORMATION: $E = 69$ GPa $= 69000$ MPa $= 69000\ \text{N/mm}^2$

REQ'D. $A = \frac{FL}{E\delta} = \frac{(3500)(630)}{(69000)(0.12)} = 266.3\ \text{mm}^2$ GOVERNS

REQ'D $D = \sqrt{4A/\pi} = \underline{18.4\ \text{mm}}$ (CONT. NEXT PAGE)

3-27 (CONTINUED) MASS = VOL × DENSITY = A × L × DENSITY

STEEL: $MASS = (88.8\,mm^2)(630\,mm)(7680\,kg/m^3) \times \frac{1\,m^3}{(10^3\,mm)^3} = \underline{0.430\,kg}$

ALUM: $MASS = (266.3)(630)(2770)/10^9 = \underline{0.465\,kg}$

3-28 $A = \pi D^2/4 = \pi(12)^2/4 = 113.1\,mm^2$; $\sigma = F/A = 17000\,N/113.1\,mm^2 = 150\,MPa$ LOW - OK FOR STEEL

$\delta = \frac{FL}{EA} = \frac{(17.0\times10^3 N)(220\,mm)}{(207000\,N/mm^2)(113.1\,mm^2)} = \underline{0.160\,mm}$

3-29 a) Ti-6AL-4V: $E = 114\,GPa = 114000\,MPa = 114000\,N/mm^2$, $S_y = 1070\,MPa$

$\delta = \frac{FL}{EA} = \frac{(5000\,N)(1250\,mm)}{(114000\,N/mm^2)(64\,mm^2)} = \underline{0.857\,mm}$; $\sigma = \frac{F}{A} = 78.1\,MPa$ LOW-OK

b) AISI 501 OQT 1000 STEEL: $E = 200\,GPa = 200000\,N/mm^2$, $S_y = 931\,MPa$

$\delta = \frac{FL}{EA} = \frac{(5000)(1250)}{(200000)(64)} = \underline{0.488\,mm}$

3-30 REQ'D. $A = \frac{F}{\sigma_d} = \frac{35000\,LB}{21600\,LB/IN^2} = 1.620\,IN^2$: USE $\underline{L4 \times 4 \times 1/4}$; $A = 1.94\,IN^2$

$\delta = \frac{FL}{AE} = \frac{(35000\,LB)(13.0\,FT)(12\,IN/FT)}{(1.94\,IN^2)(29\times10^6\,LB/IN^2)} = \underline{0.097\,IN}$

3-31 ELONGATION: $\delta = \frac{FL}{EA} = \frac{(450\,LB)(8.40\,IN)}{(30\times10^6\,LB/IN^2)(.25)(.125)\,IN^2} = \underline{0.0040\,IN}$

COMPRESSION: $\delta = \frac{FL}{EA} = \frac{(50)(8.40)}{(30\times10^6)(.25)(.125)} = \underline{0.00045\,IN}$

STRESS $\sigma_t = F_t/A = (450)/0.03125 = 14400\,PSI < S_y$ FOR ANY STEEL

REPEATED LOAD: $\sigma_d = S_u/8$; $S_{u_{MIN}} = 8(14.4\,KSI) = 115\,KSI$; AISI 1141 OQT 1100, $S_u = 116\,KSI$

3-32 LOWER SECTION: $\delta_1 = \frac{F_1 L_1}{EA} = \frac{(5000)(10)}{(30\times10^6)(.50)} = 0.0033\,IN$

MID SECTION: $\delta_2 = \frac{F_2 L_2}{EA} = \frac{(7000)(15)}{(30\times10^6)(.50)} = 0.0070\,IN$

TOP SECTION: $\delta_3 = \frac{F_3 L_3}{EA} = \frac{(10500)(25)}{(30\times10^6)(0.50)} = 0.0175\,IN$

TOTAL: $\delta_T = \delta_1 + \delta_2 + \delta_3 = \underline{0.0278\,IN}$

MAX. STRESS IN TOP SECTION
ASSUME STATIC LOAD
$\sigma = \frac{F}{A} = \frac{10500\,LB}{0.50\,IN^2} = 21000\,PSI$
$\sigma_d = S_y/2$; $S_{y_{MIN}} = 2(21\,KSI) = 42\,KSI$
OK FOR ANY STEEL IN APPENDIX A-14.

3-33 $A = \pi(D_o^2 - D_i^2)/4 = \pi(1.25^2 + 1.126^2)/4 = 0.2314\,IN^2$

$F = \frac{\delta EA}{L} = \frac{(0.050\,IN)(10\times10^6\,LB/IN^2)(0.2314\,IN^2)}{36.0\,IN} = 3214\,LB$

$\sigma = F/A = 3214\,LB/0.2314\,IN^2 = \underline{13900\,PSI}$ $\underline{\text{TOO HIGH}}$

$\sigma_d = \frac{S_u}{8} = \frac{45000\,PSI}{8} = 5625\,PSI$

3-34 $A = \pi(D_o^2 - D_i^2)/4 = \pi(0.75^2 - 0.563^2)/4 = 0.1928\,IN^2$

$\sigma = F/A = 2500\,LB/0.1928\,IN^2 = 12964\,PSI = \sigma_d = S_y/2$; $S_y = 25930\,PSI$

ALUMINUM 3003-H18 HAS $S_y = 27000\,PSI$

$\delta = \frac{FL}{EA} = \frac{(2500\,LB)(8.75\,FT)(12\,IN/FT)}{(10\times10^6\,LB/IN^2)(0.1928\,IN^2)} = \underline{0.136\,IN}$

3-35 $A = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}(56^2 - 48^2) = 653\ mm^2$

$$\delta = \frac{FL}{EA} = \frac{(18.2\times10^3 N)(40\ mm)}{(69\,000\ N/mm^2)(653\ mm^2)} = \underline{0.016\ mm}$$

$$\sigma = \frac{F}{A} = \frac{18.2\times10^3 N}{653\ mm^2} = \underline{27.9\ MPa}$$

3-36 $A = \pi D^2/4 = \pi(0.375)^2/4 = 0.1104\ IN^2$

$$\sigma = \frac{F}{A} = \frac{1600\ LB}{0.1104\ IN^2} = \underline{14\,500\ PSI}$$

$$\delta = \frac{FL}{EA} = \frac{(1600\ LB)(135\ FT)(12\ IN/FT)}{(30\times10^6 LB/IN^2)(0.1104\ IN^2)} = \underline{0.782\ IN}$$

3-37 $A = (30\ mm)^2 = 900\ mm^2$; $E = 114\ GPa = 114\,000\ N/mm^2$

$F_{AB} = 110 + 80 - 40 = 150\ kN$ TENSION

$F_{BC} = 110 - 40 = 70\ kN$ TENSION

$F_{CD} = 110\ kN$ TENSION

$\sigma_{MAX} = \frac{150\,000\ N}{900\ mm^2} = 167\ MPa$

$s_y = 1070\ MPa$ - OK

$$\delta_T = \delta_{AB} + \delta_{BC} + \delta_{CD} = \frac{F_{AB}L}{EA} + \frac{F_{BC}L}{EA} + \frac{F_{CD}L}{EA}$$

$$\delta_T = \frac{(150\times10^3 N)(250\ mm)}{(114\,000\ N/mm^2)(900\ mm^2)} + \frac{(70\times10^3)(250)}{(114\,000)(900)} + \frac{(110\times10^3)(250)}{(114\,000)(900)}$$

$$\delta_T = 0.365\ mm + 0.171\ mm + 0.268\ mm = \underline{0.804\ mm}$$

3-38 $$E = \frac{FL}{\delta A} = \frac{(10\,000\ LB)(10.0\ IN)}{(0.023\ IN)(\pi(0.75)^2/4)\ IN^2} = \underline{9.84\times10^6\ PSI}$$

PROBABLY ALUMINUM: $E \cong 10.0\times10^6$ PSI

$\sigma = F/A = 10000\ LB/0.4418\ IN^2 = 22\,635\ PSI$ SEE APP. A-18

$\sigma < s_y$ FOR MOST TEMPERS OF MOST ALLOYS. ENSURE NO YIELDING OCCURED.

3-39 $A_{AB} = A_{BC} = \frac{\pi(25)^2}{4} = 491\ mm^2$; $A_{CD} = \frac{\pi(16)^2}{4} = 201\ mm^2$

$F_{AB} = -9.65 - 12.32 + 4.45 = -17.52\ kN$ (COMP.); $L_{AB} = 80\ mm$

$F_{BC} = -9.65 - 12.32 = -21.97\ kN$ (COMP.); $L_{BC} = 100\ mm$

$F_{CD} = -9.65\ kN$ (COMP.) ; $L_{CD} = 120\ mm$

$E = 2960\ N/mm^2$

$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \underline{44.7\ MPa}$

$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \underline{48.0\ MPa}$ $\underline{MAX}$

$s_u = 72.0\ MPa$

$N = \frac{s_u}{\sigma_{CD}} = \frac{72}{48} = 1.50$

OK FOR STATIC LOAD

$$\delta_{AD} = \frac{F_{AB}L_{AB}}{EA_{AB}} + \frac{F_{BC}L_{BC}}{EA_{BC}} + \frac{F_{CD}L_{CD}}{EA_{CD}}$$

$$\delta_{AD} = \frac{(-17.52\times10^3 N)(80\ mm)}{(2960\ N/mm^2)(491\ mm^2)} + \frac{(-21.97\times10^3)(100)}{(2960)(491)} + \frac{(-9.65\times10^3)(120)}{(2960)(201)}$$

$$\delta_{AD} = -0.9644\ mm - 1.512\ mm - 1.946\ mm = \underline{\underline{-4.42\ mm\ \text{SHORTER}}}$$

3-40 $\sigma_c = F/A_c = 64000\,LB/50.27\,IN^2 = 1273\,PSI$; LET $\sigma_a > 4\sigma_c = 5092\,PSI$

USE $f_c' = 6000\,PSI$ RATED: THEN $E = 4.7 \times 10^6\,PSI$ (SEC 2-10)

$A_c = \pi(8)^2/4 = 50.27\,IN^2$; $A_T = 6.02\,IN^2$ (APP. A-9); $E_T = 29 \times 10^6\,psi$ STRUCTURAL STEEL

$$\delta = \delta_c + \delta_T = \frac{(64000\,LB)(3.0\,FT)(12\,IN/FT)}{(4.7\times10^6\,LB/IN^2)(50.27\,IN^2)} + \frac{(64000)(8.6)(12)}{(29\times10^6)(6.02)}$$

$\delta = 0.0098\,IN + 0.0378\,IN = \underline{0.0476\,IN}$; $\sigma_T = \frac{64000}{6.02} = 10631\,PSI < S_y$ FOR ASTM A36

3-41 a) $\delta = \frac{FL}{EA} = \frac{(120\,LB)(10.5\,FT)(12\,IN/FT)}{(17\times10^6\,LB/IN^2)(0.00322\,IN^2)} = \underline{0.276\,IN.}$

$A = \pi D^2/4 = \pi(0.064)^2/4 = 0.00322\,IN^2$

$\sigma = F/A = 120\,LB/0.00322\,IN^2 = 37300\,PSI$; CLOSE TO $S_y = 44000\,PSI$

b) $\sigma = \frac{F}{A} = 200\,LB/0.00322\,IN^2 = 62200\,PSI$: THIS EXCEEDS THE ULTIMATE STRENGTH OF THE WIRE. IT SHOULD BREAK.

3-42 $\sigma = F/A = 25\,LB/(0.006)(0.75)\,IN^2 = \underline{5556\,PSI}$

$$\delta = \frac{FL}{EA} = \frac{(25\,LB)(25\,FT)(12\,IN/FT)}{(30\times10^6\,LB/IN^2)(0.0045\,IN^2)} = \underline{0.0556\,IN}$$

3-43 $F = \sigma_{allow} \cdot A = (550\,LB/IN^2)(12.25\,IN^2) = \underline{6737\,LB}$

$$\delta = \frac{-FL}{EA} = \frac{-(6737\,LB)(10.75\,FT)(12\,IN/FT)}{(1300000\,LB/IN^2)(12.25\,IN^2)} = \underline{-0.055\,IN}$$

3-44 $A = (200^2 - 150^2) = 17500\,mm^2$

$F = \sigma \cdot A = (-200\,N/mm^2)(17500\,mm^2) = -3.5\times10^6\,N = \underline{-3.50\,MN}$

$$\delta = \frac{\sigma L}{E} = \frac{(-200\,N/mm^2)(1800\,mm)}{(165000\,N/mm^2)} = \underline{-2.18\,mm}$$

3-45 $A = \pi D^2/4 = \pi(3.0)^2/4 = 7.07\,mm^2$: (ASSUME MASS OF PLATE IS SMALL)

$$F = \frac{\delta EA}{L} = \frac{(6.0\,mm)(110000\,N/mm^2)(7.07\,mm^2)}{3600\,mm} = \underline{1296\,N}$$

$\sigma = F/A = 1296\,N/7.07\,mm^2 = \underline{183\,MPa}$ (LESS THAN S_y)

$m = W/g = 1296\,N/9.81\,m/s^2 = 132\,N\cdot s^2/m = \underline{132\,kg}$

3-46 $\sigma = 50\,MPa$ (PROB 1-53): $\delta = \sigma L/E = \frac{(50\,N/mm^2)(1250\,mm)}{69000\,N/mm^2} = \underline{0.906\,mm}$

Thermal Deformation and Thermal Stress

3-47 $\delta = \alpha L(\Delta t) = (6.0\times10^{-6}\,°F^{-1})(80\,FT)(140°F)\times(12\,IN)/FT = \underline{0.806\,IN}$

3-48 $\delta = \alpha L(\Delta t) = (11.3\times10^{-6}\,°C^{-1})(12000\,mm)(77°C) = \underline{10.4\,mm}$

3-49 $\sigma = E\alpha(\Delta t) = (207\times10^9\,Pa)(11.3\times10^{-6}\,°C^{-1})(77°C) = \underline{180\,MPa}$ HIGH!

3-50 $\delta = \alpha L(\Delta t) = (11.3\times10^{-6}\,°C^{-1})(625\,mm)(156°C) = \underline{1.10\,mm}$

3-51 a) $\delta = \alpha L(\Delta t) = (11.3\times10^{-6}\,°C^{-1})(625\,mm)(65°C) = \underline{0.459\,mm}$

b) $\sigma = E\alpha(\Delta t) = (207\times10^9\,Pa)(11.3\times10^{-6}\,°C^{-1})(91°C) = \underline{213\,MPa}$

3-52 $\delta = \alpha L(\Delta t) = (6.0\times10^{-6}\,°F^{-1})(140\,FT)(80°F)(12\,IN/FT) = \underline{0.806\,IN.}$

THIS IS THE TOTAL WIDTH. EACH END COULD BE 0.41 IN.

3-53 DECK COULD EXPAND BY A TOTAL OF 0.50 IN WITHOUT STRESS.

REQ'D. $\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.50\ IN}{(6.0 \times 10^{-6}\ ^\circ F^{-1})(140\ FT)(12\ IN/FT)} = 49.6\ ^\circ F$

$t_2 = t_1 + \Delta t = 30 + 49.6 = 79.6\ ^\circ F$

REMAINING $\Delta t = 110\ ^\circ F - 79.6\ ^\circ F = 30.4\ ^\circ F$

$\sigma = E\alpha(\Delta t) = (3.8 \times 10^6\ PSI)(6.0 \times 10^{-6}\ ^\circ F^{-1})(30.4\ ^\circ F) = \underline{693\ PSI}$

3-54 $\Delta t = 110\ ^\circ F - 60\ ^\circ F = 50\ ^\circ F$

$\delta = \alpha L(\Delta t) = (6.0 \times 10^{-6}\ ^\circ F^{-1})(140 \times 12\ IN)(50\ ^\circ F) = \underline{0.504\ IN}$

3-55 $\delta = \pi(55.300) - \pi(55.100) = 0.2\pi\ mm$ (CHANGE IN CIRCUMFERENCE)

$\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.2\pi\ mm}{(16.9 \times 10^{-6}\ ^\circ C^{-1})(\pi)(55.100\ mm)} = 214.8\ ^\circ C$

$t_2 = t_1 + \Delta t = 20 + 214.8 = \underline{234.8\ ^\circ C}$

3-56 FOR FIRST PART OF COOLING, RING IS UNRESTRAINED UNTIL ITS DIAMETER GETS TO 55.200 mm. $\delta = 55.300 - 55.200 = 0.100\ mm$

$\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.100\ mm}{(16.9 \times 10^{-6}\ ^\circ C^{-1})(55.200\ mm)} = 107.2\ ^\circ C$

$t_2 = t_1 - \Delta t = 234.8\ ^\circ C - 107.2\ ^\circ C = 127.6\ ^\circ C$

ADDITIONAL $\Delta t = 127.6 - 20 = 107.6\ ^\circ C$

$\sigma = E\alpha(\Delta t) = (193 \times 10^9\ Pa)(16.9 \times 10^{-6}\ ^\circ C^{-1})(107.6\ ^\circ C) = \underline{351\ MPa}$

3-57 $\delta_{BRASS} = \alpha L(\Delta t) = (20.5 \times 10^{-6}\ ^\circ C^{-1})(4200\ mm)(75\ ^\circ C) = \underline{6.46\ mm}$

$\delta_{SS} = \alpha L(\Delta t) = (10.4 \times 10^{-6}\ ^\circ C^{-1})(4500\ mm)(75\ ^\circ C) = \underline{3.51\ mm}$

3-58 $\delta = \alpha L(\Delta t) = (6.5 \times 10^{-6}\ ^\circ F^{-1})(40 \times 12\ IN)(190\ ^\circ F) = \underline{0.593\ IN}$

3-59 INITIAL EXPANSION OF 0.10 mm IS UNRESTRAINED.

REQ'D $\Delta t = \dfrac{\delta}{\alpha L} = \dfrac{0.10\ mm}{(25.2 \times 10^{-6}\ ^\circ C^{-1})(250\ mm)} = 15.9\ ^\circ C$

$t_2 = t_1 + \Delta t = 20\ ^\circ C + 15.9\ ^\circ C = 35.9\ ^\circ C$

ADD. $\Delta t = 70\ ^\circ C - 35.9\ ^\circ C = 34.1\ ^\circ C$ – RESTRAINED

$\sigma = E\alpha\Delta t = (45 \times 10^9\ Pa)(25.2 \times 10^{-6}\ ^\circ C^{-1})(34.1\ ^\circ C) = \underline{38.7\ MPa}$

3-60 $\sigma = E\alpha\Delta t = (207 \times 10^9\ Pa)(11.3 \times 10^{-6}\ ^\circ C^{-1})(90\ ^\circ C) = \underline{211\ MPa}$

3-61 $\delta_1 = 10.505 - 10.500 = 0.005\ IN$ UNRESTRAINED

$\Delta t_1 = \dfrac{\delta}{\alpha L} = \dfrac{0.005\ IN}{(13.0 \times 10^{-6}\ ^\circ F^{-1})(10.500\ IN)} = 36.6\ ^\circ F$

$t_2 = t_1 + \Delta t_1 = 75 + 36.6 = 111.6\ ^\circ F$

ADD. $\Delta t = 400 - 111.6 = 288.4\ ^\circ F$ – RESTRAINED

$\sigma = E\alpha(\Delta t) = (10 \times 10^6\ PSI)(13.0 \times 10^{-6}\ ^\circ F^{-1})(288.4\ ^\circ F) = \underline{37\,500\ PSI}$

FOR 6061-T4, $S_u = 35\,000\ PSI$; BAR SHOULD FAIL.

ALSO, BECAUSE BAR IS IN COMPRESSION IT MAY BUCKLE.

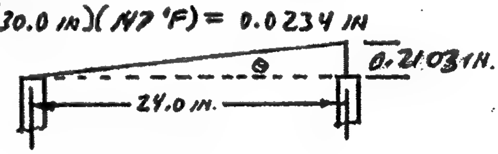
3-62 $\delta_P = \alpha L(\Delta t) = (53.0 \times 10^{-6}\ ^\circ F^{-1})(30.0\ IN)(212 - 65)\ ^\circ F = 0.2337\ IN.$

$\delta_{Ti} = \alpha L(\Delta t) = (5.3 \times 10^{-6}\ ^\circ F^{-1})(30.0\ IN)(147\ ^\circ F) = 0.0234\ IN$

$\delta_P - \delta_{Ti} = 0.2103\ IN$

$\tan\theta = \dfrac{0.2103}{24} = 0.00876$

$\underline{\theta = 0.502\ deg.}$



3-63 $\delta = \alpha L(\Delta t) = (6.3\times10^{-6}\,°F^{-1})(25\,FT)(12\,IN/FT)(68-(-15))°F = \underline{0.157\,IN}$

3-64 TOTAL $\delta_T = 0.50\,mm = 0.00050\,m = \delta_S + \delta_B$

$\delta_T = \alpha_S L_S(\Delta t) + \alpha_B L_B(\Delta t) = \Delta t[\alpha_S L_S + \alpha_B L_B]$

$$\Delta t = \frac{\delta_T}{\alpha_S L_S + \alpha_B L_B} = \frac{0.0005\,m}{(10.4\times10^{-6}\,°C^{-1})(2.800\,m) + (20.5\times10^{-6}\,°C^{-1})(1.900\,m)}$$

$\Delta t = 7.35°C$; $t_2 = t_1 + \Delta t = 20 + 7.35 = \underline{27.35°C}$

3-65 ADDED $\sigma = E\alpha\,\Delta t = (193\times10^9\,Pa)(16.9\times10^{-6}\,°C^{-1})(35°C) = 114\,MPa$

FINAL $\sigma = 40 + 114 = \underline{154\,MPa}$

3-66 WHEN HEATED, WIRE WOULD RELAX.

$$\Delta t = \frac{\sigma}{E\alpha} = \frac{40\times10^6\,Pa}{(193\times10^9\,Pa)(16.9\times10^{-6}\,°C^{-1})} = 12.3\,°C$$

$t_2 = t_1 + \Delta t = 20°C + 12.3°C = \underline{32.3\,°C}$

Members Made from Two Materials

3-67 $A_S = 150^2 - 130^2 = 5600\,mm^2$; $A_C = 130^2 = 16900\,mm^2$

$$\sigma_C = \frac{FE_C}{A_S E_S + A_C E_C} = \frac{(900\times10^3\,N)(32.4\,GPa)}{(5600\,mm^2)(207\,GPa) + (16900\,mm^2)(32.4\,GPa)} = \underline{17.1\,MPa}$$

$\sigma_S = \sigma_C E_S/E_C = (17.1\,MPa)(207\,GPa)/32.4\,GPa = \underline{109\,MPa}$

3-68 $A_S = 9.74\,IN^2$ (APP A-9); $A_C = (5.0\,IN)^2 = 25.0\,IN^2$

$E_S = 30\times10^6\,PSI$; $E_C = 4.7\times10^6\,PSI$ (SEC 2-10)

LET $\sigma_C = \sigma_{dC} = 1500\,PSI$; THEN $\sigma_S < \sigma_{dS} = 21600\,PSI$

$$F = \frac{\sigma_{dC}[A_S E_S + A_C E_C]}{E_C} = \frac{1500\,PSI\,[(9.74)(30\times10^6) + 25(4.7\times10^6)]}{4.7\times10^6}$$

$\underline{F = 130\,800\,LB}$; CHECK $\sigma_S = \sigma_C\,E_S/E_C = 1500(30/4.7) = 9575\,PSI$ $\underline{OK}$

3-69 LET $\sigma_C = 1500\,PSI$; $\sigma_S = \sigma_C \frac{E_S}{E_C} = 1500\,\frac{30}{4.7} = 9575\,PSI$ - $\underline{OK}$

$$\frac{F}{\sigma_C} = \frac{A_S E_S + A_C E_C}{E_C} = \frac{A_S E_S}{E_C} + A_C = A_S\frac{30}{4.7} + A_C = 6.38A_S + A_C$$

$A_S = b^2 - (b-2t)^2 = b^2 - [b-2(0.5)]^2 = b^2 - (b-1)^2 = b^2 - b^2 + 2b - 1 = 2b - 1$

$A_C = (b-1)^2 = b^2 - 2b + 1$

$$\frac{F}{\sigma_C} = 6.38[2b-1] + b^2 - 2b + 1 = b^2 + 10.77b - 5.38$$

$$\frac{F}{\sigma_C} = \frac{500\,000}{1500} = 333.3 = b^2 + 10.77b - 5.38$$

$b^2 + 10.77b - 338.7 = 0$

BY QUADRATIC EQN., $\underline{b = 13.81\,IN}$

3-70 $A_S = A_A = 2\,\pi(6)^2/4 = 56.55\ mm^2$

$$\sigma_A = \frac{PE_A}{A_S E_S + A_A E_A} = \frac{(11.3 \times 10^3\ N)(69\ GPa)}{(56.55\ mm^2)(207\ GPa) + (56.55\ mm^2)(69\ GPa)} = \underline{49.5\ MPa}$$

$$\sigma_S = \sigma_A \cdot E_S/E_A = 49.5\ MPa \cdot 207/69 = \underline{148.5\ MPa}$$

3-71 $W = mg = 2265\ kg \cdot 9.81\ m/s^2 = 22.22 \times 10^3\ N$; $A_S = 2A_C$

IF $\sigma_S = 552\ MPa/2 = 276\ MPa$; THEN $\sigma_C = \sigma_S \cdot \frac{E_C}{E_S} = 276 \cdot \frac{131}{200} = 181\ MPa$

$\sigma_{C\,ALLOW} = S_Y/2 = 1000/2 = 500\ MPa$ $\underline{OK}$

$$\sigma_C = \frac{PE_C}{A_S E_S + A_C E_C} = \frac{PE_C}{2A_C E_S + A_C E_C} = \frac{PE_C}{A_C[2E_S + E_C]}$$

$$A_C = \frac{PE_C}{\sigma_C[2E_S + E_C]} = \frac{(22.22 \times 10^3\ N)(131\ GPa)}{(181\ N/mm^2)[2(200) + 131]\ GPa} = 30.3\ mm^2$$

$D = \sqrt{4A/\pi} = \sqrt{4(30.3)/\pi} = \underline{6.20\ mm = \text{WIRE DIA.}}$

3-72 1) COMPUTE FORCE REQ'D. TO DEFLECT ALUM. ROD 0.12 mm

$$P_1 = \frac{\delta EA}{L} = \frac{(0.12\ mm)(69000\ N/mm^2)(15\,394\ mm^2)}{1650.12\ mm} = 77.24\ kN$$

2) ADDITIONAL LOAD AVAILABLE: $P_2 = 350\ kN - 77.24\ kN = 272.76\ kN$

3) BOTH MEMBERS DEFLECT THE SAME AMOUNT UNDER P_2

$$\delta_A = \delta_S : \frac{P_A \cancel{L_A}}{E_A A_A} = \frac{P_S \cancel{L_S}}{E_S A_S} \quad [L_A = L_S]$$

$$P_A = P_S \frac{E_A A_A}{E_S A_S} = P_S \frac{(69\ GPa)(15394\ mm^2)}{(207\ GPa)(3600\ mm^2)} = 1.425\ P_S$$

6 IN SCH 40 PIPE: $A_S = 5.581\ in^2 (25.4)^2 mm^2/in^2 = 3600\ mm^2$

APP A-12 (SI)

4) $P_S + P_A = P_2 = 272.76\ kN$

$P_S + 1.425 P_S = 2.425 P_S = 272.76\ kN$

$\underline{P_S = 112.48\ kN}$

$P_A = 272.76 - 112.48 = 160.28\ kN$

5) TOTAL LOAD ON ALUM: $P_{AT} = P_1 + P_A = 77.24 + 160.28 = 237.52\ kN$

6) STRESSES:

$$\sigma_S = \frac{P_S}{A_S} = \frac{112.48 \times 10^3\ N}{3600\ mm^2} = \underline{31.24\ MPa}$$

$$\sigma_A = \frac{P_{AT}}{A_A} = \frac{237.52 \times 10^3\ N}{15\,394\ mm^2} = \underline{15.43\ MPa}$$

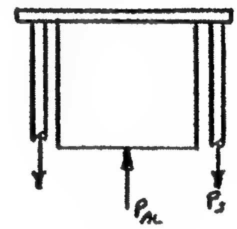
3-73

1) THE NUT MOVES 1.25 mm IN ONE TURN

2) THE FORCE CREATED CAUSES THE TUBE TO SHORTEN AND THE BOLTS TO GET LONGER. $\delta_{AL} + \delta_S = 1.25\,mm$; AND $L_{AL} = L_S$

3) THE COMPRESSIVE FORCE IN THE TUBE EQUALS THE TENSILE FORCE IN THE BOLTS

$P_{AL} = P_S$ (ON ALL FOUR BOLTS)



4) $A_{AL} = \frac{\pi}{4}(150^2 - 138^2) = 2714\,mm^2$

$A_S = 4\pi(10^2)/4 = 314\,mm^2$

5) $\frac{P_{AL} L_{AL}}{E_{AL} A_{AL}} + \frac{P_S L_S}{E_S A_S} = 1.25\,mm$

$$\frac{P_S\,(450)\,mm}{(69 \times 10^3\,N/mm^2)(2714\,mm^2)} + \frac{P_S\,(450)}{(207 \times 10^3)(314)} = 1.25$$

$P_S\,[2.40 \times 10^{-6} + 6.92 \times 10^{-6}] = P_S\,[9.32 \times 10^{-6}] = 1.25$

$P_S = 134\,kN = P_{AL}$

6) STRESSES:

$$\sigma_S = \frac{P_S}{A_S} = \frac{134\,000\,N}{314\,mm^2} = \underline{427\,MPa}$$

$$\sigma_{AL} = \frac{P_{AL}}{A_{AL}} = \frac{134\,000\,N}{2714\,mm^2} = \underline{49.4\,MPa}$$

3-74

$$\sigma_c = \frac{P E_c}{A_s E_s + A_c E_c} = \frac{(50000\,lb)(2.7 \times 10^6\,psi)}{(4.43\,in^2)(29 \times 10^6 psi) + (108.7\,in^2)(2.7 \times 10^6 psi)} = 320\,psi$$

$A_s = 4.43\,in^2$; $A_c = \pi(12)^2/4 - 4.43 = 108.7\,in^2$

$\sigma_s = \sigma_c \cdot E_s/E_c = (320\,psi)(29/2.7) = \underline{3437\,psi}$

NOTE: $E_s = 29 \times 10^6$ psi FOR STRUCTURAL STEEL

$E_c = 2.7 \times 10^6$ psi FOR CONCRETE WITH $f'_c = 2000$ psi RATED

(SEE SEC. 2-10)

3-75

NO. 2 SOUTHERN PINE. $\sigma_{c\,ALLOWABLE} = 1.59\,MPa$; $E = 9.0\,GPa$

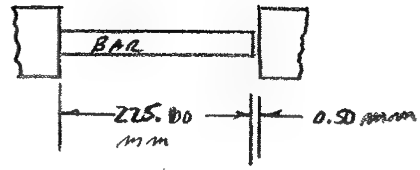
4x4 POST. $A = 7.90 \times 10^3\,mm^2$, $L = 4.25\,m$

$\sigma = \frac{F}{A}$; $\underline{F_{MAX}} = \sigma_{c\,ALL} \cdot A = \frac{1.59\,N}{mm^2} \times 7.90 \times 10^3\,mm^2 = \underline{12\,560\,N}$
$= \underline{12.56\,kN}$

$$\delta = \frac{FL}{EA} = \frac{(12560\,N)(4.25\,m)}{(9.0 \times 10^9\,N/m^2)(7.90 \times 10^3\,mm^2)} \times \frac{(10^3\,mm)^2}{m^2}$$

$\underline{\delta} = 7.51 \times 10^{-4}\,m = \underline{0.751\,mm}$

3-76 $t_1 = 20°C$ $L_1 = 225.0\ mm$



$t_{FINAL} = 205°C$. ALUM 6061-T4, $S_y = 110\ MPa$

a) TEMP. AT WHICH BAR TOUCHES PLATE

$\delta = 0.50\ mm = \alpha L(\Delta t)$

$$\Delta t_1 = \frac{\delta}{\alpha L} = \frac{0.50\ mm}{(23.4 \times 10^{-6}\ °C^{-1})(225.0\ mm)} = 95.0°C$$

$t_2 = t_1 + \Delta t = 20 + 95 = 115°C$ NO STRAIN AT THIS TEMP.

b) ADDITIONAL $\Delta t_2 = 205°C - 115°C = 90.0°C$ RESTRAINED.

$\sigma = E\alpha(\Delta t) = (69 \times 10^9\ Pa)(23.4 \times 10^{-6}\ °C^{-1})(90.0°C)$

$\underline{\sigma} = 145 \times 10^6\ Pa = \underline{145\ MPa}$ COMPRESSION $> S_y$ <u>MATERIAL WOULD YIELD-FAILURE</u>

3-77 (a) $t_1 = 20°C$. $L_1 = 2.400\ m$. ALUM. 2014-T4, $L_2 = 2.405\ m$

$\delta = L_2 - L_1 = 2.405 - 2.400 = 0.005\ m = 5.00\ mm = \alpha L(\Delta t)$

$$\Delta t_1 = \frac{\delta}{\alpha L_1} = \frac{0.005\ m}{(23.0 \times 10^{-6}\ °C^{-1})(2.400\ m)} = 90.6°C$$

$\underline{t_2} = t_1 + \Delta t_1 = 20 + 90.6 = \underline{110.6°C}$

(b) INCREASE 30°C. $t_3 = 110.6°C + 30 = 140.6°C$ RESTRAINED.

$\sigma = E\alpha\Delta t_2 = (73 \times 10^9\ Pa)(23.0 \times 10^{-6}\ °C^{-1})(30°C)$

$\underline{\sigma} = 50.4 \times 10^6\ Pa = \underline{50.4\ MPa}$ COMPRESSION

(c) $S_y = 290\ MPa$ FOR 2014-T4. SAFE AGAINST YIELDING. BUT BUCKLING SHOULD BE CHECKED.

3-78 $\delta_{MAX} = 0.50\ mm$. REPEATED AXIAL TENSILE LOAD.

SPECIFY MAX. PERMISSIBLE LOAD. 4140 OQT 1300. $E = 207\ GPa$

a) <u>DEFORMATION:</u> $\delta = \frac{FL}{EA}$. THEN $F_{MAX} = \frac{\delta EA}{L}$

$A = 30 \times 20 = 600\ mm^2$, $L = 700\ mm$

$$F_{MAX} = \frac{(0.50\ mm)(207 \times 10^9\ N)\ 600\ mm^2}{(700\ mm)\ m^2} \times \frac{1\ m^2}{(10^3\ mm)^2}$$

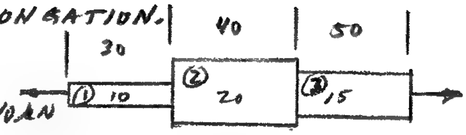
$\underline{F_{MAX}} = 88.7 \times 10^3\ N = \underline{88.7\ kN}$

(b) <u>STRESS:</u> $\sigma_d = S_u/8 = 814\ MPa/8 = 101.8\ MPa = F/A$

$\underline{F_{MAX}} = \sigma_d \cdot A = (101.8\ N/mm^2)(600\ mm^2) = 61\,000\ N = \underline{61.0\ kN}$

STRESS GOVERNS THE DESIGN. <u>$F_{MAX} = 61.0\ kN$</u>

3-79 FIGURE P3-79. COMPUTE TOTAL ELONGATION.



① $\delta_1 = \dfrac{F_1 L_1}{E A_1} = \dfrac{(40\times10^3 N)(30 mm)}{(73\times10^3 N/mm^2)(100 mm^2)}$

$\delta_1 = 0.164\ mm$

2014-T6 ALUM.
$E = 73 GPa = 73\times10^9 Pa$
$E = 73\times10^3\ MPa$
$E = 73\times10^3\ N/mm^2$
$S_y = 414\ MPa$

② $\delta_2 = \dfrac{F_2 L_2}{E A_2} = \dfrac{(40\times10^3)(40)}{(73\times10^3)(400)} = 0.0548\ mm$

③ $\delta_3 = \dfrac{F_3 L_3}{E A_3} = \dfrac{(40\times10^3)(50)}{(73\times10^3)(225)} = 0.1218\ mm$

$\delta_{TOTAL} = \delta_1 + \delta_2 + \delta_3 = \underline{0.341\ mm}$

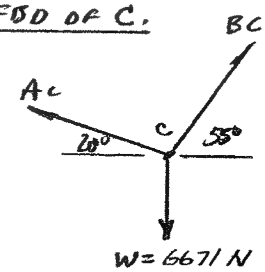
CHECK STRESS IN ①: $\sigma_1 = \dfrac{F_1}{A_1} = \dfrac{40\times10^3 N}{100 mm^2} = 400 MPa$ – CLOSE TO SY
NOT SAFE.

3-80 FORCE ANALYSIS: $m = 680 kg.$

$W = mg = 680 kg \cdot 9.81 m/s^2 = 6671 N$ = DOWNWARD FORCE AT C.

SINE LAW: $\dfrac{6671 N}{\sin 75°} = \dfrac{AC}{\sin 35°} = \dfrac{BC}{\sin 70°}$

FBD OF C.



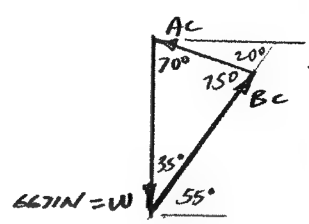
$AC = \dfrac{6671 N(\sin 35°)}{\sin 75°} = 3961\ N$

$BC = \dfrac{6671 N(\sin 70°)}{\sin 75°} = 6490\ N$

STRESS: ROD AREA $= A = \pi(8.00 mm)^2/4 = 50.27\ mm^2$

$\sigma_{BC} = BC/A = 6490 N/50.27 mm^2 = 129\ MPa$

$\sigma_{AC} = AC/A = 3961 N/50.27 mm^2 = 78.8\ MPa$

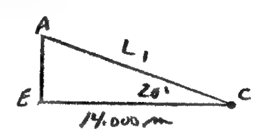


VECTOR TRIANGLE

ASSUME STATIC LOAD; SAFE FOR ANY STEEL

DEF. $\delta_{AC} = \dfrac{(AC)(L_1)}{E A}$, $\delta_{BC} = \dfrac{(BC)(L_2)}{E A}$

$L_1 = 14.000 m/\cos 20° = 14.898\ m$

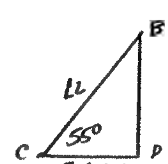


$L_2 = 7.000 m/\cos 55° = 12.204\ m$

ORIGINAL VERTICAL DISTANCE FROM CEILING TO C:

$\tan 55° = BD/CD$; $BD = CD \tan 55° = (7.00)(\tan 55°)$

$BD = 9.997\ m$



$\delta_{AC} = \dfrac{AC\ L_1}{E A} = \dfrac{(3961 N)(14.898 m)}{(207\times10^9 N/m^2)(50.27 mm^2)} \times \dfrac{(10^3 mm)^2}{m^2}$

$\delta_{AC} = 5.67\times10^{-3} m = \underline{5.67\ mm}$

$\delta_{BC} = \dfrac{BC\ L_2}{E A} = \dfrac{(6490 N)(12.204 m)}{(207\times10^9 N/m^2)(50.27 mm^2)} \times \dfrac{(10^3 mm)^2}{m^2} = 7.61\times10^{-3} m$

$\delta_{BC} = 7.61\ mm$

3-81

FORCE ANALYSIS. $m = 4200\ kg$.

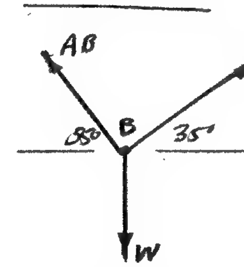
$W = mg = (4200\ kg)(9.81\ m/s^2) = 41200\ N = 41.2\ kN$

VECTOR TRIANGLE IS A RIGHT TRIANGLE,

$BC = W \sin 35° = (41.2\ kN)(\sin 35°) = 23.63\ kN$

$AB = W \cos 35° = (41.2\ kN)(\cos 35°) = 33.75\ kN$

FBD OF B



STRESS: $D = 10.0\ mm$; $A = \pi D^2/4 = 78.54\ mm^2$

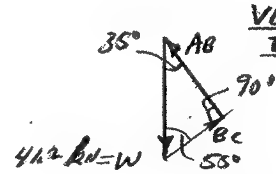
$$\sigma_{AB} = \frac{F}{A} = \frac{AB}{A} = \frac{33.75 \times 10^3\ N}{78.54\ mm^2} = 429.7\ MPa$$

ASSUME STATIC LOAD, $\sigma_d = S_y/2$

REQ'D $S_y = 2\sigma = 2(429.7\ MPa) = 859\ MPa$

SPECIFY AISI 4140 OQT 1100, $S_y = 903\ MPa$

VECTOR TRIANGLE

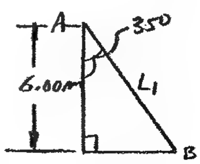


DEFORMATION:

$$\delta_{AB} = \frac{(AB)L_1}{EA} \quad ; \quad \delta_{BC} = \frac{(BC)(L_2)}{EA}$$

$L_1 = 6.00\ m/\cos 35° = 7.32\ m$

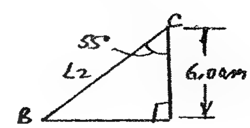
$$\delta_{AB} = \frac{(33.75 \times 10^3 N)(7.32\ m)}{(207 \times 10^9\ N/m^2)(78.54\ mm^2)} \times \frac{(10^3\ mm)^2}{m^2}$$



$\delta_{AB} = 0.0152\ m = 15.2\ mm$

$$\delta_{BC} = \frac{(BC)L_2}{EA} = \frac{(23.63 \times 10^3\ N)(10.46\ m)}{(207 \times 10^9\ N/m^2)(78.54\ mm^2)} \times \frac{(10^3\ mm)^2}{m^2}$$

$[L_2 = 6.00\ m/\cos 55° = 10.46\ m]$



$\delta_{BC} = 0.0152\ m = 15.2\ mm$

3-82 AISI 1040 CD, $S_y = 82\,ksi$. LET $\sigma = 0.9\,S_y = 73.8\,ksi = F/A$

$L = 2.00\,IN$; $A = \frac{\pi D^2}{4} = \frac{\pi(0.505\,IN)^2}{4} = 0.200\,IN^2$

$E = 30\times10^6\,psi = 30\times10^3\,ksi$

$$\delta = \frac{FL}{EA} = \frac{\sigma L}{E} = \frac{(73.8\,ksi)(2.00\,IN.)}{30\times10^3\,ksi} = \underline{0.00492\,IN} = \delta$$

STRAIN $= \delta/L = 0.00492\,IN/2.00\,IN = \underline{0.00246\,IN/IN.} = \epsilon$

$F = \sigma\cdot A = (73.8\times10^3\,LB/IN^2)(0.200\,IN^2) = \underline{14760\,LB} = F$

DISTANCE BETWEEN GAGE MARKS $= 2.00\,IN + \delta = \underline{2.0049\,IN}$

PROBLEMS 83-90 FOLLOW SIMILAR SOLUTION PRICEDURE.

RESULTS SUMMARIZED ON FOLLOWING PAGE.

3-91 NYLON 66 PLASTIC. TENSILE STRENGTH $= S_u = 83\,MPa$

LET $\sigma = 0.5\,S_u = 0.5(83\,MPa) = 41.5\,MPa$

$L = 50\,mm$, $A = t\cdot w = (12.5\,mm)(16.0\,mm) = 200\,mm^2$

$E = 2900\,MPa = 2900\,N/mm^2$

$$\delta = \frac{FL}{EA} = \frac{\sigma L}{E} = \frac{(41.5\,MPa)(50\,mm)}{2900\,MPa} = \underline{0.7155\,mm} = \delta$$

STRAIN $= \epsilon = \delta/L = 0.7155\,mm/50\,mm = \underline{0.0143\,mm/mm} = \epsilon$

$F = \sigma\cdot A = (41.5\,N/mm^2)(200\,mm^2) = 8300\,N = \underline{8.30\,kN} = F$

DISTANCE BETWEEN GAGE MARKS $= 50\,mm + \delta = \underline{50.716\,mm}$

PROBLEMS 92-99 FOLLOW SIMILAR SOLUTION PROCEDURE.

RESULTS SUMMARIZED ON FOLLOWING PAGE.

NOTE: DATA FROM TABLE 2-13 FOR PROBLEMS 96-99 REQUIRE CONVERSIONS AS SHOWN IN LOWER TABLE ON FOLLOWING PAGE.

SOLUTIONS TO PROBLEMS 3-82 TO 3-90										
	Metals									
Prob. No.	Material	s_y (ksi)	$0.9*s_y$ (ksi)	E (ksi)	L (in)	A (in^2)	Elong. (in)	Length betw. gage marks (in)	Strain (in/in)	Force (lb)
3-82	AISI 1040 CD	82	73.8	30000	2.000	0.200	0.00492	2.0049	0.00246	14760
3-83	AISI 5160 OQT 700	238	214	30000	2.000	0.200	0.01428	2.0143	0.00714	42840
3-84	AISI 501 OQT 1000	135	122	29000	2.000	0.200	0.00838	2.0084	0.00419	24300
3-85	C17200 Ber. Copper, hard	145	131	19000	2.000	0.200	0.01374	2.0137	0.00687	26100
3-86	Magnesium AZ63A-T6	19	17.1	6500	2.000	0.200	0.00526	2.0053	0.00263	3420
3-87	Zinc ZA 12	47	42.3	12000	2.000	0.200	0.00705	2.0071	0.00353	8460
3-88	Steel ASTM A572 Gr 65	65	58.5	29000	2.000	0.200	0.00403	2.0040	0.00202	11700
3-89	ADI Grade 4	155	140	24000	2.000	0.200	0.01163	2.0116	0.00581	27900
3-90	Aluminum 5154-H38	39	35.1	10200	2.000	0.200	0.00688	2.0069	0.00344	7020

SOLUTIONS TO PROBLEMS 3-91 to 3-101										
	Plastics and Composites									
Prob. No.	Material	s_u MPa	$0.5*s_u$ MPa	E MPa	L (mm)	A (mm^2)	Elong. (mm)	Length betw. gage marks (mm)	Strain (mm/mm)	Force (kN)
3-91	Nylon 66, dry+	83	41.5	2900	50.0	200	0.71552	50.716	0.01431	8.30
3-92	ABS, high impact+	34	17	1720	50.0	200	0.49419	50.494	0.00988	3.40
3-93	Acetal copolymer+	55	27.5	2830	50.0	200	0.48587	50.486	0.00972	5.50
3-94	Polyurethane elastomer+	34	17	690	50.0	200	1.23188	51.232	0.02464	3.40
3-95	Phenolic+	45	22.5	7580	50.0	200	0.14842	50.148	0.00297	4.50
3-96	Glass/epoxy composite#	786	393	27580	50.0	200	0.71250	50.713	0.01425	78.6
3-97	Aramid/epoxy composite#	1379	690	75845	50.0	200	0.45455	50.455	0.00909	137.9
3-98	Graphite/epoxy, High s_u#	1917	958	135832	50.0	200	0.35279	50.353	0.00706	191.7
3-99	Graphite/epoxy, High E#	1103	552	330960	50.0	200	0.08333	50.083	0.00167	110.3

+	From Appendix A20
#	From Table 2-13, Chapter 2

Problems 3-96 to 3-99: Data conversion of units					
Prob. No.	Material	s_u ksi	s_u MPa	E ksi	E MPa
3-96	Glass/epoxy composite	114	786	4000	27580
3-97	Aramid/epoxy composite	200	1379	11000	75845
3-98	Graphite/epoxy, High s_u	278	1917	19700	135832
3-99	Graphite/epoxy, High E	160	1103	48000	330960

Stress Concentrations for Direct Axial Stresses

3-100 $D = 40.0\,mm$, $d_g = 35.0\,mm$, $r = 3.0\,mm$, $F = 46\,kN = 46000\,N$

$D/d_g = 40/35 = 1.14$; $r/d_g = 3.0/35 = 0.086$; $K_t = 2.3$ APP. A 22-1

$\sigma_{NOM} = \dfrac{F}{\pi d_g^2/4} = \dfrac{46000\,N}{\pi(35)^2/4\,mm^2} = 47.8\,MPa$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (2.3)\,47.8 = \underline{110\,MPa}$

3-101 $D = 1.50\,IN$, $d_g = 1.25\,IN$, $r = 0.12\,IN$, $F = 10300\,LB$

$D/d_g = 1.50/1.25 = 1.20$; $r/d_g = 0.12/1.25 = 0.096$, $K_t = 2.40$ APP. A22-1

$\sigma_{NOM} = \dfrac{F}{\pi d_g^2/4} = \dfrac{10300\,LB}{\pi(1.25)^2/4} = 8393\,psi$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = 2.40(8393) = \underline{20\,140\,psi}$

3-102 $D = 0.40\,IN$ $d_g = 0.35\,IN$ $r = 0.040\,IN$ $F = 1250\,LB$ USE APP. A22-1

$D/d_g = 0.40/0.35 = 1.14$; $r/d_g = 0.04/0.35 = 0.114$; $K_t = 2.05$

$\sigma_{NOM} = \dfrac{F}{\pi d_g^2/4} = \dfrac{1250\,LB}{\pi(0.35)^2/4} = 12990\,psi$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (2.05)(12990) = \underline{26\,635\,psi}$

3-103 $D = 10.0\,mm$, $d_g = 8.0\,mm$, $r = 1.20\,mm$, $F = 5500\,N$ USE APP. A22-1

$D/d_g = 10/8 = 1.25$; $r/d_g = 1.2/8 = 0.15$; $K_t = 2.05$; $\sigma_{NOM} = F/\pi d_g^2/4$

$\sigma_{NOM} = \dfrac{5500\,N}{\pi(8)^2/4\,mm^2} = 109.4\,MPa$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = 2.05(109.4) = \underline{224\,MPa}$

3-104 $H = 2.50\,IN$, $h = 2.20\,IN$, $r = 0.25/2 = 0.125\,IN$, $F = 17{,}500\,LB$ USE APP A-22-3

$H/h = 2.50/2.20 = 1.14$; $r/h = 0.125/2.20 = 0.057$; $\sigma_{NOM} = F/th$; $t = 0.400\,IN$; $K_t = 1.96$

$\sigma_{NOM} = \dfrac{17\,500\,LB}{(0.40)(2.20)\,IN^2} = 19886\,psi$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (1.96)(19\,886) = \underline{38\,980\,psi}$

3-105 $H = 60\,mm$, $h = 55\,mm$, $t = 10\,mm$, $r = 6/2 = 3.0\,mm$, $F = 75\,kN$, USE APP. A22-3

$H/h = 60/55 = 1.09$; $r/h = 3/55 = 0.055$; $K_t = 1.75$; $\sigma_{NOM} = F/th$

$\sigma_{NOM} = \dfrac{75\,000\,N}{(10)(55)\,mm^2} = 136.4\,MPa$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (1.75)(136.4) = \underline{239\,MPa}$

3-106 $H = 25\,mm$, $h = 22\,mm$, $t = 3.0\,mm$, $r = 5/2 = 2.5\,mm$, $F = 6800\,N$ USE APP A22-3

$H/h = 25/22 = 1.14$, $r/h = 2.5/22 = 0.114$; $K_t = 1.67$; $\sigma_{NOM} = F/th$

$\sigma_{NOM} = \dfrac{6800\,N}{(3)(22)\,mm^2} = 103.0\,MPa$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (1.67)\,103 = \underline{172\,MPa}$

3-107 $H = 0.80\,IN$; $h = 0.50\,IN$; $r = 0.2/2 = 0.10\,IN$; $t = 0.12\,IN$, $F = 1800\,LB$, USE APP. A22-3

$H/h = 0.80/0.50 = 1.60$; $r/h = 0.10/0.50 = 0.20$; $K_t = 1.76$; $\sigma_{NOM} = F/th$

$\sigma_{NOM} = \dfrac{1800\,LB}{(0.12)(0.50)\,IN^2} = 30\,000\,psi$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (1.76)(30000) = \underline{52800\,psi}$

3-108 $D = 50\,mm$, $d = 40\,mm$, $r = 6.0\,mm$, $F = 230\,kN = 230\,000\,N$, USE APP A22-2

$D/d = 50/40 = 1.25$; $r/d = 6/40 = 0.15$; $K_t = 1.65$; $\sigma_{NOM} = F/\pi d^2/4$

$\sigma_{NOM} = \dfrac{230\,000\,N}{\pi(40)^2/4\,mm^2} = 183\,MPa$; $\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (1.65)(183) = \underline{302\,MPa}$

3-109 $D = 2.50\ \text{IN},\ d = 1.75\ \text{IN},\ r = 0.25\ \text{IN},\ F = 48\ \text{K} = 48\,000\ \text{LB}$, USE APP A22-2

$D/d = 2.5/1.75 = 1.43;\ r/d = 0.25/1.75 = 0.143;\ K_t = 1.66;\ \sigma_{NOM} = F/\pi d^2/4$

$\sigma_{NOM} = \dfrac{48\,000\ \text{LB}}{\pi(1.75)^2/4\ \text{IN}^2} = 19956\ \text{psi};\ \underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (1.66)(19956) = \underline{33\,127\ \text{psi}}$

3-110 $D = 0.38\ \text{IN},\ d = 0.32\ \text{IN},\ r = 0.02\ \text{IN},\ F = 375\ \text{LB}$ USE APP. A22-2.

$D/d = 0.38/0.32 = 1.19;\ r/d = 0.02/0.32 = 0.063;\ K_t = 1.91\quad \sigma_{NOM} = F/\pi d^2/4$

$\sigma_{NOM} = \dfrac{375\ \text{LB}}{\pi(0.32)^2/4\ \text{IN}^2} = 4663\ \text{psi};\ \underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (1.91)(4663) = \underline{8906\ \text{psi}}$

3-111 $D = 10.0\ \text{mm},\ d = 8.0\ \text{mm},\ r = 0.50\ \text{mm},\ F = 1600\ \text{N}$, USE APP. A22-2

$D/d = 10/8 = 1.25;\ r/d = 0.5/8 = 0.063;\ K_t = 2.00;\ \sigma_{NOM} = F/\pi d^2/4$

$\sigma_{NOM} = \dfrac{1600\ \text{N}}{\pi(8)^2/4\ \text{mm}^2} = 31.83\ \text{MPa};\ \underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (2.00)(31.83) = \underline{63.7\ \text{MPa}}$

3-112 $w = 2.50\ \text{IN},\ t = 0.400\ \text{IN},\ d = 1.75\ \text{IN},\ F = 14200\ \text{LB}$. USE APP A22-4 CURVE A

$d/w = 1.75/2.50 = 0.70;\ K_t = 2.05,\ \sigma_{NOM} = F/(w-d)t$

$\sigma_{NOM} = \dfrac{14200\ \text{LB}}{(2.50-1.75)(0.4)} = 47333\ \text{psi};\ \underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (2.05)(47333) = \underline{97\,030\ \text{psi}}$

3-113 $w = 60\ \text{mm},\ t = 8.00\ \text{mm},\ d = 40\ \text{mm},\ F = 65\ \text{kN}$, USE APP A22-4, CURVE A

$d/w = 40/60 = 0.67;\ K_t = 2.05;\ \sigma_{NOM} = F/(w-d)t = \dfrac{65\,000\ \text{N}}{(60-40)(8)} = 406\ \text{MPa}$

$\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (2.05)(406) = \underline{833\ \text{MPa}}$

3-114 $w = 18\ \text{mm},\ t = 2.50\ \text{mm},\ d = 8.00\ \text{mm},\ F = 2250\ \text{N}$ USE APP. 22-4, CURVE A

$d/w = 8/18 = 0.444;\ K_t = 2.20;\ \sigma_{NOM} = F/(w-d)t = \dfrac{2250\ \text{N}}{(18-8)(2.5)} = 90.0\ \text{MPa}$

$\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (2.20)(90) = \underline{198\ \text{MPa}}$

3-115 $w = 0.60\ \text{IN},\ t = 0.088\ \text{IN},\ d = 0.25\ \text{IN},\ F = 475\ \text{LB}$ USE APP. 22-4, CURVE A

$d/w = 0.25/0.60 = 0.417;\ K_t = 2.22;\ \sigma_{NOM} = F/(w-d)t = \dfrac{475\ \text{LB}}{(0.6-0.25)(0.088)} = 15{,}422\ \text{psi}$

$\underline{\sigma_{MAX}} = K_t\ \sigma_{NOM} = (2.22)(15\,422) = \underline{34{,}240\ \text{psi}}$

3-116 $D = 50\ \text{mm},\ d = 20\ \text{mm},\ F = 120\ \text{kN} = 120{,}000\ \text{N}$ USE APP. A22-5, CURVE A

$d/D = 20/50 = 0.40;\ K_{t_g} = 5.0;\ \sigma_{NOM} = F/\pi D^2/4 = \dfrac{120\,000\ \text{N}}{\pi(50)^2/4} = 61.1\ \text{MPa}$

$\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (5.0)(61.1) = \underline{306\ \text{MPa}}$

3-117 $D = 2.00\ \text{IN},\ d = 0.75\ \text{IN},\ F = 22500\ \text{LB.}$, USE APP. A22-5, CURVE A

$d/D = 0.75/2.00 = 0.375;\ K_{t_g} = 4.75;\ \sigma_{NOM} = F/\pi D^2/4 = \dfrac{22\,500\ \text{LB}}{\pi(2.0)^2/4\ \text{IN}^2} = 7162\ \text{psi}$

$\underline{\sigma_{MAX}} = K_t\,\sigma_{NOM} = (4.75)(7162) = \underline{34\,020\ \text{psi}}$

3-118 $D = 0.63$ IN, $d = 0.35$ IN, $F = 2800$ LB, USE APP. A22-5, CURVE A

$d/D = 0.35/0.63 = 0.556$; $K_{tg} = 6.95$; $\sigma_{NOM} = \frac{F}{\pi D^2/4} = \frac{2800\,LB}{\pi(0.63)^2/4} = 8982$ PSI

$\sigma_{MAX} = K_t\,\sigma_{NOM} = (6.95)(8982) = 62\,425$ PSI

3-119 $D = 12$ mm $d = 7.25$ mm $F = 7500$ N USE APP. A22-5, CURVE A

$d/D = 7.25/12 = 0.604$; $K_t = 8.00$; $\sigma_{NOM} = \frac{F}{\pi D^2/4} = \frac{7500\,N}{\pi(12)^2/4} = 66.3$ MPa

$\sigma_{MAX} = K_t\,\sigma_{NOM} = (8.00)(66.3) = 531$ MPa

3-120 $F = 25$ kN $= 25\,000$ N, REPEATED, AISI 4140 OQT 1100 STEEL, $S_u = 1014$ MPa

LET $\sigma_{MAX} = \sigma_d = S_u/N$ THEN $N = S_u/\sigma_{MAX}$

HOLE: $d/D = 10/25 = 0.40$; $K_t = 5.0$ APP. A22-5 $\sigma_{NOM} = \sigma_g = \frac{F}{\pi D^2/4}$

$\sigma_{NOM} = \frac{25000\,N}{\pi(25)^2/4} = 50.93$ MPa; $\sigma_{MAX} = K_t\,\sigma_{NOM} = (5.0)(50.93) = 255$ MPa

$N = \frac{S_u}{\sigma_{MAX}} = \frac{1014\,MPa}{255\,MPa} = 3.98$ LOW SHOULD BE 8 FOR REPEATED LOAD.

FILLET: $D/d = 25/20 = 1.25$; $r/d = 2/20 = 0.10$; $K_t = 1.79$ APP. A22-2.

$\sigma_{NOM} = \frac{F}{\pi d^2/4} = \frac{25000\,N}{\pi(20)^2/4\,mm^2} = 79.6$ MPa; $\sigma_{MAX} = K_t\,\sigma_{NOM} = (1.79)(79.6) = 142.4$ MPa

$N = \frac{S_u}{\sigma_{MAX}} = \frac{1014\,MPa}{142.4\,MPa} = 7.12$ LOW SHOULD BE 8 FOR REPEATED LOAD

3-121 USE APP. A22-2. $D = 9$ mm; $d = 6$ mm, $r = 0.50$ mm, $D/d = 9/6 = 1.50$, $r/d = 0.5/6 = 0.083$

$K_t = 1.95$; $\sigma_{MAX} = K_t\,\sigma_{NOM} = 1.95\frac{900\,N}{\pi(6)^2/4} = 62.1$ MPa

3-122 $F = 36$ kN $= 36000$ N, USE APP. A22-2, $D = 85$ mm, $d = 75$ mm, $r = 3.0$ mm

$D/d = 85/75 = 1.13$; $r/d = 3/75 = 0.04$; $K_t = 1.95$; $\sigma_{NOM} = \frac{F}{\pi d^2/4}$

$\sigma_{MAX} = K_t\,\sigma_{NOM} = 1.95\frac{36000\,N}{\pi(75)^2/4\,mm^2} = 15.9$ MPa

3-123 HOLE: $D = 1.00$ IN; $d = a = 0.50$ IN; $d/D = 0.5/1.0 = 0.50$; $K_t = 6.1$ APP. A22-5

$\sigma_{MAX} = K_t\,\sigma_g = 6.1\frac{F}{\pi D^2/4} = \frac{6.1\,F}{\pi(1.0)^2/4} = 7.77\,F$

FILLET/STEP: LET $K_t = 1.70$; $\sigma \le 7.77\,F = K_t\,F/A$;

REQ'D $A = \frac{K_t F}{7.77F} = \frac{1.7\,F}{7.77F} = 0.219\,IN^2 = \pi d^2/4$

$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.219\,IN^2)}{\pi}} = 0.528$ IN SPECIFY

THEN $D/d = 1.00/0.528 = 1.89$ AND $K_t = 1.7 \Longrightarrow r/d = 0.17$

THEN $r = 0.17d = 0.17(0.528) = 0.090$ IN SPECIFY

3-124 $F = 8.25$ kN REPEATED. CASE B - SPECIFY MATERIAL.

$\sigma_{MAX} = K_t\,\sigma_{NOM} = K_t \cdot \frac{F}{\pi d_g^2/4} = K_t\frac{8.25\times10^3\,N}{\pi(22\,mm)^2/4} = K_t(21.7\,MPa)$

APP A21-1: $r/d_g = 4.0\,mm/22\,mm = 0.182$; $D/d_g = 30/22 = 1.36$; $K_t = 1.65$

$\sigma_{MAX} = K_t \cdot \sigma_{NOM} = 1.65(21.7\,MPa) = 35.8$ MPa

LET $\sigma_{MAX} = \sigma_d = S_u/8$, REQ'D $S_u = 8\sigma_d = 8(35.8\,MPa) = 286$ MPa

SPECIFY ALUMINUM 6061-T6. $S_u = 310$ MPa, 17% ELONG. — OR ANY STEEL

3-125

FIGURE P3-125 AISI 1141 OQT 1100, $S_u = 800\,MPa$, $\sigma_d = S_u/8$ REPEATED LOAD.

$\sigma_{max} = K_t F/A$. $F_{ALLOW} = \sigma_d A/K_t$ $\sigma_d = 100\,MPa$

MIDDLE SECTION: $A = (10\,mm)(6\,mm) = 60\,mm^2$. $\frac{r}{h} = \frac{1.5\,mm}{10\,mm} = 0.15$

APP. A22-3. $H/h = 16\,mm/10\,mm = 1.60$, $K_t = 1.90$

$F_{ALLOW} = \frac{\sigma_d \cdot A}{K_t} = \frac{(100\,N/mm^2)(60\,mm^2)}{1.90} = 3158\,N$

AT PIN: APP. A22-4: CURVE B. $w = 16\,mm$, $d = 6\,mm$, $\frac{d}{w} = \frac{6}{16} = 0.375$

$K_t = 3.15$. $A = (w-d)t = (16-6)(6) = 60\,mm^2$

$F_{ALLOW} = \frac{\sigma_d \cdot A}{K_t} = \frac{(100\,N/mm^2)(60\,mm^2)}{3.15} = 1967\,N = F_{ALLOW}$

3-126

SPECIFY MATERIAL. $\sigma_{max} = K_t F/A$. $\sigma_d = S_u/12$ SHOCK

AT HOLE: APP A22-5 CURVE A. $d/D = 12/30 = 0.40$. $K_{tg} = 5.0$

$A = \frac{\pi D^2}{4} = \frac{\pi(30\,mm)^2}{4} = 707\,mm^2$ GROSS AREA

LET $\sigma_{MAX} = \sigma_d = \frac{S_u}{12} = \frac{K_t F}{A}$, REQ.D $S_u = \frac{12 K_t F}{A} = \frac{12(5.0)(12.6 \times 10^3 N)}{707\,mm^2}$

REQ'D. $S_u = 1070\,MPa$

AT FILLET: APP A22-2. $r/d = 1.2\,mm/18\,mm = 0.067$. $D/d = 30/18 = 1.67$

$K_t = 2.10$. $A = \pi d^2/4 = \frac{\pi(18\,mm)^2}{4} = 254\,mm^2$

REQ'D. $S_u = \frac{12 K_t F}{A} = \frac{12(2.10)(12.6 \times 10^3 N)}{254\,mm^2} = 1248\,MPa$

STRESS AT FILLET GOVERNS: $S_{u\,min} = 1248\,MPa$ WITH GOOD DUCTILITY

SPECIFY: AISI 4140 OQT 900. $S_u = 1289\,MPa$; 15% ELONGATION

Bearing Stress

3-127

A) W6x15 ON STEEL PLATE: $A_b = 4.43\,in^2$ (APP. A-7)

$\sigma_b = F/A_b = 26000\,lb/4.43\,in^2 = 5869\,psi$

B) STEEL PLATE ON CONCRETE: $A_b = (12)^2 = 144\,in^2$

$\sigma_b = F/A_b = 26000\,lb/144\,in^2 = 181\,psi$

C) CONCRETE PIER ON CONCRETE FOOTING: $A_b = (18)^2 = 324\,in^2$

$\sigma_b = F/A_b = 26000\,lb/324\,in^2 = 80.2\,psi$

D) CONCRETE FOOTING ON SOIL: $A_b = (36)^2 = 1296\,in^2$

$\sigma_b = F/A_b = 26000\,lb/1296\,in^2 = 20.1\,psi$

3-128

a) PIPE ON FLOOR: $A_b = \frac{\pi}{4}(2.375^2 - 2.067^2) = 1.075\,in^2$

$\sigma_b = F/A_b = 2350\,lb/1.075\,in^2 = 2187\,psi$ (APP. A-12)

b) 2.375 IN DIA. ROUND PLATE : $A_b = 2.375^2\left(\frac{\pi}{4}\right) = 4.43\,in^2$

$\sigma_b = F/A_b = 2350\,lb/4.430\,in^2 = 530\,psi$

3-129

a) BOLT HEAD ON WASHER: $A_b = A_{HEX} - A_{SD}$ (SEE APP. A-1)

$A_b = 0.866(0.75)^2 - \pi(0.562)^2/4 = 0.239\ IN^2$

$\sigma_b = F/A_b = 385\ LB/0.239\ IN^2 = \underline{1610\ PSI}$

b) WASHER ON WOOD: $A_b = \frac{\pi}{4}(1.375^2 - 0.562^2) = 1.237\ IN^2$

$\sigma_b = F/A_b = 385\ LB/1.237\ IN^2 = \underline{311\ PSI}$

3-130

DATA FROM PROB. 1-64: F = 8000 LB, L = 2.25 IN, h = 0.375 IN.

$\sigma_b = F/A_b = 8000\ LB/(2.25)(0.375/2)\ IN^2 = \underline{18\,963\ PSI}$

3-131

DATA FROM PROB. 1-65: F = 20 000 LB, FIG. P1-65

a) PIN/TUBE: $A_b = D_p(D-d) = (0.50)(1.25-0.875) = 0.1875\ IN^2$

$\sigma_b = F/A_b = 20\,000\ LB/0.1875\ IN^2 = \underline{106\,700\ PSI}$ (VERY HIGH)

b) COLLAR/TUBE: $A_b = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(1.25^2 - 0.875^2) = 0.626\ IN^2$

$\sigma_b = F/A_b = 20000\ LB/0.626\ IN^2 = \underline{31\,950\ PSI}$

3-132

FROM FIG. P1-70: $A_b = 2(d)(t) = 2(12)(15) = 360\ mm^2$

$\sigma_b = F/A_b = 10.2 \times 10^3\ N/360\ mm^2 = \underline{28.3\ MPa}$

3-133

FROM FIG. P1-71

a) ON MIDDLE PART: $A_b = 2dt_1 = (2)(12)(15) = 360\ mm^2$

$\sigma_b = F/A_b = 10.2 \times 10^3\ N/360\ mm^2 = \underline{28.3\ MPa}$

b) ON OUTER PARTS: $A_b = 4dt_2 = (4)(12)(10) = 480\ mm^2$

$\sigma_b = F/A_b = 10.2 \times 10^3\ N/480\ mm^2 = \underline{21.25\ MPa}$

3-134

FIG. P3-134: $A_b = (10)(6) + \frac{1}{2}(\frac{\pi}{4})(10)^2 = 99.3\ mm^2$

$\sigma_b = F/A_b = 535\ N/99.3\ mm^2 = \underline{5.39\ MPa}$

3-135

W = 90 kN TOTAL; 45 kN ON TWO LEGS; 22.5 kN ON EACH LEG

a) STEEL PLATE: $\sigma_b = \dfrac{22.5 \times 10^3\ N}{(0.10\ m)^2} = 2.25\ MPa$

(EQ. 3-22)

FOR A36 STEEL: $\sigma_{bd} = 0.9 S_y = 0.9(248\ MPa) = 223\ MPa$ $\underline{ok}$

b) TOP OF CONCRETE: $A_b = 2(0.20\ m)^2 = 0.080\ m^2$ (TWO LEGS)

$\sigma_b = \dfrac{45 \times 10^3\ N}{0.08\ m^2} = 0.563\ MPa$

(CONTINUED)

3-135 (CONTINUED)

CONCRETE: 2000 PSI = 2.0 KSI × 6.895 MPa/KSI = 13.79 MPa = f_c'

(TABLE 3-6) — $\sigma_{bd} = 0.34 f_c' \sqrt{A_2/A_1} = 0.34(13.79\,MPa)\sqrt{\frac{0.30\,m^2}{0.08\,m^2}} = 9.08\,MPa$ OK

C) SOIL: $\sigma_b = \frac{45 \times 10^3\,N}{(.8)(2.5)\,m^2} = 22.5\,kPa$

ON COMPACT GRAVEL $\sigma_{bd} = 380\,kPa$ OK (TABLE 3-7)

3-136 ON SOFT ROCK: $\sigma_{bd} = 480\,kPa = 480 \times 10^3\,N/m^2$ (TABLE 3-7)

REQ'D. $A = \frac{P}{\sigma_{bd}} = \frac{160 \times 10^3\,N}{480 \times 10^3\,N/m^2} = 0.333\,m^2 = S^2$

REQ'D. SIDE DIMENSION: S = 0.577 m

3-137 $R_a = (S_y - 13)(0.03\,d\,L) = (36-13)(0.03)(3.00)(16.0) = 33.1$ KIPS

(EQ. 3-23)

3-138 $R_a = (46-13)(0.03)(3.00)(16.0) = 47.5$ KIPS

3-139 a) $R_a = (36-13)(0.03)(5.00)(8.00) = 27.6$ KIPS

b) $R_a = (46-13)(0.03)(5.00)(8.00) = 39.6$ KIPS

3-140 LOAD ON EACH FOOT = F = 10 000/4 = 2500 LB

$A_b = 1.51\,IN^2$ (TABLE A-9 2×2×1/4)

$\sigma_b = F/A_b = 2500\,LB / 1.51\,IN^2 = 1656$ PSI

REQ'D $A_b = F/\sigma_{bd} = 2500/400 = 6.25\,IN^2$. USE SQUARE PLATE

SIDE = $\sqrt{A_b} = \sqrt{6.25\,IN^2} = 2.50$ IN

3-141 ALLOWABLE REACTION = R_a FROM EQ. 3-23.

$R_a = (S_y - 13)(0.03)(d)(L)$ U.S. CUSTOMARY UNITS

$S_y = 36$ KSI

d = 2(200 mm)(1.0 IN/25.4 mm) = 15.75 IN.

L = (150 mm)(1.0 IN/25.4 mm) = 5.91 IN.

$R_a = (36-13)(0.03)(15.75)(5.91) = 64.2$ KIPS

$R_a = (64.2\,KIPS)(4.448\,kN/KIP) = 285\,kN$

OR – USING EQ 3-24 FOR SI UNITS

d = 2h = 2(200 mm) = 400 mm, L = 150 mm

$S_y = 248$ MPa

$R_a = (248 - 90)(3.0 \times 10^{-5})(400)(150) = 284\,kN$

3-142 $\sigma_b = \dfrac{F_b}{A_b}$. $F_b = 28\,500\ lb/4\ legs = 7125\ lb/leg$. $A = 1.44\ in^2$

$\sigma_b = 7125\ lb/1.44\ in^2 = \underline{4948\ psi = \sigma_b}$

3-143 CONCRETE FLOOR. $f'_c = 3000\ psi$. $\sigma_{bd} = 0.34\, f'_c \sqrt{A_2/A_1}$

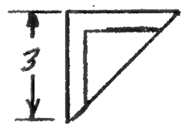
BUT $\sigma_{bd\,max} = 0.68\, f'_c$ BECAUSE $A_2/A_1 \gg 4$. $F_b = 7125\ lb$

$\sigma_{bd} = 0.68(3000) = 2040\ psi = F_b/A_b$. REQ'D $A_b = F_b/\sigma_{bd}$

$A_{b\,min} = 7125\ lb / 2040\ lb/in^2 = 3.49\ in^2$

TRIANGULAR AREA: $A = \frac{1}{2}(3)(3) = 4.50\ in^2$

WELD PAD TO BOTTOM OF EACH LEG.



3-144 $A_b = (3.50)(7.50) = 26.25\ in^2$, $F_b = \sigma_{bd} A_b$. $\sigma_{bd} = 55\ psi$

$F_b = (55\ lb/in^2)(26.25\ in^2) = \underline{1444\ lb = F_b\ \text{ALLOWABLE}}$

3-145 4-IN SCH. 40 PIPE. $A_b = 3.174\ in^2$. $\sigma_{bd} = 0.68\, f'_c$ ON LARGE FLOOR

$\sigma_{bd} = 0.68\ (4000\ psi) = 2720\ psi$

$F_b = \sigma_{bd} \cdot A_b = (2720\ lb/in^2)(3.174\ in^2) = \underline{8633\ lb = F_b\ \text{ALLOWABLE}}$

3-146 DATA FROM PROB 3-145 $\sigma_{bd} = 2720\ psi = F_b/A_b$

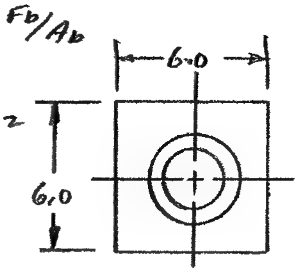
$F_b = 10(8633\ lb) = 86{,}330\ lb$

REQ'D. $A_b = F_b/\sigma_{bd} = \dfrac{86330\ lb}{2720\ lb/in^2} = 31.74\ in^2$

TRY SQUARE PLATE: $A_b = b^2$.

$b_{min} = \sqrt{A_{b\,min}} = \sqrt{31.74\ in^2} = \underline{5.63\ in}$

USE $\underline{b = 6.0\ \text{IN SQUARE PLATE.}}$



Direct Shear Stress

3-147 $\tau = F/A_s = F/(8in)(a)$; LET $\tau = \tau_d = 6000\ psi$

REQ'D $a = \dfrac{F}{(8in)(\tau_d)} = \dfrac{21000\ LB}{(8\ IN)(6000\ LB/IN^2)} = \underline{0.438\ IN}$

3-148 $\tau_d = 0.5\, s_y/6 = (0.5)(565\ MPa)/6 = 47.1\ MPa$

$\tau = F/A_s = P/2[\pi D^2/4]$

$F_{allow} = \tau_d \cdot A_s = 47.1\ \dfrac{N}{mm^2} \cdot \dfrac{2\pi(16\ mm)^2}{4} = \underline{18.9\ kN}$

3-149 FROM PROB. 1-59: $F = 23\,695\ N$; $\tau = 151\ MPa$ ON PIN.
LET $\tau = \tau_d = \frac{S_y}{8} = 151\ MPa$ $\therefore$ REQ'D. $S_y = 8\tau = 8(151) = 1208\ MPa$

POSSIBLE STEEL: AISI 4140 OQT 700, $S_y = 1462\ MPa$; 12% ELONG.

3-150 $A_s = (\pi D)(t) = \pi(20)(8)\ mm^2 = 503\ mm^2$
$\tau = S_{us} \cong 0.82\ S_u = 0.82(448\ MPa) = 367\ MPa = 367\ N/mm^2$
REQ'D. $F = \tau \cdot A_s = \frac{367\ N}{mm^2} \cdot 503\ mm^2 = \underline{185\ kN}$

3-151 $\tau = S_{us} = 165\ MPa$ - GIVEN IN APP. A17
REQ'D. $F = \tau \cdot A_s = \frac{165\ N}{mm^2} \cdot 503\ mm^2 = \underline{83\ kN}$

3-152 $\tau = S_{us} \cong 0.90\ S_u = 0.90(331\ MPa) = 298\ MPa$ COPPER
REQ'D. $F = \tau \cdot A_s = \frac{298\ N}{mm^2} \cdot 503\ mm^2 = \underline{150\ kN}$

3-153 $\tau = S_{us} \cong 0.82\ S_u = 0.82(621\ MPa) = 509\ MPa$
REQ'D. $F = \tau \cdot A_s = \frac{509\ N}{mm^2} \cdot 503\ mm^2 = \underline{256\ kN}$

3-154 $F = \tau \cdot A_s$: LET $\tau = S_{us} \cong 0.82\ S_u = 0.82(448\ MPa) = 367\ MPa$
$A_s = [2(35\ mm) + \pi(8\ mm)][5.0\ mm] = 476\ mm^2$
$F = (367\ N/mm^2)(476\ mm^2) = \underline{175\ kN}$

3-155 $F = \tau \cdot A_s$: LET $\tau = S_{us} = 16\ ksi$ (FROM APP. A-18)
$A_s = 1.144\ in^2$ (SEE PROB. 1-62)
$F = (16\,000\ lb/in^2)(1.144\ in^2) = \underline{18\,300\ lb}$

3-156 $A_s = (3.5\ in)(3.0\ in) = 10.5\ in^2$
$\tau = \frac{F}{A_s} = \frac{1800\ lb}{10.5\ in^2} = 171\ psi$ (UNSAFE)
MAXIMUM ALLOWABLE SHEAR STRESS LISTED IN TABLE A-19 IS $\underline{95\ psi}$.

3-157 $F = \tau \cdot A_s$: LET $\tau = S_{us} \cong 0.82\ S_u = 0.82(97\,000\ psi) = 79\,540\ psi$
$F = \tau \cdot A_s = (79\,540\ lb/in^2)(7.5\ in \cdot 0.105\ in) = \underline{62\,650\ lb}$

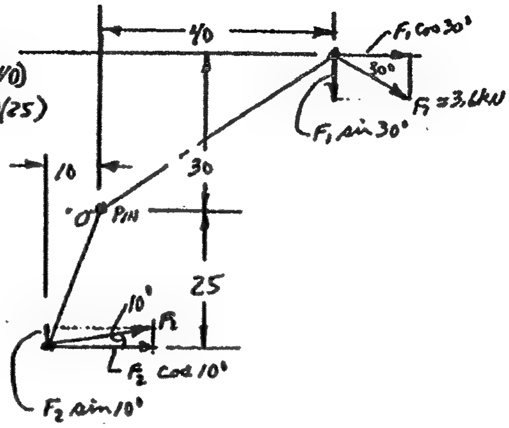
3-158 LET $\tau = S_{us} \cong 0.82(S_u) = 0.82)(263\,000\ psi) = 215\,660\ psi$
$F = \tau \cdot A_s = (215\,660\ lb/in^2)(7.5 \cdot 0.105)\ in^2 = \underline{169\,800\ lb}$

3-159 LET $\tau = S_{us} \cong 0.82\ S_u = 0.82(185\,000\ psi) = 151\,700\ psi$
$F = \tau \cdot A_s = (151\,700\ lb/in^2)(7.5 \cdot 0.105)\ in^2 = \underline{119\,500\ lb}$

3-160 C36000 BRASS: LET $\tau = S_{us} \cong 0.9(70\,000\ psi) = 63\,000\ psi$
$F = \tau \cdot A_s = (63\,000\ lb/in^2)(7.5 \cdot 0.105)\ in^2 = \underline{49{,}600\ lb}$

3-161 ALUM. 5154-H32: LET $\tau = S_{us} = 152\,000\ psi$
$F = \tau \cdot A_s = (152\,000\ lb/in^2)(7.5 \cdot 0.105)\ in^2 = \underline{119\,700\ lb}$

3-162



1. FIND F_2:

$$\Sigma M_0 = 0 = (F_1 \cos 30)(30) + (F_1 \sin 30)(40) + (F_2 \sin 10°)(10) - (F_2 \cos 10°)(25)$$

$$0 = 165.5 - 22.88\,F_2$$

$$F_2 = 7.23\,kN$$

2. FIND RESULTANT F ON PIN

$$R_p = \left[(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2\right]^{1/2}$$

$$R_p = \left[(3.12 + 7.12)^2 + (1.8 - 1.26)^2\right]^{1/2}$$

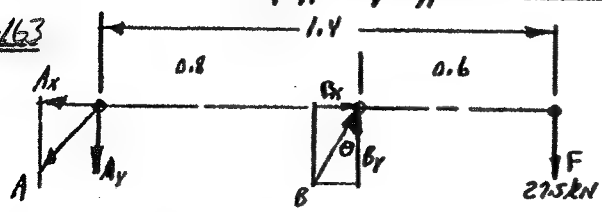
$$R_p = 10.7\,kN$$

3. DESIGN SHEAR STRESS: $\tau_d = \dfrac{S_{ys}}{4} \cong \dfrac{S_y}{8} = \dfrac{1000\,MPa}{8} = 125\,MPa$

4. REQ'D $A_s = \dfrac{R}{\tau_d} = \dfrac{10.7 \times 10^3\,N}{125\,N/mm^2} = 85.6\,mm^2 = \dfrac{\pi D^2}{4}$

5. REQ'D. $D = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{4(85.6)}{\pi}} = 10.4\,mm$

3-163



$$\Sigma M_A = 0 = 27.5(1.4) - B_y(0.8)$$

$$B_y = 48.125\,kN$$

$$A_y = B_y - F = 20.625\,kN$$

$$B = B_y/\cos\theta = 54.5\,kN$$

$$B_x = B \sin\theta = 25.7\,kN$$

$$A_x = B_x = 25.7\,kN$$

$$A = \sqrt{A_x^2 + A_y^2} = 32.95\,kN$$

$$C = B = 54.5\,kN$$

$$\tan\theta = 0.8/1.50$$

$$\theta = 28.1°$$

$$\tau_d = \frac{0.5\,S_y}{2} = \frac{0.5(441\,MPa)}{2} = 110\,MPa$$

PIN A: REQ'D. $A_s = \dfrac{A}{\tau_d} = \dfrac{32.95 \times 10^3\,N}{110\,N/mm^2} = 299\,mm^2 = \left[\dfrac{\pi D_A^2}{4}\right]2 = \dfrac{\pi D_A^2}{2}$

REQ'D. $D_A = \sqrt{2A/\pi} = \sqrt{2(299)/\pi} = 13.8\,mm$

PINS B AND C: REQ'D. $A_s = \dfrac{54.5 \times 10^3\,N}{110\,N/mm^2} = 494\,mm^2 = \dfrac{\pi D_B^2}{2}$

REQ'D. $D_B = D_C = \sqrt{2A_s/\pi} = \sqrt{2(494)/\pi} = 17.7\,mm$

3-164 FORCES ON JOINTS FOUND FROM PROBLEM 1-51.

JOINT A: $F_s = AD = 10.5\,kN$ = JOINT C

JOINT B: $F_s = \sqrt{10.5^2 + 9.09^2} = 13.9\,kN$ = JOINT D

$$\tau_d = \frac{0.5\,S_y}{2} = \frac{(0.5)(345)}{2} = 86.3\,MPa$$

REQ'D. $A_s = \dfrac{F_s}{\tau_d} = \dfrac{10.5 \times 10^3\,N}{86.3\,N/mm^2} = 121.7\,mm^2 = \dfrac{\pi D^2}{4}$

$D = \sqrt{4A_s/\pi} = 12.4\,mm$ FOR JOINTS A AND C

REQ'D. $A_s = \dfrac{F_s}{\tau_d} = \dfrac{13.9 \times 10^3\,N}{86.3\,N/mm^2} = 161.1\,mm^2$

$D = \sqrt{4A_s/\pi} = 14.3\,mm$ FOR JOINTS B AND D.

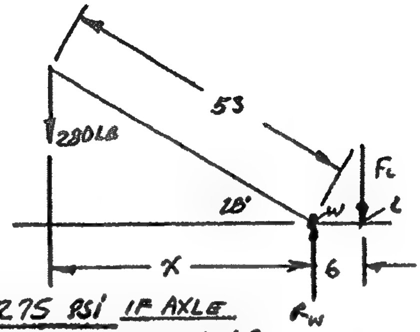
3-165 $x = 53\cos 28^\circ = 46.8\ \text{IN.}$

$\Sigma M_W = 0 = 280(46.8) - F_L(6)$

$F_L = 2184\ \text{LB}$

$\Sigma M_L = 0 = 280(52.8) - R_W(6)$

$R_W = 2464\ \text{LB}$



SHEAR STRESS ON AXLE:

$\tau = \dfrac{R_W}{A_s} = \dfrac{2464\ \text{LB}}{2(\pi(0.5)^2/4)\ \text{IN}^2} = \underline{6275\ \text{PSI}}$ IF AXLE IS IN DOUBLE SHEAR

3-166 $F = \tau \cdot A = S_{us} \cdot A$, ALUM. 3003-H12. $S_{us} = 83\ \text{MPa}$

$A = p \cdot t$, $t = 1.40\ \text{mm}$.

$p = 2(90) + 2(18) + 3[\pi(6)] + 2[2(12) + \pi(6)] = 358\ \text{mm}$

$A = p \cdot t = (358)(1.40) = 502\ \text{mm}^2$

$F = S_{us} \cdot A = (83\ \text{N/mm}^2)(502\ \text{mm}^2) = \underline{41.6\ \text{kN} = F}$

3-167 IMPACT: $\tau_d = \dfrac{S_{ys}}{12} = \dfrac{90\ \text{KSI}}{12} = 7.50\ \text{KSI} = 7500\ \text{PSI}$

$A_s = 2\pi D^2/4 = \pi D^2/2$

REQ'D. $A_s = F/\tau_d = \dfrac{500\ \text{LB}}{7500\ \text{LB/IN}^2} = 0.0667\ \text{IN}^2 = \pi D^2/2$

REQ'D. $D = \sqrt{\dfrac{2A_s}{\pi}} = \sqrt{\dfrac{2(0.0667)}{\pi}} = \underline{0.206\ \text{IN}}$; SPECIFY $\underline{D = 0.250\ \text{IN.}}$

3-168 $F_{\text{ALLOW.}}$ ON EACH BOLT: $F = A_s\,\tau_d = \dfrac{\pi(1.25)^2\ \text{IN}^2}{4} \cdot \dfrac{6000\ \text{LB}}{\text{IN}^2} = 7363\ \text{LB}$

TORQUE $= F \cdot \text{RADIUS} = \dfrac{7363\ \text{LB}}{\text{BOLT}} \cdot 8\ \text{BOLTS} \cdot 4.50\ \text{IN} = \underline{2.65 \times 10^5\ \text{LB-IN}}$

3-169 COMPUTE FORCE REQUIRED TO PUNCH OUT THE SHAPE IN FIGURE P3-169 $\tau = F/A$; $F = \tau A$; LET $\tau = S_{us}$

AISI 1020 CD, $S_u = 75\ \text{ksi}$. $S_{us} = 0.82 S_u = 0.82(75) = 61.5\ \text{ksi}$

A = SHEAR AREA = PERIMETER × THICKNESS $= p \cdot t$; $t = 0.085\ \text{IN}$

$p = 1.0 + 1.0 + 2.0 + \sqrt{1^2 + 1^2} + 0.50 + \pi(0.5)/2 = 6.70\ \text{IN}$

$A = p \cdot t = (6.70)(0.085) = 0.569\ \text{IN}^2$.

$F = S_{us} \cdot A = (61.5 \times 10^3\ \text{lb/in}^2)(0.569\ \text{in}^2) = \underline{35\,020\ \text{lb} = F}$

3-170 $F = \tau A = S_{us} \cdot A$. 6061-T4. $S_{us} = 24\ \text{ksi}$ – APP. A-17. $t = 0.10\ \text{IN}$

$p = 4(1.25) + 3(0.5) + \pi(1.50)/2 = 8.86\ \text{IN}^2$; $A = p \cdot t = 0.886\ \text{IN}^2$

$F = S_{us} \cdot A = (24 \times 10^3\ \text{lb/in}^2)(0.886\ \text{in}^2) = \underline{21\,260\ \text{lb} = F}$

3-171 $F = \tau \cdot A = S_{us} \cdot A$. ALUM. 3003-H18 $S_{us} = 110\ \text{MPa}$. $A = p \cdot t$, $t = 3.0\ \text{mm}$

$p = 30 + 60 + \sqrt{20^2 + 40^2} + \sqrt{10^2 + 40^2} = 176\ \text{mm}$. $A = 527.9\ \text{mm}^2$

$F = S_{us} \cdot A = (110\ \text{N/mm}^2)(527.9\ \text{mm}^2) = \underline{58.06\ \text{kN} = F}$

3-172 $F = \tau \cdot A = S_{us} \cdot A$. AISI 1040 CD. $S_u = 669$ MPa. $S_{us} = .82 S_u = 549$ MPa

$A = p \cdot t$, $t = 1.60$ mm. $p = 50 + 30 + 2(20) + \pi(20)/2 = 151.4$ mm

$A = (151.4\text{ mm})(1.60\text{ mm}) = 242\text{ mm}^2$. $\underline{F} = \underline{549\frac{N}{mm^2}} \cdot 242\text{ mm}^2 = \underline{133\text{ kN}}$

3-173 $F = \tau \cdot A = S_{us} \cdot A$. AISI 1080 OQT 900. $S_u = 1234$ MPa

$S_{us} = 0.82 S_u = 0.82(1234\text{ MPa}) = \underline{1012\text{ MPa}}$

$A = p \cdot t$. $t = 0.80$ mm. $p = 60 + 2\sqrt{15^2 + 15^2} + 22 + \pi(4) = 137$ mm

$A = (137\text{ mm})(0.80\text{ mm}) = 109.6\text{ mm}^2$. $F = (1012\text{ N/mm}^2)(109.6\text{ mm}^2)$

$\underline{F = 110.9\text{ kN}}$

3-174 $F = \tau \cdot A = S_{us} \cdot A$ - ALUM 5154-H38. $S_{us} = 193$ MPa

$A = p \cdot t$. $t = 2.00$ mm. $p_{OUTSIDE} = 2(128) + 2(50) = 356$ mm

$p_{INSIDE} = \pi(15) + 2[2(15) + 2(10)] = 147.1$ mm

TOTAL $p = p_o + p_i = 503.1$ mm. $A = p \cdot t = (503.1)(2.0) = 1006\text{ mm}^2$

$F = S_{us} \cdot A = (193\text{ N/mm}^2)(1006\text{ mm}^2) = \underline{194.2\text{ kN} = F}$

3-175 DATA FROM PROB. 3-174

<u>FIRST STEP:</u> $p = p_{INSIDE} = 147.1$ mm. $A_1 = p \cdot t = (147.1)(2.0) = 294.2\text{ mm}^2$

$F_1 = S_{us} \cdot A_1 = (193\text{ N/mm}^2)(294.2\text{ mm}^2) = \underline{56.8\text{ kN} = F_1}$

<u>2ND STEP:</u> $p = p_{OUTSIDE} = 356$ mm. $A_2 = (356)(2.0) = 712\text{ mm}^2$

$F_2 = S_{us} \cdot A_2 = (193\text{ N/mm}^2)(712\text{ mm}^2) = \underline{137.4\text{ kN} = F_2} = \underline{\text{ANSWER}}$

Problems with More Than One Kind of Direct Stress and Design Problems

3-176 $T = 5000$ LB-IN; FORCE ON SIDE OF KEY $= F = T/R = 5000/1.125 = 4444$ LB.

SHEAR: $\tau = F/A_s = F/bL$

LET $\tau = \tau_d = \dfrac{0.5S_y}{2} = \dfrac{(0.5)(64000\,PSI)}{2} = 16000\,PSI$

REQ'D. $L = \dfrac{F}{b\,\tau_d} = \dfrac{4444\,LB}{(0.50\,IN)(16000\,LB/IN^2)} = \underline{0.556\,IN}$

BEARING: $\sigma = \dfrac{F}{A_b} = \dfrac{F}{(h/2)(L)}$

LET $\sigma = \sigma_{bd} = 0.9S_y = 0.9(64000\,PSI) = 57600\,PSI$

REQ'D. $L = \dfrac{F}{(h/2)\,\sigma_{bd}} = \dfrac{4444}{(0.50/2)(57600\,LB/IN^2)} = \underline{0.309\,IN.}$

3-177 SHEAR: TWO PINS, EACH IN DOUBLE SHEAR; $A_s = 4[\pi D^2/4] = \pi D^2$

$\tau = F/A_s$: FOR STATIC LOAD, $\tau = \tau_d = 0.5S_y/2 = \dfrac{0.5(82000)}{2} = 20500\,PSI$

REQ'D. $A_s = F/\tau_d = 42000\,LB/20500\,LB/IN^2 = 2.049\,IN^2 = \pi D^2$

REQ'D. $D = \sqrt{A_s/\pi} = \sqrt{2.049\,IN^2/\pi} = \underline{0.808\,IN.}$

SPECIFY $D = 1.00$ IN.

CHECK BEARING: $\sigma_{bd} = (0.9)(S_y) = 0.9(82000\,PSI) = 73800\,PSI$

$\sigma_b = \dfrac{F}{A_b} = \dfrac{F}{(D)(2L)}$

REQ'D. $L = \dfrac{F}{(D)(2)(\sigma_{bd})} = \dfrac{42000\,LB}{(1.0\,IN)(2)(73800\,LB/IN^2)} = \underline{0.285\,IN}$ $\underline{\text{VERY SMALL}}$

3-178

FORCE $= 20\,kN = 12.5\,kN\,(4.0\,m/2.5\,m)$

a) TENSION IN MEMBER 1 AT PIN HOLES:

$A_t = 2[(20-12)(14)] = 224\,mm^2$

$\sigma = \dfrac{P}{A} = \dfrac{20\times10^3\,N}{224\,mm^2} = \underline{89.3\,MPa \quad \text{TOO HIGH}}$

FOR 6061-T4: $\sigma_d = S_y/2 = 145\,MPa/2 = 72.5\,MPa$ (A-18)

b) BEARING AT THE PIN — MEMBER 1

$\sigma_b = \dfrac{P}{A_b} = \dfrac{20\times10^3\,N}{(12)(14)(2)\,mm^2} = 59.5\,MPa$

FOR 6061-T4; $\sigma_{bd} = 0.65\,S_y = 0.65(145\,MPa) = 94.3\,MPa$ $\underline{OK}$

c) BEARING AT THE PIN — MEMBER 2

$\sigma_b = \dfrac{P}{A_b} = \dfrac{20\times10^3\,N}{(12)(20)\,mm^2} = 83.3\,MPa$

FOR 2014-T4; $\sigma_{bd} = 0.65\,S_y = 0.65(290\,MPa) = 189\,MPa$ $\underline{OK}$ (EQ. 3-12)

d) PIN IN BEARING: - MEMBER 3

$\sigma_{b_{MAX}} = 83.3\,MPa$ AT MEMBER 2

FOR 2014-T6; $\sigma_d = 0.65\,S_y = 0.65(414\,MPa) = 269\,MPa$ $\underline{OK}$

e) PIN IN SHEAR — DOUBLE SHEAR

$A_s = \dfrac{\pi}{4}(12)^2(2) = 226\,mm^2$

$\tau = P/A_s = 20\times10^3\,N/226\,mm^2 = 88.4\,MPa$

FOR 2014-T6; $\tau_d = \dfrac{0.5S_y}{2} = \dfrac{0.5(414\,MPa)}{2} = 103\,MPa$ $\underline{OK}$

3-179

a) SHEAR OF PIN: $\tau = F/A_s$; LET $\tau = \tau_d = \frac{S_{us}}{8} = \frac{0.82 S_u}{8} = \frac{0.82(97)}{8} = 9.94\ \text{KSI}$

$F_{ALLOW} = \tau_d \cdot A_s = 9940 \frac{LB}{IN^2} \cdot \frac{2\pi(0.63)^2}{4} IN^2 = \underline{6260\ LB}$

b) BEARING: $\sigma_b = F/A_b$; LET $\sigma_b = \sigma_{bd} = 0.9 S_y = 0.9(82) = 73.8\ \text{KSI}$

$F_{ALLOW} = \sigma_{bd} \cdot A_b = 73800 \frac{LB}{IN^2} \cdot (0.63)(2)(0.38) IN^2 = \underline{35{,}340\ LB}$

c) TENSION: $\sigma = \frac{F}{A_t} K_t$; LET $\sigma = \sigma_d = \frac{S_u}{8} = \frac{97}{8} = 12.125\ \text{KSI}$

$A_t = (1.50 - 0.63)(2)(0.38) = 0.661\ IN^2$

K_t FROM APP. A-22-4 CURVE B; $d/w = 0.63/1.50 = 0.42$; $K_t = 2.83$

$F_{ALLOW} = \frac{\sigma_d \cdot A_t}{K_t} = \frac{(12125\ LB/IN^2)(0.661\ IN^2)}{2.83} = \underline{2832\ LB}$

3-180

FIG. P3-180 RIVETED PLATES. ASSUME STATIC LOAD

6061-T6 PLATES: $S_y = 40\ KSI$, $S_u = 45\ KSI$, 17% ELONGATION

2014-T4 RIVETS: $S_{us} = 38\ KSI$, $S_y = 42\ KSI$

a) SHEAR OF RIVETS: $\tau_d = S_{us}/4 = 38\ KSI/4 = 9.50\ KSI = F/A_s$

$A_s = 2(\pi(0.5\ IN)^2/4) = 0.393\ IN^2$ TWO CROSS SECTIONS

$F = \tau_d \cdot A_s = (9.5 \times 10^3 \frac{lb}{IN^2})(0.393\ IN^2) = \underline{3730\ lb}$

b) TENSILE STRESS ON PLATE: $\sigma_d = S_y/3 = 40\ KSI/3 = 13.3\ KSI$

$\sigma = F/A$, $A = [3.0\ IN - 2(0.5\ IN)]\ 0.375\ IN = 0.75\ IN^2$

$F = \sigma_d \cdot A = (13\,333 \frac{lb}{IN^2})(0.75\ IN^2) = \underline{10000\ lb}$

c) BEARING AT RIVETS/HOLES: $\sigma_b = \frac{F}{A_b} = \sigma_{bd} = 0.65\ S_y$

$\sigma_{bd} = 0.65(40\ KSI) = 26\ KSI$ ON PLATE

$A_b = 2(D)(t) = (0.50)(.375)(2) = 0.375\ IN^2$ - PROJECTED AREA

$F = \sigma_{bd} \cdot A_b = (26\,000 \frac{lb}{IN^2})(0.375\ IN^2) = \underline{9750\ lb}$

SHEAR STRESS GOVERNS: $\underline{F_{ALLOW} = 3730\ lb}$

3-181

DATA FROM PROBLEM 3-180; USE FIGURE P3-181

a) SHEAR OF RIVETS: $\tau_d = 9500\ lb/IN^2 = F/A_s$

$A_s = 3(\pi(0.375)^2/4) = 0.3313\ IN^2$

$F = \tau_d \cdot A_s = (9500\ lb/IN^2)(0.3313\ IN^2) = \underline{3148\ lb}$

b) TENSILE STRESS IN PLATE: $\sigma_d = 13\,333\ PSI = F/A$

$A = [3.0 - 3(0.375)](0.375) = 0.703\ IN^2$

$F = \sigma_d \cdot A = (13{,}333\ lb/IN^2)(0.703\ IN^2) = \underline{9375\ lb}$

c) BEARING: $\sigma_{bd} = 26\,000\ lb/IN^2$ ON PLATE

$A_b = 3[D \cdot t] = 3[(0.375)(0.375)] = 0.422\ IN^2$

$F = \sigma_{bd} \cdot A_b = (26\,000\ lb/IN^2)(0.422\ IN^2) = \underline{10{,}969\ lb}$

SHEAR STRESS GOVERNS: $\underline{F_{ALLOW} = 3148\ lb}$

3-182 DATA FROM PROBLEM 3-180

a) SHEAR OF RIVETS: $\tau_d = 9500\ lb/in^2 = F/A_s$

$A_s = 4[\pi(0.375)^2/4] = 0.4418\ in^2$

$F = \tau_d \cdot A_s = (9500\ lb/in^2)(0.4418\ in^2) = \underline{4197\ lb}$

b) TENSION ON PLATE: $\sigma_d = 13\,333\ lb/in^2 = F/A$

$A = [3.0 - 2(0.375)]\,0.375 = 0.844\ in^2$

$F = \sigma_d \cdot A = (13\,333\ lb/in^2)(0.844\ in^2) = \underline{11\,250\ lb}$

c) BEARING AT RIVETS:

ON PLATES: $\sigma_{bd} = 0.65(40\ ksi) = 26\ ksi$

$A_b = 2(0.25)(0.375)(2) = 0.375\ in^2$

$F_b = \sigma_{bd} \cdot A_b = (26000\ lb/in^2)(0.375\ in^2) = \underline{9750\ lb}$

ON RIVETS: $\sigma_{bd} = 0.65(42\ ksi) = 27.3\ ksi$

$A_b = 2(0.375)(0.375) = 0.281\ in^2$

$F_b = (27300\ lb/in^2)(0.281\ in^2) = \underline{7678\ lb}$

SHEAR OF RIVETS GOVERNS: $\underline{F_{ALLOW} = 4197\ lb}$

3-183 REPEATED FORCE.

AISI 4140 OQT 1100

TENSION IN LINK A: $\sigma_d = s_u/8 = 147\ ksi/8 = 18.375\ ksi$

$\sigma = \dfrac{K_t F}{A_{NET}}$; $F = \dfrac{\sigma_d A_{NET}}{K_t} = \dfrac{(18375\ lb/in^2)(1.5-0.75)(1.25)\ in^2}{2.60}$

APP A22-4, CURVE B: $d/w = 0.75/1.5 = 0.50$, $K_t = 2.60$

$F_{ALLOW} = \underline{6626\ lb}$

AISI 1141 OQT 1100

SHEAR STRESS IN PIN: $\tau_d = s_y/8 = 97\ ksi/8 = 12.125\ ksi$

$\tau = \dfrac{F}{A_s}$: $F = \tau_d \cdot A_s = \left(12125\ \dfrac{lb}{in^2}\right)\left(2[\pi(0.75)^2/4]\right) = \underline{10{,}713\ lb}$

BEARING STRESS AT PIN: $\sigma_{bd} = 0.90\,s_y = (0.90)(97\ ksi) = 87.3\ ksi$

$A_b = (1.25\ in)(0.75\ in) = 0.9375\ in^2$

$\sigma_b = F/A_b$: $F = \sigma_{bd} \cdot A_b = (87300\ lb/in^2)(0.9375\ in^2) = \underline{81843\ lb}$

TENSION IN LINK GOVERNS: $\underline{F_{ALLOW} = 6626\ lb}$

3-184 FORCES IN MEMBERS: AB = 2465 LB (T), AC = 1925 LB (C)
BC = 1375 LB (T), BD = 1200 LB (T), CE = 650 LB (C),
CD = 750 LB (C), DE = 961 LB (T).

SUPPORT FORCES: A_y = 1540 LB ↓, B_y = 2640 LB ↑, B_x = 100 LB ←

MATERIAL: ASTM A36 STRUCTURAL STEEL, S_y = 36 KSI
ASSUME STATIC LOAD. $\sigma_d = S_y/2 = 18$ KSI

REQ'D AREA: $A_{MIN} = F/\sigma_d$

MEMBER	F (LB)	σ_d (KSI)	A_{MIN} (IN^2)	SQUARE b_{MIN}	ROUND D_{MIN}	THREAD
AB	2465	18	0.137	.374	.418	1/2-13
BC	1375	18	0.0764	.276	.312	3/8-16
BD	1200	18	0.0667	.258	.291	3/8-16
DE	961	18	0.0534	.231	.261	3/8-16

ALTERNATIVE DESIGNS:

SQUARE ROD: $A = b^2$; $b_{MIN} = \sqrt{A}$
ROUND ROD: $A = \pi D^2/4$; $D_{MIN} = \sqrt{4A/\pi}$
THREADED ROD: LET A_{MIN} < TENSILE STRESS AREA OF THREAD FROM APP A-3, COARSE THREADS

FOR THREADED ROD, ATTACH TO CLEVIS AT EACH END. DESIGN PIN FOR CLEVIS FOR SAFE SHEAR STRESS.
AT B: PIN JOINT ATTACHED TO FRAME
AT A: PROVIDE ROLLER ON PIN THAT BEARS ON FRAME.
AT C AND E: PROVIDE AN ADDITIONAL CLEVIS FROM WHICH TO ATTACH LOADS.
NOTE: COMPRESSION MEMBERS MUST BE DESIGNED WITH COLUMN BUCKLING ANALYSIS. SEE CHAPTER 11.

3-185 FORCES IN MEMBERS:

AB = 4687 N (T)	BE = 0	BF = 2241 N (T)	CG = 1097 N (T)
AD = 1400 N (T)	DE = 2300 N (C)	CF = 800 N (C)	FG = 500 N (C)
BD = 2597 N (C)	BC = 750 N (T)	EF = 2300 N (C)	

SUPPORT FORCES: A_y = 1400 N ↑, A_x = 4683 N ←, D_x = 4488 N →

DESIGNS COULD BE SIMILAR TO PROBLEM 3-184
FORCES ARE GENERALLY SMALLER. SMALL WIRES MAY BE USED FOR TENSION MEMBERS. COMPRESSION MEMBERS MUST BE DESIGNED FOR BUCKLING. SEE CHAPTER 11.

3-186

FORCE ANALYSIS:

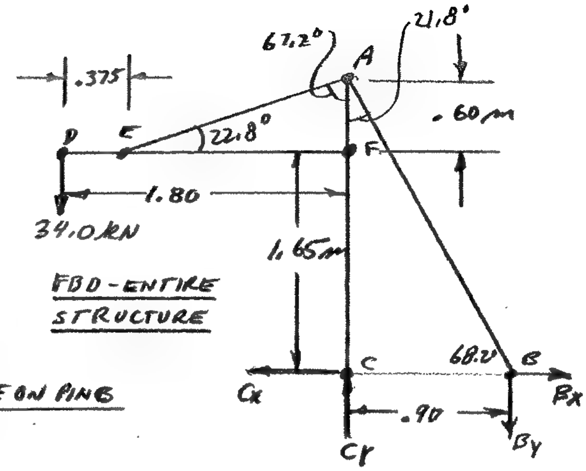
USING FBD OF ENTIRE STRUCTURE:

$\Sigma M_c = 0 = (34.0\ kN)(1.8\ m) - B_y(.90)$

$B_y = (34)(1.8)/.9 = 68.0\ kN \downarrow$

$\Sigma F_y = 0 = 34.0 + 68.0 - C_y$

$C_y = 102\ kN \uparrow$



FBD - ENTIRE STRUCTURE

AB IS A TWO-FORCE MEMBER

$AB_y = B_y = 68.0\ kN = AB\cos 21.8°$

$AB = 68/\cos 21.8 = \underline{73.2\ kN\ \text{TENSION}}$

$AB_x = AB\sin 21.8° = 27.2\ kN$ └ = FORCE ON PINS

$B_x = AB_x = 27.2\ kN$

$\Sigma F_x = 0 = B_x - C_x$

$C_x = B_x = 27.2\ kN$

FORCE ON PIN C $= \sqrt{C_x^2 + C_y^2} = \sqrt{27.2^2 + 102^2} = \underline{105.6\ kN}$

FBD OF BOOM:

$\Sigma M_F = 0 = (34.0\ kN)(1.80\ m) - AE_y(1.425)$

$AE_y = (34)(1.8)/1.425 = 42.95\ kN$

$AE = AE_y/\sin 22.8° = \underline{110.8\ kN\ \text{TENSION}}$

$AE_x = AE\cos 22.8° = 102.2\ kN$

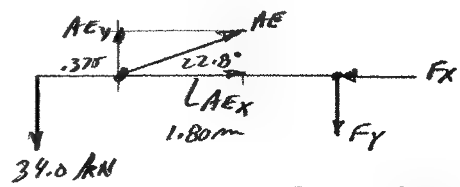
$\Sigma F_x = 0 = AE_x - F_x$

$F_x = AE_x = 102.2\ kN$

$\Sigma M_E = 0 = (34.0\ kN)(.375) - F_y(1.425)$

$F_y = (34)(.375)/1.425 = 8.95\ kN$

FORCE ON PIN F $= \sqrt{F_x^2 + F_y^2} = \sqrt{102.2^2 + 8.95^2} = 102.6\ kN$



FBD OF BOOM

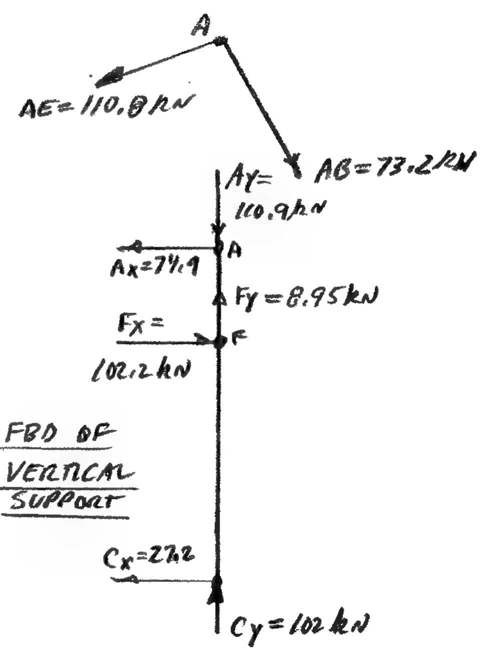
PIN A:

$A_y = AE_y + AB_y = (110.8)\cos 67.2° + (73.2)\cos 21.8°$

$A_y = 42.95 + 68.0 = 110.9\ kN \downarrow$

$A_x = AB_x - AE_x = 73.2\sin 21.8° - (110.8)\sin 67.2$

$A_x = 27.18 - 102.1 = -74.9\ kN \leftarrow$



SUMMARY OF RESULTS:

a) FORCES IN WIRES: $AE = 110.8\ kN$; $AB = 73.2\ kN$

c) SHEARING FORCE IN EACH PIN:

PIN A: $F_A = \sqrt{74.9^2 + 110.9^2} = 134\ kN$

PIN F: $F_F = \sqrt{102.2^2 + 8.95^2} = 102.6\ kN$

PIN C: $F_C = \sqrt{27.2^2 + 102^2} = 105.6\ kN$

PIN E: $F_E = AE = 110.8\ kN$

PIN D: $F_D = 34.0\ kN$

[NEXT PAGE FOR PARTS b, d, e.]

3-186 (CONTINUED)

b) DESIGN OF RODS: $AE = 110.8\ kN$, $AB = 73.2\ kN$, MODERATE SHOCK.

$\sigma_d = S_u/12$. SPECIFY AISI 4140 OQT 900. $S_u = 1289\ MPa$, 15% ELONG.

HIGH STRENGTH, GOOD DUCTILITY.

$\sigma_d = 1289\ MPa/12 = 107.4\ MPa = 107.4\ N/mm^2$

LET $\sigma_d = \sigma_{MAX} = F/A$. THEN REQ'D $A = \frac{F}{\sigma_d}$

FOR AE: $A = \frac{F}{\sigma_d} = \frac{110.8 \times 10^3 N}{107.4\ N/mm^2} = 1031\ mm^2 = \pi D^2/4$

REQ'D $D = \sqrt{4A/\pi} = \sqrt{4(1031\ mm^2)/\pi} = 36.2\ mm$

SPECIFY $D = 40\ mm$ - PREFERRED BASIC SIZE. APP. 2

FOR AB: $A = \frac{F}{\sigma_d} = \frac{73.2 \times 10^3 N}{107.4\ N/mm^2} = 681.6\ mm^2 = \pi D^2/4$

REQ'D $D = \sqrt{4A/\pi} = \sqrt{4(681.6\ mm^2)/\pi} = 29.5\ mm$

SPECIFY $D = 30.0\ mm$ - PREFERRED BASIC SIZE

d) DESIGN OF PINS: ALL PINS TO BE IN DOUBLE SHEAR USING A CLEVIS-TYPE CONNECTION. [SEE FIG. 3-17]

FROM TABLE 3-8; $\tau_d = s_{ys}/6 = s_y/12$

SPECIFY AISI 4140 OQT 900. $S_y = 1193\ MPa$, 15% ELONG.

$\tau_d = s_y/12 = 1193\ MPa/12 = 99.4\ MPa = 99.3\ N/mm^2$

LET $\tau_d = \tau_{MAX} = F/A_s$. THEN REQ'D $A_s = F/\tau_d$

PIN A: $F_A = 134\ kN$. REQ'D. $A_s = \frac{F}{\tau_d} = \frac{134 \times 10^3 N}{99.3\ N/mm^2} = 1348\ mm^2$

$A_s = 2A = 2\ \pi D^2/4 = \pi D^2/2$. REQ'D $D_A = \sqrt{2A_s/\pi}$

$D_{MIN} = \sqrt{\frac{2(1348\ mm^2)}{\pi}} = 29.3\ mm$. SPECIFY $D = 30.0\ mm$

PIN F: $F = 102.6\ kN$. $A_s = \frac{F}{\tau_d} = \frac{102.6 \times 10^3}{99.3} = 1032\ mm^2$

$D_{MIN} = \sqrt{2A_s/\pi} = \sqrt{\frac{2(1032\ mm^2)}{\pi}} = 25.6\ mm$; $D_F = 28\ mm$

PIN C: $F = 105.6\ kN$. $A_s = \frac{105.6 \times 10^3}{99.3} = 1062\ mm^2$

$D_{MIN} = \sqrt{\frac{2(1062)}{\pi}} = 26.0\ mm$. SPECIFY $D_c = 28.0\ mm$

PIN E: $F = 110.8\ kN$. $A_s = \frac{110.8 \times 10^3}{99.3} = 1115\ mm^2$

$D_{MIN} = \sqrt{\frac{2(1115)}{\pi}} = 26.6\ mm$. SPECIFY $D_E = 28.0\ mm$

PIN D: $F = 34.0\ kN$. $A_s = \frac{34 \times 10^3}{99.3} = 342.4\ mm^2$

$D_{MIN} = \sqrt{\frac{2(342.4)}{\pi}} = 14.8\ mm$. SPECIFY $D = 16\ mm$

[NEXT PAGE FOR BEARING STRESS]

3-186 (CONTINUED)

BEARING STRESS ON PINS: [SEE FIG 3-17 FOR DESIGN OF JOINT]

$$\sigma_b = \frac{F}{A_b} = \frac{F}{D \cdot t_1} \quad \text{AND } t_2 \geq t_1/2$$

$$\text{REQ'D. } t_1 = \frac{F}{D\,\sigma_{bd}}$$

$\sigma_{bd} = 0.90\,s_y$ EQ. 3-22 FOR STEEL.

$s_y = 1193$ MPa - AISI 4140 OQT 900 FOR PINS

MATERIAL FOR MATING PARTS MUST BE AT LEAST AS STRONG.

$\sigma_{bd} = 0.90(1193\text{ MPa}) = 1074\text{ MPa} = 1074\text{ N/mm}^2$

PIN A: $D = 30.0$ mm. $F = 134$ kN

$$t_{1\,min} = \frac{134 \times 10^3\ N}{(30\ mm)(1074\ N/mm^2)} = \underline{4.16\ mm}$$

THE REQ'D THICKNESS IS QUITE SMALL. IT IS HIGHLY LIKELY THAT ACTUAL DIMENSIONS FOR t_1 AND t_2 ARE MUCH LARGER FOR OTHER STRESS CONDITIONS.

PINS F, C, AND E: ALL HAVE $D = 28$ mm. $F_E = 110.8$ kN

$$t_{1\,MIN} = \frac{110 \times 10^3\ N}{(28\ mm)(1074\ N/mm^2)} = \underline{3.66\ mm} \quad \text{PIN E}$$

THIS IS ALSO VERY SMALL. PINS F AND C HAVE SLIGHTLY LOWER FORCES, SO REQ'D t IS SIMILAR.

NOTE: IF BOOM OR COLUMN ARE MADE FROM A MATERIAL WITH LOWER STRENGTH (SUCH AS STRUCTURAL STEEL), BEARING STRESS CALCULATIONS MUST BE REDONE.

PIN D: $F_D = 34.0$ kN. $D = 16$ mm

$$t_{1\,MIN} = \frac{34.0 \times 10^3\ N}{(16\ mm)(1074\ N/mm^2)} = \underline{1.98\ mm\ \text{SMALL}}$$

4-1
$$\tau = \frac{Tc}{J} = \frac{(280\ N\cdot m)(10\ mm)}{\pi(20)^4/32\ mm^4} \times \frac{10^3\ mm}{m} = \underline{178\ MPa}$$

4-2
$$J = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi}{32}(35^4 - 25^4) \doteq 109 \times 10^3\ mm^4$$
$$\tau = \frac{Tc}{J} = \frac{(560\ N\cdot m)(35/2)\ mm}{109\times 10^3\ mm^4} \times \frac{10^3\ mm}{m} = \underline{89.9\ MPa}$$

4-3
$$\tau = \frac{Tc}{J} = \frac{(1550\ LB\cdot IN)(1.25/2)\ IN}{\pi(1.25)^4/32\ IN^4} = \underline{4042\ psi}$$

4-4
$$D_i = D_o - 2t = 1.75 - 2(0.125) = 1.50\ IN$$
$$J = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi(1.75^4 - 1.50^4)}{32} = 0.424\ IN^4$$
$$\tau_o = \frac{Tc}{J} = \frac{(5500\ LB\cdot IN)(1.75/2)\ IN}{0.424\ IN^4} = \underline{11\,360\ psi}$$
$$\tau_i = \frac{Tr_i}{J} = \frac{(5500\ LB\cdot IN)(1.50/2)\ IN}{0.424\ IN^4} = \underline{9734\ psi}$$

4-5
$$T = \frac{P}{n} = \frac{0.08\times 10^3\ N\cdot m/s}{180\ rad/s} = 0.444\ N\cdot m$$
$$\tau = \frac{Tc}{J} = \frac{(0.440\ N\cdot m)(1.50\ mm)}{\pi(3.0)^4/32\ mm^4} \times \frac{10^3\ mm}{m} = \underline{83.8\ MPa}$$

4-6
$$T = \frac{P}{n} = \frac{35\times 10^3\ N\cdot m/s}{42\ rad/s} = 833\ N\cdot m$$
$$J = \frac{\pi(40^4 - 25^4)}{32}\ mm^4 = 213\times 10^3\ mm^4$$
$$\tau = \frac{Tc}{J} = \frac{(833\ N\cdot m)(20\ mm)}{213\times 10^3\ mm^4} \times \frac{10^3\ mm}{m} = \underline{78.3\ MPa}$$

4-7
$$T = \frac{63\,000\,(P)}{n} = \frac{(63\,000)(15.0\ hp)}{240\ rpm} = 3938\ LB\cdot IN$$
$$\tau = \frac{Tc}{J} = \frac{3938\ LB\cdot IN\ (1.44/2)\ IN}{\pi(1.44)^4/32\ IN^4} = \underline{6716\ psi}$$
$$\tau_d = \frac{s_y}{2N} = \frac{101\,000\ psi}{2(6)} = \underline{8417\ psi}\quad \underline{OK}$$

4-8 FROM PROBLEM 4-7, $T = 3938$ LB·IN, $\tau_d = 8417$ PSI

$C = 1.44\ \text{IN.}/2 = 0.72$ IN.

$J = \dfrac{\pi D^4}{32} = \dfrac{\pi(1.44\ \text{IN.})^4}{32} = 0.422\ \text{IN}^4$

FOR PROFILE KEYSEAT, $K_t = 2.0$

$\tau = \dfrac{K_t T C}{J} = \dfrac{(2.0)(3938\ \text{LB·IN})(0.72\ \text{IN})}{0.422\ \text{IN}^4} = \underline{13\,433\ \text{PSI}}$

BECAUSE $\tau > \tau_d$ DESIGN IS <u>NOT SAFE</u>.

4-9 $T = \dfrac{63000(7.5\ \text{HP})}{2200\ \text{RPM}} = 215$ LB·IN

AT FILLET: $r/d = 0.05/0.75 = 0.066$; $D/d = 1.25/0.75 = 1.67$; $K_t = 1.55$

AT KEYSEAT: $K_t = 2.0$

$\tau_{MAX} = \dfrac{K_t T}{Z_p} = \dfrac{(2.0)(215\ \text{LB·IN})}{\pi(0.75)^3/16\ \text{IN}^3} = 5190$ PSI

BLADE WOULD SEE SHOCK: $\tau_d = \dfrac{S_y}{2(6)}$

REQ'D. $S_y = 2(6)(5190) = \underline{62300\ \text{PSI}}$: AISI 1040 WQT 1300
$S_y = 63$ KSI; 32% ELONG.

4-10 AT FILLET: $r/d = 0.08/1.50 = 0.053$; $D/d = 2.00/1.50 = 1.33$; $K_t = 1.58$

a) $\tau_{MAX} = \dfrac{K_t T}{Z_p} = \dfrac{(1.58)(7500\ \text{LB·IN})}{\pi(1.50)^3/16\ \text{IN}^3} = \underline{17880\ \text{PSI}}$

b) AT HOLE: $Z_{p_{GROSS}} = \pi(2.00)^3/16 = 1.57\ \text{IN}^3$

$\tau_{NOM} = \dfrac{T}{Z_p} = \dfrac{7500}{1.57} = 4775$ PSI

$K_{t_{MAX}} = \dfrac{\tau_{MAX}}{\tau_{NOM}} = \dfrac{17880\ \text{PSI}}{4775\ \text{PSI}} = 3.74$

FROM APP. A-21-5 $(d/D)_{MAX} = 0.18$; THEN $d_{MAX} = 0.18(2.0) = \underline{0.360\ \text{IN}}$

4-11 $J = \pi(D_o^4 - D_i^4)/32 = \pi(80^4 - 60^4)/32 = 2.75 \times 10^6\ \text{mm}^4$

$\tau_{max} = \dfrac{TC}{J} = \dfrac{(4500\ \text{N·m})(40\ \text{mm})}{2.75 \times 10^6\ \text{mm}^4} \times \dfrac{10^3\ \text{mm}}{\text{m}} = \underline{65.5\ \text{MPa}}$

$\theta = \dfrac{TL}{GJ} = \dfrac{(4500\ \text{N·m})(600\ \text{mm})}{(26 \times 10^9\ \text{N/m}^2)(2.75 \times 10^6\ \text{mm}^4)} \times \dfrac{(10^3\ \text{mm})^3}{\text{m}^3} = \underline{0.0378\ \text{rad}}$

ASSUME STEADY TORQUE: REQ'D. $S_y = 2(2)(\tau_{max})$

$S_y = 4(65.5) = \underline{262\ \text{MPa}}$

6061-T6 HAS $S_y = 276$ MPa

4-12 SOLID: $J = \pi D^4/32 = \pi(50)^4/32\ mm^4 = 613.6\times10^3\ mm^4$

$$\tau = \frac{Tc}{J} = \frac{(850\ N\cdot m)(25\ mm)}{613.6\times10^3\ mm^4}\times\frac{10^3\ mm}{m} = \underline{34.6\ MPa}$$

$$\theta = \frac{TL}{GJ} = \frac{(850\ N\cdot m)(600\ mm)}{(80\times10^3\ N/mm^2)(613.6\times10^3\ mm^4)}\times\frac{(10^3\ mm)^3}{m^3} = \underline{0.0104\ rad}$$

$$MASS = (VOL)(DENS.) = A\cdot L\cdot DENS. = \frac{\pi(50)^2}{4}\times600\ mm^3\times\frac{7680\ kg}{m^3}\times\frac{1\ m^3}{(10^3\ mm)^3}$$

$\underline{m = 9.05\ kg}$

HOLLOW: $J = \frac{\pi}{32}(50^4 - 40^4) = 362.3\times10^3\ mm^4$

$$\tau = \frac{(850)(25)(10^3)}{362.3\times10^3} = \underline{58.7\ MPa}\ [1.69\ \text{TIMES}\ \tau_{SOLID}]$$

$$\theta = \frac{(850)(600)(10^9)}{(80\times10^9)(362.3\times10^3)} = \underline{0.0176\ rad}\ [1.69\ \text{TIMES}\ \theta_{SOLID}]$$

$$M = \frac{\pi(50^2 - 40^2)}{4}\times\frac{(600)(7680)}{10^9} = \underline{3.26\ kg}\ \boxed{\text{SOLID IS 2.78 TIMES HIGHER}}$$

4-13 REQ'D. $Z_p = \frac{T}{\tau_d} = \frac{1200\ N\cdot m}{45\ N/mm^2}\times\frac{10^3\ mm}{m} = 26667\ mm^3$

$$Z_p = \frac{\pi}{16}\,\frac{D_o^4 - D_i^4}{D_o} = \frac{\pi}{16}\,\frac{(1.25D_i)^4 - D_i^4}{1.25D_i} = 0.226\,D_i^3$$

REQ'D. $D_i = \sqrt[3]{26667/0.226} = \underline{49.0\ mm}$

$D_o = 1.25D_i = \underline{61.3\ mm}$

4-14 $T = 63000(7.5)/240 = 1969\ LB\cdot IN$

$$\tau = \frac{T}{Z_p} = \frac{1969\ LB\cdot IN}{\pi(0.86)^3/16\ IN^3} = \underline{15764\ PSI}$$

4-15 $T = 63000(7.5)/1140 = 414\ LB\cdot IN$

REQ'D. $Z_p = \frac{T}{\tau} = \frac{414\ LB\cdot IN}{15764\ LB/IN^2} = 0.0263\ IN^3 = \pi D^3/16$

REQ'D. $D = \sqrt[3]{16Z_p/\pi} = \sqrt[3]{16(0.0263)/\pi} = \underline{0.512\ IN}$

4-16 $T = F\cdot d = (80\ LB)(18\ IN) = 1440\ LB\cdot IN$; $Z_p = 0.6524\ IN^3$ (APP. A-12)

$\tau = T/Z_p = 1440\ LB\cdot IN/0.6524\ IN^3 = \underline{2207\ PSI}$

4-17 $n = (1\ REV/5\ SEC)(60\ SEC/MIN) = 12.0\ RPM$; $T = (30\ LB\cdot FT)(12\ IN/FT) = 360\ LB\cdot IN$

$$P = \frac{Tn}{63000} = \frac{(360)(12)}{63000} = \underline{0.0686\ hP}$$

$\tau = T/Z_p = 360\ LB\cdot IN/\pi(0.60)^3/16\ IN^3 = \underline{8488\ PSI} = S_y/2N = S_y/8$

REQ'D $S_y = 8\tau = 8(8488) \doteq \underline{67900\ PSI}$

POSSIBLE STEEL: AISI 1040 WQT 1100, $S_y = 80\ KSI$, 24% ELONG.

4-18 $\tau = S_{ys} = S_y/2 = 441\ MPa/2 = 220\ MPa$

$T = \tau \cdot Z_P = \frac{220\ N}{mm^2} \times \frac{\pi (15)^3}{16} mm^3 \times \frac{m}{10^3\ mm} = \underline{146\ N \cdot m}$

4-19 $T = 63000(2500)/75 = 2.1 \times 10^6\ LB \cdot IN$

$\tau_d = \frac{S_y}{2N} = \frac{63000\ PSI}{2(6)} = 5250\ PSI$

REQ'D. $Z_P = \frac{T}{\tau_d} = \frac{2.1 \times 10^6\ LB \cdot IN}{5250\ LB/IN^2} = 400\ IN^3 = \frac{\pi}{16}\left[\frac{D_o^4 - D_i^4}{D_o}\right]$

BUT $D_i = 0.8\ D_o$; $D_i^4 = 0.4096 D_o^4$

$Z_P = \frac{\pi}{16}\frac{(D_o^4 - 0.4096 D_o^4)}{D_o} = \frac{\pi(0.5904\ D_o^4)}{16\ D_o} = 0.1159\ D_o^3$

REQ'D. $D_o = \sqrt[3]{\frac{Z_P}{0.1159}} = \sqrt[3]{\frac{400}{0.1159}} = \underline{15.11\ IN}$

$D_i = 0.8\ D_o = \underline{12.09\ IN}$

4-20 REQD $Z_P = 400\ IN^3 = \pi D^3/16$: $D = \sqrt[3]{16 Z_P/\pi} = \underline{12.68\ IN}$

$\frac{WT_S}{WT_H} = \frac{A_S \cdot L \cdot DENS}{A_H \cdot L \cdot DENS} = \frac{A_S}{A_H} = \frac{\pi(12.68)^2/4}{\pi(15.11^2 - 12.09^2)/4} = \underline{1.96}$

4-21 $T = \tau \cdot Z_P = \frac{80\ N}{mm^2} \cdot \frac{\pi(5.0)^3\ mm^3}{16} \cdot \frac{1\ m}{10^3\ mm} = \underline{1.96\ N \cdot m}$

4-22 $J = \pi D^4/32 = \pi(6.0)^4/32 = 127.2\ mm^4$

$\tau = \frac{Tc}{J} = \frac{(5.5\ N \cdot m)(3.0\ mm)}{127.2\ mm^4} \times \frac{10^3\ mm}{1\ m} = \underline{130\ MPa}$

$\theta = \frac{TL}{GJ} = \frac{(5.5\ N \cdot m)(250\ mm)}{(80 \times 10^9\ N/m^2)(127.2\ mm^4)} \times \frac{(10^3\ mm)^3}{m^3} = \underline{0.135\ RAD}$ $\underline{(7.74\ deg)}$

4-23 $\theta = \frac{TL}{GJ} = \frac{(240\ N \cdot m)(250\ mm)}{(80 \times 10^9\ N/m^2)(\pi(15)^4/32)\ mm^4} \times \frac{(10^3\ mm)^3}{m^3} = \underline{0.1509\ RAD}$ $\underline{(8.65\ deg.)}$

4-24 $J = \frac{\pi}{32}(80^4 - 60^4) = 2.75 \times 10^6\ mm^4$

$\theta = \frac{TL}{GJ} = \frac{(2250\ N \cdot m)(1200\ mm)}{(26 \times 10^9\ N/m^2)(2.75 \times 10^6\ mm^4)} \times \frac{(10^3\ mm)^3}{m^3} = \underline{0.0378\ RAD}$ $\underline{(2.16\ DEG)}$

4-25 $\theta = \frac{TL}{GJ} = \frac{(40\ LB \cdot FT)(8\ FT)}{(11.5 \times 10^6\ LB/IN^2)(\pi(0.625)^4/32)\ IN^4} \times \frac{144\ IN^2}{FT^2} = \underline{0.267\ RAD}$ $\underline{(15.3\ DEG)}$

4-26 $\theta = (2.0\ DEG)(\pi\ RAD/180\ DEG.) = 0.0349\ RAD$

$J = \frac{TL}{G\theta} = \frac{(40\ LB \cdot FT)(8\ FT)}{(11.5 \times 10^6\ LB/IN^2)(0.0349\ RAD)} \times \frac{144\ IN^2}{FT^2} = 0.1148\ IN^4 = \pi D^4/32$

REQ'D. $D = \sqrt[4]{32(0.1148)/\pi} = \underline{1.04\ IN}$

4-27 $\theta = \theta_1 + \theta_2 = \dfrac{(200\,N\cdot m)(400\,mm)}{(80\times10^9\,N/m^2)(15\,708\,mm^4)} \times \dfrac{(10^3)^3\,mm^3}{m^3} + \dfrac{(200)(1200)(10^9)}{(80\times10^9)(251\,300)}$

$J_1 = \pi(20)^4/32 = 15\,708\,mm^4$ & $J_2 = \pi(40)^4/32 = 251\,300\,mm^4$

$\theta = 0.0637 + 0.0119 = \underline{0.0756\,RAD}$ (4.33 DEG.)

4-28 $\theta = (10.0\,DEG)(\pi\,RAD/180\,DEG) = 0.1745\,RAD$

REQ'D $J = \dfrac{TL}{G\theta} = \dfrac{(5.0\,N\cdot m)(150\,mm)}{(26\times10^9\,N/m^2)(0.1745)} \times \dfrac{10^9\,mm^3}{m^3} = 165.3\,mm^4$

REQ'D. $D = \sqrt[4]{32J/\pi} = \underline{6.40\,mm}$

$\tau = \dfrac{Tc}{J} = \dfrac{(5000\,N\cdot mm)(3.20\,mm)}{165.3\,mm^4} = \underline{96.8\,MPa}$

$N = \dfrac{S_y}{2(\tau)} = \dfrac{276\,MPa}{2(96.8\,MPa)} = \underline{1.43\ LOW}$

COULD USE STRONGER ALUMINUM OR LONGER BAR

4-29 $T = \tau \cdot Z_p = \dfrac{250\,N}{mm^2} \times \dfrac{\pi(1.50)^3\,mm^3}{16} = 165.7\,N\cdot mm$

$J = \pi(1.50)^4/32 = 0.497\,mm^4$

$\theta = \dfrac{TL}{GJ} = \dfrac{(165.7\,N\cdot mm)(40\,mm)}{(48\times10^9\,N/m^2)(0.497\,mm^4)} \times \dfrac{10^6\,mm^2}{m^2} = \underline{0.278\,RAD}$ (15.9 DEG)

4-30 $J = \dfrac{\pi}{32}(18^4 - 16^4) = 3872\,mm^4$; $\theta = 40\,DEG \times \pi\,RAD/180\,DEG = 0.698\,RAD$

$T = \dfrac{\theta GJ}{L} = \dfrac{(0.698\,RAD)(43\times10^9\,N/m^2)(3872\,mm^4)}{1650\,mm} \times \dfrac{m^2}{10^6\,mm^2} = 70.4\times10^3\,N\cdot mm$

$\tau = \dfrac{Tc}{J} = \dfrac{(70.4\times10^3\,N\cdot mm)(9\,mm)}{3872\,mm^4} = \underline{164\,MPa}$

$N = \dfrac{S_y}{2\tau} = \dfrac{1070\,MPa}{2(164\,MPa)} = \underline{3.27}$

4-31 $J = \pi(35)^4/32 = 147.3\times10^3\,mm^4$

$T_{BC} = T_3 = 500\,N\cdot m = 500\times10^3\,N\cdot mm$; $T_{AB} = T_2 + T_3 = 1500\,N\cdot m = 1.5\times10^6\,N\cdot mm$

$G = 80\,GPa = (80\times10^9\,N/m^2)(m^2/10^6\,mm^2) = 80\times10^3\,N/mm^2$

$\theta_{AC} = \theta_{AB} + \theta_{BC} = \dfrac{T_{AB}L_1}{GJ} + \dfrac{T_{BC}L_2}{GJ}$

$\theta_{AC} = \dfrac{(1.5\times10^6\,N\cdot mm)(500\,mm)}{(80\times10^3\,N/mm^2)(147.3\times10^3\,mm^4)} + \dfrac{(500\times10^3)(800)}{(80\times10^3)(147.3\times10^3)}$

$\theta_{AC} = 0.0636 + 0.0339 = \underline{0.0976\,RAD}$ (5.59 DEG.) | $\theta_{AB} = 0.0636\,RAD$ (3.64 DEG.)

4-32 $\theta = (2.2\,DEG)(\pi\,RAD/180\,DEG) = 0.0384\,RAD$

REQ'D $J = \dfrac{TL}{G\theta} = \dfrac{(1360\,N\cdot m)(820\,mm)}{(80\times10^9\,N/m^2)(0.0384)} \times \dfrac{(10^3\,mm)^3}{m^3} = 363\times10^3\,mm^4 = \dfrac{\pi D^4}{32}$

REQ'D $D = \sqrt[4]{32J/\pi} = \underline{43.9\,mm}$

$\tau = \dfrac{Tc}{J} = \dfrac{(1360\,N\cdot m)(21.95\,mm)}{363\times10^3\,mm^4} \times \dfrac{10^3\,mm}{m} = \underline{82.1\,MPa}$

4-33

$$T = \frac{P}{n} = \frac{120\times10^3\ \text{N}\cdot\text{m/s}}{225\ \text{RAD/s}} = 533\ \text{N}\cdot\text{m}$$

$$J = \frac{\pi}{32}(75^4 - 55^4) = 2.208\times10^6\ \text{mm}^4$$

$$\tau = \frac{Tc}{J} = \frac{(533\ \text{N}\cdot\text{m})(37.5\ \text{mm})}{2.208\times10^6\ \text{mm}^4}\times\frac{10^3\ \text{mm}}{\text{m}} = \underline{9.06\ \text{MPa}}$$

$$\theta = \frac{TL}{GJ} = \frac{(533\ \text{N}\cdot\text{m})(1525\ \text{mm})}{(80\times10^9\ \text{N/m}^2)(2.208\times10^6\ \text{mm}^4)}\times\frac{(10^3\ \text{mm})^3}{\text{m}^3} = \underline{0.0046\ \text{RAD}}\ \underline{(0.264\ \text{DEG})}$$

4-34

$$T = \frac{P}{n} = \frac{60\times10^3\ \text{N}\cdot\text{m/s}}{70\ \text{RAD/s}} = 857\ \text{N}\cdot\text{m}$$

$$Z_p = \pi d^3/16 = \pi(35)^3/16 = 8418\ \text{mm}^3$$

$r/d = 4/35 = 0.114$; $D/d = 50/35 = 1.43$; $K_t = 1.35$ FROM APP. A-22-7

$$\tau = \frac{TK_t}{Z_p} = \frac{(857\ \text{N}\cdot\text{m})(1.35)}{8418\ \text{mm}^3}\times\frac{10^3\ \text{mm}}{\text{m}} = \underline{137\ \text{MPa}}$$

4-35

$$T = \frac{P}{n} = \frac{105\times10^3\ \text{N}\cdot\text{m/s}}{220\ \text{RAD/s}} = 477\ \text{N}\cdot\text{m}$$

$$Z_p = \pi d^3/16 = \pi(40)^3/16 = 12566\ \text{mm}^3$$

$r/d = 6/40 = 0.150$; $D/d = 70/40 = 1.75$; $K_t = 1.29$ FROM APP. A-22-7

$$\tau = \frac{TK_t}{Z_p} = \frac{(477\ \text{N}\cdot\text{m})(1.29)}{12566\ \text{mm}^3}\times\frac{10^3\ \text{mm}}{\text{m}} = \underline{49.0\ \text{MPa}}$$

FOR PROBLEMS 4-36 THROUGH 4-39: $\tau_d = \frac{s_y}{2N} = \frac{669\ \text{MPa}}{2(4)} = 83.6\ \text{MPa}$

OR $\tau_d = \frac{97000\ \text{PSI}}{2(4)} = 12125\ \text{PSI}$

$\tau = TK_t/Z_p$; ALLOW. $T = (\tau_d)(Z_p)/K_t$

4-36 LEFT END: $Z_p = \pi(12)^3/16 = 339.3\ \text{mm}^3$

$r/d = 2/12 = 0.167$; $D/d = 24/12 = 2.0$; $K_t = 1.27$ (A-22-

$$T = \frac{(83.6\ \text{N/mm}^2)(339.3\ \text{mm}^3)}{1.27} = \underline{22.3\ \text{N}\cdot\text{m}}\quad \underline{\text{CRITICAL VALUE}}$$

RIGHT END: $Z_p = \pi(16)^3/16 = 804.2\ \text{mm}^3$

$r/d = 1/16 = 0.063$; $D/d = 24/16 = 1.50$; $K_t = 1.53$ (A-22-7)

$$T = \frac{(83.6\ \text{N/mm}^2)(804.2\ \text{mm}^3)}{1.53} = \underline{43.9\times10^3\ \text{N}\cdot\text{mm}} = \underline{43.9\ \text{N}\cdot\text{m}}$$

4-37 GROOVE $Z_p = \pi(1.20)^3/16 = 0.339\ \text{in}^3$

LEFT GROOVE: $r/d = 0.008/1.20 = 0.0067$; $D/d = 1.50/1.20 = 1.25$; $K_t \cong 3.0$ EST. (A-22-7)

RIGHT GROOVE: $r/d = 0.08/1.20 = 0.067$; $D/d = 1.25$; $K_t = 1.63$ (A-22-6)

LEFT GROOVE CRITICAL: $T = \frac{(12125\ \text{lb/in}^2)(0.339\ \text{in}^3)}{3.0} = \underline{1370\ \text{LB}\cdot\text{IN}}$

4-38 GROOVE: $Z_p = \pi(25)^3/16 = 3068\ mm^3$

$r/d = 1.50/25 = 0.060$; $D/d = 30/25 = 1.20$; $K_t = 1.66$ (A-22-6)

$T = \tau_d \cdot Z_p/K_t = (83.6 N/mm^2)(3068\ mm^3)/1.66 = 154.5 \times 10^3 N \cdot mm = 154.5 N \cdot m$

FILLET: $Z_p = \pi(20)^3/16 = 1571\ mm^3$

$r/d = 1.50/20 = 0.075$; $D/d = 30/20 = 1.50$; $K_t = 1.47$ (A-22-7)

$T = (83.6)(1571)/1.47 = 89.3 \times 10^3 N \cdot mm = 89.3 N \cdot m$

HOLE: $Z_p = 1571\ mm^3$; $d/D = 4/20 = 0.200$; $K_{tg} = 3.8$ (A-22-5C)

$T = (83.6)(1571)/3.8 = 34.5 \times 10^3 N \cdot mm = 34.5 N \cdot m$ CRITICAL

4-39 LEFT PART: $Z_p = \pi(1.25)^3/16 = 0.383\ in^3$

FILLET — $r/d = 0.188/1.25 = 0.150$; $D/d = 2.00/1.25 = 1.60$; $K_t = 1.26$ (A-22-7)

KEYSEAT: $K_t = 1.60$

$T = \frac{\tau_d \cdot Z_p}{K_t} = \frac{(12125 LB/IN^2)(0.383 IN^3)}{1.60} = 2902$ LB·IN CRITICAL

OTHER PARTS OBVIOUSLY STRONGER

Note concerning Problems 4-40 to 4-57:
Torsion of Noncircular sections

These problems involve the analysis of torsional shear stress and torsional deformation of load-carrying members having noncircular cross sections. Data for the factors J and Z_p are computed from the equations in Figure 4-27.

4-40 $\tau = T/Z_p$; $T = \tau Z_p = (50\ N/mm^2)(1664\ mm^3) = 83.2\times10^3\ N\cdot mm = \underline{83.2\ N\cdot m}$

$Z_p = 0.208\,a^3 = 0.208(20)^3 = 1664\ mm^3$

4-41 $J = 0.141\,a^4 = 0.141(20)^4 = 22.56\times10^3\ mm^4$

$\theta = \dfrac{TL}{GJ} = \dfrac{(83.2\times10^3\ N\cdot mm)(1800\ mm)}{(80\,000\ N/mm^2)(22.56\times10^3\ mm^4)} = \underline{0.083\ RAD}\ (4.75\ DEG)$

4-42 $Z_p = 0.208\,a^3 = 0.208(1.25)^3 = 0.406\ IN^3$

$T = \tau Z_p = (7500\ LB/IN^2)(0.406\ IN^3) = 3047\ LB\cdot IN$

4-43 $J = 0.141\,a^4 = 0.141(1.25)^4 = 0.344\ IN^4$

$\theta = \dfrac{TL}{GJ} = \dfrac{(3047\ LB\cdot IN)(48\ IN)}{(3.75\times10^6\ LB/IN^2)(0.344\ IN^4)} = \underline{0.112\ RAD}\ (6.42\ DEG)$

4-44 $Z_p = \dfrac{bh^2}{[3+1.8(h/b)]} = \dfrac{(3.0)(1.25)^2}{[3+1.8(1.25/3)]} = 1.25\ IN^3$

$T = \tau Z_p = (7500\ LB/IN^2)(1.25\ IN^3) = 9375\ LB\cdot IN$

4-45 $J = (3.0)(1.25)^3\left[\frac{1}{3} - 0.21\,\frac{1.25}{3.0}\left(1 - \frac{(1.25/3.0)^4}{12}\right)\right] = 1.44\ IN^4$

$\theta = \dfrac{TL}{GJ} = \dfrac{(7500)(48)}{(3.75\times10^6)(1.44)} = \underline{0.0667\ RAD}\ (3.82\ DEG.)$

4-46 $J = 0.0217\,a^4 = 0.0217(30)^4 = 17.58\times10^3\ mm^4$

$\theta = (0.80\ DEG)(\pi\ RAD/180\ DEG.) = 0.0140\ RAD$

$T = \dfrac{\theta GJ}{L} = \dfrac{(0.0140\ RAD)(26000\ N/mm^2)(17.58\times10^3\ mm^4)}{2600\ mm} = 2461\ N\cdot mm = \underline{2.46\ N\cdot m}$

4-47 $Z_p = 0.050\,a^3 = 0.050(30)^3 = 1350\ mm^3$

$\tau = \dfrac{T}{Z_p} = \dfrac{2461\ N\cdot mm}{1350\ mm^3} = \underline{1.82\ MPa}$

4-48 CIRCULAR PART: $Z_p = \pi D^3/16 = \pi(1.75)^3/16 = 1.052\ IN^3$

$\tau = \dfrac{T}{Z_p} = \dfrac{850\ LB\cdot IN}{1.052\ IN^3} = \underline{808\ PSI}$

SHAFT WITH FLAT: $h = 1.50 - 0.875 = 0.625\ IN$; $R = D/2 = \frac{1.75}{2} = 0.875\ IN$

$h/R = 0.625/0.875 = 0.714$; $C_2 = 1.069$ (INTERPOLATION–FIG. 4-27)

$Z_p = C_2 R^3 = (1.069)(0.875)^3 = 0.716\ IN^3$

$\tau = T/Z_p = 850\ LB\cdot IN/0.716\ IN^3 = \underline{1187\ PSI}$

4-49 $J_c = \pi(1.75)^4/32 = 0.9208\ IN^4$; $C_1 = 1.24$ (INTERPOLATION – FIG 4-27)

$J_F = C_1 R^4 = 1.24(0.875)^4 = 0.727\ IN^4$

$\theta = \dfrac{TL}{GJ_c} + \dfrac{TL}{GJ_F} = \dfrac{(850)(20)}{(11.5\times10^6)(0.9208)} + \dfrac{(850)(20)}{(11.5\times10^6)(.727)} = 0.00160 + 0.00203$

$\theta = \underline{0.00364\ RAD} = 0.209\ DEG.$

J_c = J FOR CIRCULAR PART

J_F = J FOR SHAFT WITH FLAT

4-50 $h = 1.25/2 = 0.625 \text{ IN} : h/r = 0.625/0.875 = 0.714 : C_4 = 0.839$

$Z_P = C_4 r^3 = 0.839(0.875)^3 = 0.562 \text{ IN}^3$

$\tau = \dfrac{T}{Z_P} = \dfrac{850 \text{ LB·IN}}{0.562 \text{ IN}^3} = \underline{1512 \text{ PSI}}$ IN SHAFT WITH FLATS

4-51 $C_3 = 0.966$ FOR $h/r = 0.714$ IN FIG 4-27 BY INTERPOLATION

$J = C_3 r^4 = 0.966(0.875)^4 = 0.566 \text{ IN}^4$

$\theta = \dfrac{TL}{GJ} = \dfrac{(850)(20)}{(11.5\times10^6)(0.566)} = 0.0026 \text{ RAD}$ IN SHAFT WITH FLATS

$\theta_{TOT} = \underset{(\text{ROUND})}{0.0016} + \underset{(\text{FLATS})}{0.0026} = \underline{0.0042 \text{ RAD}}$ (0.241 DEG.)

4-52 $\left.\begin{array}{l} Z_p = 0.208\, a^3 = 0.208(8)^3 = 106.5 \text{ mm}^3 \\ J = 0.141\, a^4 = 0.141(8)^4 = 577.5 \text{ mm}^4 \end{array}\right\}$ (FIG. 4-27)

$\tau = S_{ys} = 0.5 S_y = 0.5(1070) = 535 \text{ MPa}$

$T = \tau Z_p = (535 \text{ N/mm}^2)(106.5 \text{ mm}^3) = 57.0\times10^3 \text{ N·mm} = 57.0 \text{ N·m}$

$\theta = \dfrac{TL}{GJ} = \dfrac{(57.0\times10^3 \text{ N·mm})(200) \text{ mm}}{(43000 \text{ N/mm}^2)(577.5 \text{ mm}^4)} = \underline{0.459 \text{ RAD}}$ (26.3 DEG)

4-53 $\theta = 3.0 \text{ DEG} (\pi \text{ RAD}/180 \text{ DEG}) = 0.0524 \text{ RAD.}$; USE $t = t_{des} = 0.233 \text{ IN}$

$J = \dfrac{2t(a-t)^2(b-t)^2}{(a+b-2t)} = \dfrac{2(0.233)(3.767)^2(3.767)^2}{(4+4-2(0.233))} = 12.45 \text{ IN}^4$

$T = \dfrac{\theta G J}{L} = \dfrac{(0.0524)(11.5\times10^6 \text{ LB/IN}^2)(12.45 \text{ IN}^4)}{(8\times12) \text{ IN}} = \underline{78\,150 \text{ LB·IN}}$

4-54 $Z_p = 2t(a-t)(b-t) = 2(0.233)(3.767)(3.767) = 6.613 \text{ IN}^3$

$\tau = \dfrac{T}{Z_p} = \dfrac{78\,150 \text{ LB·IN}}{6.613 \text{ IN}^3} = \underline{11818 \text{ PSI}}$

FOR ASTM A501 STEEL, $S_y = 36000 \text{ PSI}$

$\tau_d = \dfrac{S_y}{2(2)} = \dfrac{36000 \text{ PSI}}{4} = \underline{9000 \text{ PSI}}$ NOT SAFE

4-55 $J = \dfrac{2(0.233)(3.767)^2(5.767)^2}{(4+6-2(0.233))} = 23.07 \text{ IN}^4$

$T = \dfrac{\theta G J}{L} = \dfrac{(0.0524)(11.5\times10^6)(23.07)}{96} = 144\,800 \text{ LB-IN}$

4-56 $Z_p = 2(0.233)(3.767)(5.767) = 10.12 \text{ IN}^3$

$\tau = T/Z_p = 144800/10.12 = \underline{14300 \text{ PSI}}$ NOT SAFE

4-57

TUBE	PIPE
$Z_p = 2(0.233)(5.767)^2 = 15.50 \text{ IN}^3$	$Z_p = 16.99 \text{ IN}^3$
$J = \dfrac{2(0.233)(5.767)^2(5.767)^2}{(6+6-2(0.233))} = 44.69 \text{ IN}^4$	$J = 2I = 2(28.14) = 56.28 \text{ IN}^4$
$\tau_T = T/Z_p = 0.0648\, T$	$\tau_P = T/Z_P = 0.0589\, T$
$\theta_T = \dfrac{TL}{GJ} = 0.0224(TL/G)$	$\theta_P = \dfrac{TL}{GJ} = 0.0177(TL/G)$

$\dfrac{\tau_T}{\tau_P} = \dfrac{0.0648}{0.0589} = \underline{1.10}$ $\dfrac{\theta_T}{\theta_P} = \dfrac{0.0224}{0.0177} = \underline{1.266}$

4-58 $P = T \cdot n \quad T = P/n \quad D = 25\text{ mm}$

$$n = 1150\ \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\ \text{rad}}{\text{rev}} \cdot \frac{1\text{ min}}{60\text{ s}} = 120.4\ \text{rad/s}$$

$$T = \frac{P}{n} = \frac{125 \times 10^3\ \text{N}\cdot\text{m/s}}{120.4\ \text{rad/s}} = 1038\ \text{N}\cdot\text{m}$$

$$\tau_{MAX} = \frac{Tc}{J} = \frac{T}{Z_P} = \frac{1038\ \text{N}\cdot\text{m}}{\pi(35\text{ mm})^3/16} \cdot \frac{10^3\text{ mm}}{\text{m}} = 123\ \frac{\text{N}}{\text{mm}^2} = \underline{123\text{ MPa}}$$

4-59 SMOOTH POWER. $\tau_d = S_{ys}/N = S_y/2N$. LET $N = 2$

$\tau_d = S_y/4$. LET $\tau_{MAX} = \tau_d = S_y/4$

REQ'D $S_y = 4(\tau_d) = 4(123\text{ MPa}) = 493\text{ MPa}$

POSSIBLE STEEL: AISI 1040 CD, $S_y = 565\text{ MPa}$

4-60 REPEATED POWER LET $N = 4$. $\tau_d = S_y/2N = S_y/8 = \tau_{MAX}$

REQ'D $S_y = 8\,\tau_{MAX} = 8(123\text{ MPa}) = 986\text{ MPa}$

POSSIBLE STEEL: AISI 4140 OQT 900. $S_y = 1193\text{ MPa}$

15% ELONGATION. GOOD DUCTILITY.

4-61 SHOCK LOADING LET $N = 6$, $\tau_d = S_y/2N = S_y/12 = \tau_{MAX}$

REQ'D $S_y = 12\,\tau_{MAX} = 12(123\text{ MPa}) = 1480\text{ MPa}$.

POSSIBLE STEEL: AISI 4140 OQT 700. $S_y = 1462\text{ MPa}$

12% ELONGATION.

MARGINAL STRENGTH, SATISFACTORY DUCTILITY.

4-62 $P = 12.0\text{ HP}$ STEADY. $n = 1150\text{ rpm}$. $S_y = 80\text{ ksi}$

$T = 63\,000(P)/n = 63\,000(12)/1150 = 657\text{ IN}\cdot\text{LB}$

$\tau_{MAX} = T/Z_P$. REQ'D $Z_P = \dfrac{T}{\tau_{MAX}} = \dfrac{657\text{ IN}\cdot\text{LB}}{20\,000\text{ LB/IN}^2} = 0.0329\text{ IN}^3 = \dfrac{\pi D^3}{16}$

LET $\tau_{MAX} = \tau_d = S_y/4 = 80\text{ ksi}/4 = 20\text{ ksi} = 20\,000\text{ lb/in}^2$

$$D_{MIN} = \sqrt[3]{\frac{16 Z_P}{\pi}} = \sqrt[3]{\frac{16(0.0329\text{ IN}^3)}{\pi}} = 0.551\text{ IN};\ \underline{\text{SPECIFY } D = 0.60\text{ IN}}$$

4-63 $P = 20.0\text{ HP}$ STEADY. $n = 3450\text{ rpm}$. $S_y = 101\text{ KSI}$

$T = 63000(P)/n = 63000(20)/3450 = 365\text{ IN}\cdot\text{LB}$

$\tau_{MAX} = T/Z_P$. REQ'D $Z_P = \dfrac{T}{\tau_{MAX}} = \dfrac{365\text{ IN}\cdot\text{LB}}{25250\text{ LB/IN}^2} = 0.0145\text{ IN}^3 = \dfrac{\pi D^3}{16}$

LET $\tau_{MAX} = \tau_d = S_y/4 = 101\text{ ksi}/4 = 25.25\text{ ksi} = 25250\text{ psi}$

$$D_{MIN} = \sqrt[3]{\frac{16 Z_P}{\pi}} = \sqrt[3]{\frac{16(0.0145)\text{ IN}^3}{\pi}} = 0.419\text{ IN};\ \underline{\text{SPECIFY } D = 0.50\text{ IN}}$$

4-64 $D_o = 100\ mm$; $D_i = 60\ mm$; ALLOY STEEL

$\tau_{SOLID} = 200\ MPa = \frac{T}{Z_{P_S}}$ • $T = \tau \cdot Z_{P_S}$

$Z_{P_S} = \pi D_o^3/16 = \pi(100\ mm)^3/16 = 196\,350\ mm^3$

$T = \tau \cdot Z_P = (200\ N/mm^2)(196\,350\ mm^3) = 3.927 \times 10^7\ N \cdot mm$

$\tau_{HOLLOW} = \frac{T}{Z_P} = \frac{3.927 \times 10^7\ N \cdot mm}{1.709 \times 10^5\ mm^3} = 230\ N/mm^2 = \underline{230\ MPa} = \tau_H$

$Z_P = \frac{\pi}{16}\ \frac{D_o^4 - D_i^4}{D_o} = \frac{\pi[100^4 - 60^4]}{16(100)}\ mm^3 = 170\,900\ mm^3$

4-65 FIND ANGLE OF TWIST FOR SHAFT OF PROB. 5-64.

$T = 3.927 \times 10^7\ N \cdot mm$

SOLID SEGMENT: $J = \frac{\pi D_o^4}{32} = \frac{\pi(100\ mm)^4}{32} = 9.817 \times 10^6\ mm^4$

$\theta_S = \frac{TL}{GJ} = \frac{(3.927 \times 10^7\ N \cdot mm)(300\ mm)}{(80 \times 10^3\ N/mm^2)(9.817 \times 10^6\ mm^4)} = 0.0150\ rad.$

$G = 80\ GPa = \frac{80 \times 10^9\ N}{m^2} \times \frac{1\ m^2}{(10^3\ mm)^2} = 80 \times 10^3\ N/mm^2$

HOLLOW SEGMENT: $J = \frac{\pi}{32}(100^4 - 60^4)\ mm^4 = 8.545 \times 10^6\ mm^4$

$\theta_H = \frac{TL}{GJ} = \frac{(3.927 \times 10^7)(300)}{(80 \times 10^3)(8.545 \times 10^6)} = 0.0172\ rad$

TOTAL $\theta_T = \theta_S + \theta_H = 0.0150 + 0.0172 = \underline{0.0322\ rad}$ (1.85 DEG)

4-66 FIND τ IN EACH PART OF SHAFT.

$n = \frac{1750\ rev}{min} \times \frac{2\pi\ RAD}{rev} \times \frac{1\ min}{60\ s} = 183\ rad/s$

$T_A = \frac{P_A}{n} = \frac{15 \times 10^3\ N \cdot m/s}{183\ rad/s} = \underset{OUT}{81.85\ N \cdot m}$

$T_C = \frac{P_C}{n} = \frac{20 \times 10^3}{183} = \underset{OUT}{109.1\ N \cdot m}$; $T_B = \frac{P_B}{n} = \frac{35 \times 10^3}{183} = \underset{IN}{191.0\ N \cdot m}$

$T_{AB} = T_A = 81.85\ N \cdot m$ $\quad T_{BC} = T_C = 109.1\ N \cdot m$

τ AT A: $\underset{GROOVE\ DIA}{D = 9.50\ mm}$, RETAINING RING TO RIGHT OF PULLEY; $K_t = 3.0$

$\tau_A = T_{AB} \cdot K_t/Z_P = \frac{(81.85\ N \cdot m)(3.0)}{\pi(9.5\ mm)^3/16} \cdot \frac{10^3\ mm}{m} = 1251\ N/mm^2 = \underline{1459\ MPa}$ $\underline{\text{VERY HIGH}}$

τ AT BEARING TO RIGHT OF PULLEY: STEPPED SHAFT. $D = 10\ mm$

$D/d = 15/10 = 1.50$; $r/d = 0.5\ mm/10\ mm = 0.05$; $K_t = 1.60$

$\tau = T_{AB} \cdot K_t/Z_P = \frac{(81.85)(1.60)(10^3)}{\pi(10)^3/16} = \underline{667\ MPa}$

(CONTINUED NEXT PAGE)

4-66 (CONTINUED)

τ TO LEFT OF B: RETAINING RING GROOVE; D=14 mm AT GROOVE

$$\tau = \frac{T_{AB}\cdot K_t}{Z_P} = \frac{(81.85)(3.0)(1000)}{\pi(14)^3/16} = \underline{456\ MPa}$$

τ AT KEYSEAT AT B: D= 15 mm, K_t = 2.0 FOR KEYSEAT

$T = T_{BC} = 109.1$ N·m

$$\tau = \frac{T_{BC}\cdot K_t}{Z_P} = \frac{(109.1)(2.0)(1000)}{\pi(15)^3/16} = \underline{329\ MPa}$$

τ TO RIGHT OF B AT SHOULDER FILLET: D_1=15 mm, D_2=20 mm

$D_2/D_1 = 20/15 = 1.33$; $r/D_1 = 0.50\text{mm}/15\text{mm} = 0.033$; $K_t = 1.75$

$$\tau = \frac{T_{BC}\cdot K_t}{Z_P} = \frac{(109.1)(1.75)(1000)}{\pi(15)^3/16} = \underline{288\ MPa}$$

τ AT RIGHT BEARING AT SHOULDER FILLET: D_1=15 mm, D_2=20mm

SAME CONDITIONS AS AT PULLEY: $\underline{\tau = 288\ MPa}$

τ AT C AT RETAINING RING GROOVE: K_t=3.0; D_g=14.0 mm

$$\tau = \frac{T_{BC}\cdot K_t}{Z_P} = \frac{(109.1)(3.0)(1000)}{\pi(14.0)^3/16} = \underline{607\ MPa}$$

τ AT C AT SLED RUNNER KEY SEAT: D=15 mm; K_t = 1.60

$$\tau = \frac{T_{BC}\cdot K_t}{Z_P} = \frac{(109.1)(1.60)(1000)}{\pi(15)^3/16} = \underline{263\ MPa}$$

SUMMARY SEVERAL STRESSES ARE QUITE HIGH. LARGER SHAFT DIAMETERS RECOMMENDED.

4-67 FIND τ IN EACH PART OF SHAFT.

$$n = \frac{1150\text{ rev}}{\text{min}}\cdot\frac{2\pi\text{ rad}}{\text{rev}}\cdot\frac{1\text{ min}}{60\text{ s}} = 120.4\text{ rad/s}$$

$$T_A = \frac{P_A}{n} = \frac{20\times10^3\text{ N·m/s}}{120.4\text{ rad/s}} = 166\text{ N·m}\times\frac{10^3\text{ mm}}{\text{m}} = 1.66\times10^5\text{ N·mm}$$

$$T_C = \frac{P_C}{n} = \frac{12\times10^3\text{ N·m/s}}{120.4\text{ rad/s}} = 99.6\text{ N·m}\times\frac{10^3\text{ mm}}{\text{m}} = 9.96\times10^4\text{ N·mm}$$

$$T_B = \frac{P_B}{n} = \frac{32\times10^3\text{ N·m/s}}{120.4\text{ rad/s}} = 266\text{ N·m}\times\frac{10^3\text{ mm}}{\text{m}} = 2.66\times10^5\text{ N·mm}$$

$T_{AB} = T_A = 1.66\times10^5$ N·mm $T_{BC} = T_C = 9.96\times10^4$ N·mm

(CONTINUED NEXT PAGE)

4-67 (CONTINUED)

τ_1 AT A AT KEYSEAT: $D = 20.0\,mm$; $K_t = 2.0$ - PROFILE KEYSEAT

$$\tau_1 = \frac{T_{AB}\,K_t}{Z_p} = \frac{1.66\times10^5\,N\cdot mm\,(2.0)}{\pi\,(20\,mm)^3/16} = 211\,N/mm^2 = \boxed{211\,MPa = \tau_{MAX}}$$

τ_2 AT SHOULDER TO RIGHT OF A: $D = 20.0\,mm$; $D/d = \frac{30}{20} = 1.50$

$r/d = 1.0/20 = 0.05$; $K_t = 1.62$

$$\tau_2 = \frac{T_{AB}\,K_t}{Z_p} = \frac{(1.66\times10^5)(1.62)}{\pi\,(20)^3/16} = \underline{171\,MPa}$$

τ_3 AT RIGHT OF BEARING SEAT: $D = 30\,mm$; $D/d = \frac{40}{30} = 1.33$

$r/d = 1.0/30 = 0.033$; $K_t = 1.78$

$$\tau_3 = \frac{T_{AB}\,K_t}{Z_p} = \frac{(1.66\times10^5)(1.78)}{\pi\,(30)^3/16} = \underline{55.7\,MPa}$$

τ_4 AT RETAINING RING TO LEFT OF B: $D = 40.0\,mm$; $K_t = 3.0$

$$\tau_4 = \frac{T_{AB}\,K_t}{Z_p} = \frac{(1.66\times10^5)(3.0)}{\pi\,(40)^3/16} = \underline{39.6\,MPa}$$

τ_5 AT KEY SEAT AT B: $K_t = 2.0$, $D = 40\,mm$

$$\tau_5 = \frac{T_{AB}\,K_t}{Z_p} = \tau_4\cdot\frac{K_{t5}}{K_{t4}} = 39.6\,MPa\,\frac{2.0}{3.0} = \underline{26.4\,MPa}$$

τ_6 AT STEP TO RIGHT OF B: $D = 40\,mm$, $D/d = \frac{50.0}{40.0} = 1.25$

$r/d = 1.0/40 = 0.025$; $K_t = 1.85$

$$\tau_6 = \frac{T_{BC}\,K_t}{Z_p} = \frac{(9.96\times10^4)(1.85)}{\pi\,(40)^3/16} = \underline{14.7\,MPa}$$

τ_7 AT STEP FROM 50 TO 30 mm DIA.: $D = 30.0\,mm$, $D/d = 50/30 = 1.67$

$r/d = 1.0/30 = 0.033$; $K_t = 1.82$

$$\tau_7 = \frac{T_{BC}\,K_t}{Z_p} = \frac{(9.96\times10^4)(1.82)}{\pi\,(30)^3/16} = \underline{10.7\,MPa}$$

τ_8 AT LEFT OF BEARING: $D = 20.0\,mm$; $D/d = 30/20 = 1.50$

$r/d = 1.0/20 = 0.05$; $K_t = 1.62$

$$\tau_8 = \frac{T_{BC}\,K_t}{Z_p} = \frac{(9.96\times10^4)(1.62)}{\pi\,(20)^3/16} = \underline{10.3\,MPa}$$

τ_9 AT STEP TO LEFT OF C: $D = 15.0\,mm$; $D/d = 20/15 = 1.33$; $r/d = 1/15 = 0.067$

$K_t = 1.50$: $\tau_9 = \frac{T_{BC}\,K_t}{Z_p} = \frac{(9.96\times10^4)(1.50)}{\pi\,(15)^3/16} = \underline{22.5\,MPa}$

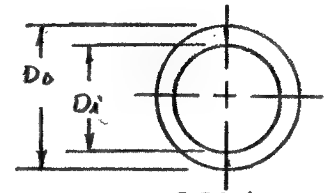
τ_{10} AT KEYSEAT AT C: $\tau_{10} = \frac{T_{BC}\,K_t}{Z_p} = \tau_9\,\frac{K_{t10}}{K_{t9}} = (22.5)\,\frac{2.0}{1.5} = \underline{30.1\,MPa}$

$K_t = 2.0$.

4-68 DESIGN SHAFT $P = 225\,kW$; $n = 80\,rpm$; $\tau_d = 60\,MPa$; $K_t = 1.0$

$$T = \frac{P}{n} = \frac{225 \times 10^3\,N \cdot m/s}{8.38\,rad/s} = 26\,857\,N \cdot m$$

$$n = 80\,\frac{REV}{MIN} \cdot \frac{2\pi\,RAD}{REV} \cdot \frac{1\,MIN}{60\,S} = 8.38\,rad/s$$



$D_o \approx 1.25 D_i$

$$\tau = \frac{T}{Z_P}\text{; REQ'D } Z_P = \frac{T}{\tau_d} = \frac{26857\,N \cdot m}{60\,N/mm^2} \cdot \frac{10^3\,mm}{m}$$

$$Z_P = 4.476 \times 10^5\,mm^3 = \frac{\pi}{16}\,\frac{D_o^4 - D_i^4}{D_o}$$

$$\text{BUT } Z_P = \frac{\pi[(1.25D_i)^4 - D_i^4]}{(16)\,1.25D_i} = \frac{\pi(1.44D_i^4)}{(16)(1.25D_i)} = 0.226\,D_i^3$$

$$\text{THEN REQ'D } D_i = \sqrt[3]{\frac{Z_P}{0.226}} = \sqrt[3]{\frac{4.476 \times 10^5\,mm^3}{0.226}} = 125.5\,mm$$

$$D_o \approx 1.25\,D_i = 1.25(125.5) = 157\,mm$$

LET $\underline{D_o = 160\,mm}$; $D_i \approx \frac{D_o}{1.25} = \frac{160}{1.25} = 128\,mm$; USE $\underline{D_i = 125\,mm}$

$$\text{CHECK } Z_P = \frac{\pi}{16}\,\frac{D_o^4 - D_i^4}{D_o} = \frac{\pi[(160^4) - (125^4)]}{(16)(160)} = 5.05 \times 10^5\,mm^3$$

OK BUT HIGH

TRY $\underline{D_o = 160\,mm}$, $\underline{D_i = 130\,mm}$

$$Z_P = \frac{\pi[(160)^4 - (130)^4]}{16\,(160)} = 4.54 \times 10^5\,mm^3 \quad \text{OK}$$

OR: LET $D_o = 160\,mm$, SOLVE FOR REQ'D. $D_{i\,MAX}$ FOR $Z_P = 4.476 \times 10^5\,mm^3$

$$Z_P = \frac{\pi[(160)^4 - D_i^4]}{16\,(160)}\text{; } (16)(160)\,Z_P = \pi[160^4 - D_i^4]$$

$$\frac{16(160)\,Z_P}{\pi} = 160^4 - D_i^4\text{; } D_i^4 = 160^4 - \frac{16(160)(4.476 \times 10^5)}{\pi}$$

$D_{i\,MAX} = 130.6\,mm$: $\underline{\text{USE } D_o = 160\,mm;\ D_i = 130\,mm}$

$$\text{CHECK FOR WALL THICKNESS : } t = \frac{D_o - D_i}{2} = \frac{160 - 130}{2} = 15\,mm$$

$$\text{MEAN RADIUS} = \frac{(D_o + D_i)/2}{2} = 72.5\,mm$$

$r_{MEAN}/t = 72.5/15 = 4.83 < 10$. SHAFT IS NOT THIN-WALLED

BUCKLING NOT LIKELY.

4-69 $D = 4.0\ mm$; $\theta = 180\ DEG \times \frac{\pi\ RAD}{180°} = \pi\ RAD$; $\tau_{MAX} = 150\ MPa = \frac{Tc}{J}$

$$J = \frac{\pi D^4}{32} = \frac{\pi(4.0\ mm)^4}{32} = 25.1\ mm^4$$

$$T_{MAX} = \frac{\tau_{MAX} J}{c} = \frac{(150\ N/mm^2)(25.1\ mm^4)}{2.0\ mm} = 1885\ N \cdot mm$$

$$\theta = \frac{TL}{GJ} ;\ L_{MIN} = \frac{\theta G J}{T_{MAX}}$$

$$G = 26\ GPa = \frac{26 \times 10^9\ N}{m^2} \times \frac{1\ m^2}{(10^3\ mm)^2} = 26.0 \times 10^3\ N/mm^2$$

$$\underline{L_{MIN}} = \frac{(\pi\ RAD)(26 \times 10^3\ N/mm^2)(25.1\ mm^4)}{1885\ N \cdot mm} = 1088\ mm = \underline{1.088\ m}$$

4-70 TORSION BAR: $L = 200\ mm = 0.200\ m$. $D_o/D_i \approx 1.50$

$$\text{TORSIONAL STIFFNESS} = \frac{\theta}{T} = \frac{0.015\ DEG}{1.0\ N \cdot m} \times \frac{\pi\ RAD}{180\ DEG} = \frac{0.2618 \times 10^{-3}\ RAD}{1.0\ N \cdot m}$$

$$\theta = \frac{TL}{GJ} ;\ REQ'D\ J = \frac{TL}{\theta G}$$

$$G = 43\ GPa = (43 \times 10^9\ N/m^2)(1.0\ m^2/10^6\ mm^2) = 43 \times 10^3\ N/mm^2$$

$$J = \frac{(1.0\ N \cdot m)(200\ mm)}{(0.2618 \times 10^{-3}\ RAD)(43 \times 10^3\ N/mm^2)} \times \frac{10^3\ mm}{1\ m} = 17766\ mm^4$$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32} = \frac{\pi[(1.5 D_i)^4 - D_i^4]}{32} = 0.3988\ D_i^4$$

$$D_i = \sqrt[4]{\frac{J}{0.3988}} = \sqrt[4]{\frac{17766\ mm^4}{0.3988}} = \underline{14.53\ mm} = D_i$$

$$D_o = 1.50(D_i) = 1.50(14.53\ mm) = \underline{21.79\ mm} = D_o$$

ALTERNATE DESIGN: PREFERRED SIZE FOR $D_o = 22.0\ mm$

FIND REQ'D D_i FOR $J = 17766\ mm^4$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32} ;\ D_o^4 - D_i^4 = \frac{32 J}{\pi} ;\ D_i^4 = D_o^4 - \frac{32 J}{\pi}$$

$$D_i = \sqrt[4]{D_o^4 - \frac{32J}{\pi}} = \sqrt[4]{22.0^4 - \frac{32(17766)}{\pi}} = \underline{15.19\ mm = D_i}$$

FOR $D_o = 22.0\ mm$

4-71 USE FIRST DESIGN FROM 4-70. $D_o = 21.79\ mm$, $D_i = 14.53\ mm$

$$\tau = \frac{Tc}{J} = \frac{T D_o}{J\,2} ;\ \theta = \frac{TL}{GJ} \text{ OR } T = \frac{\theta G J}{L}$$

$$\text{THEN } \tau = \frac{\theta G J}{L} \cdot \frac{D_o}{2J} = \frac{\theta G D_o}{2L} = \frac{(0.1745\ RAD)(43 \times 10^3\ N/mm^2)(21.79\ mm)}{2(200\ mm)}$$

$$\theta = 10° \times \frac{\pi\ RAD}{180°} = 0.1745\ RAD$$

$$\tau = 408.8\ N/mm^2 = \underline{408.8\ MPa}$$

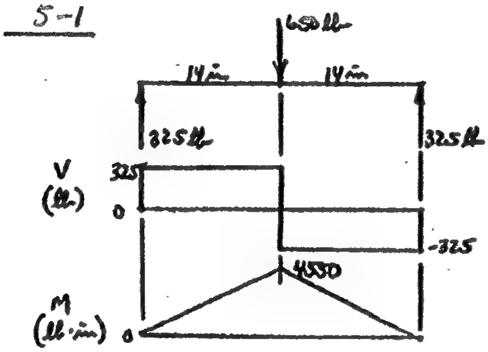
4-72 FIND θ WHEN $\tau_d = \frac{S_{ys}}{3} = \frac{S_y}{2(3)} = \frac{S_y}{6} = \frac{903\ MPa}{6} = 150.5\ MPa$

AISI 4140 OQT 1100; $S_y = 903\ MPa$, 18% ELONGATION

$$\tau = \frac{TC}{J} \; : \; T = \frac{\tau_d J}{C} = \frac{(150.5\ N/mm^2)(17766\ mm^4)}{21.79\ mm/2} \times \frac{1\ m}{1000\ mm} = 245.4\ N \cdot m$$

$$\theta = \frac{TL}{GJ} = \frac{(245.4\ N \cdot m)(200\ mm)}{(43\,000\ N/mm^2)(17766\ mm^4)} \times \frac{10^3\ mm}{m} = 0.064\ RAD \times \frac{180°}{\pi\ RAD} = 3.68\ DEG$$

5-1



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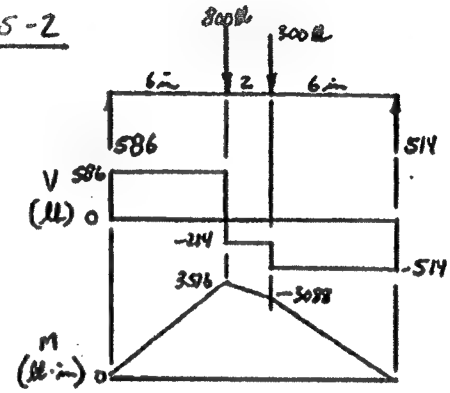

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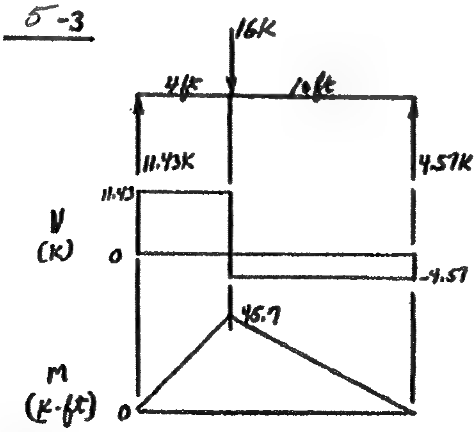

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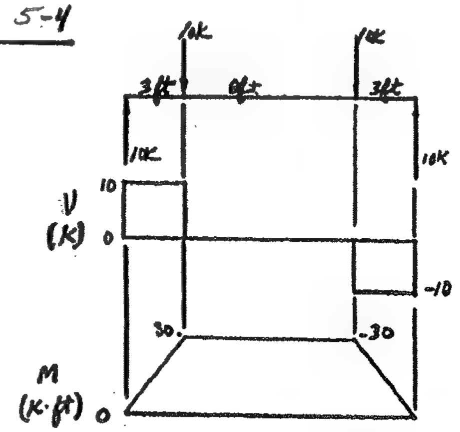

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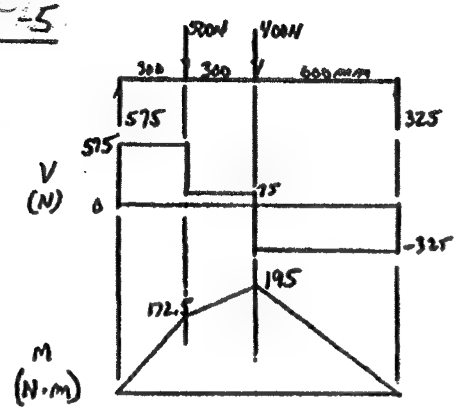

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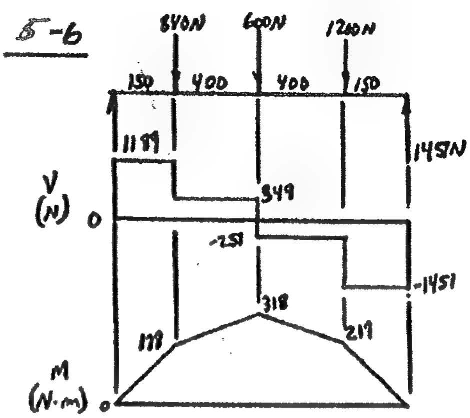

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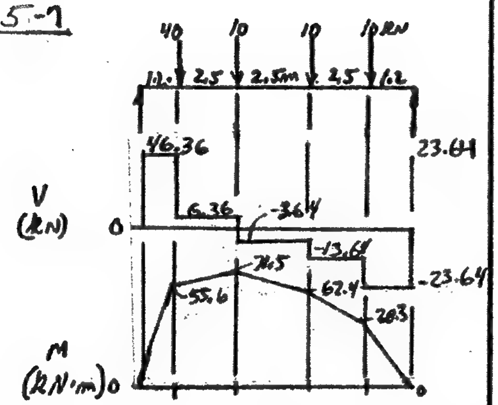

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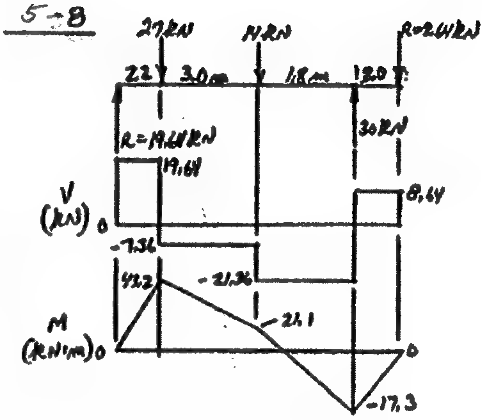

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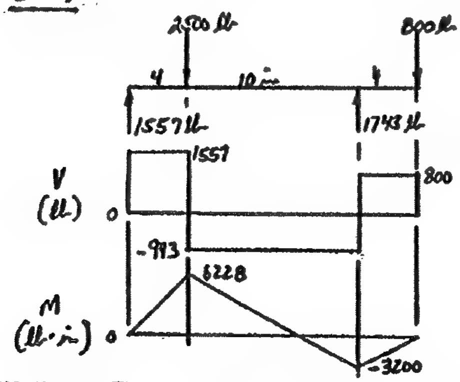

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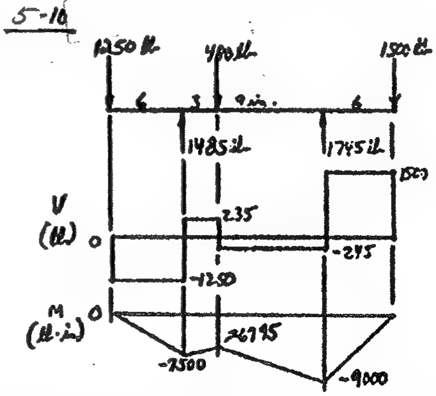

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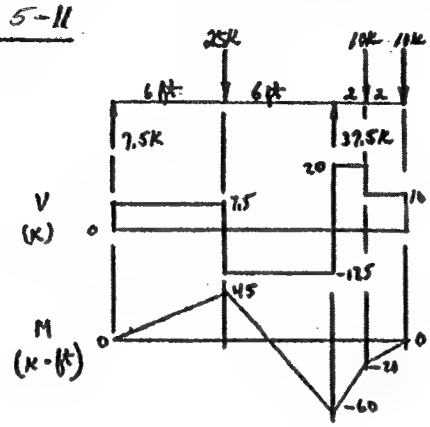

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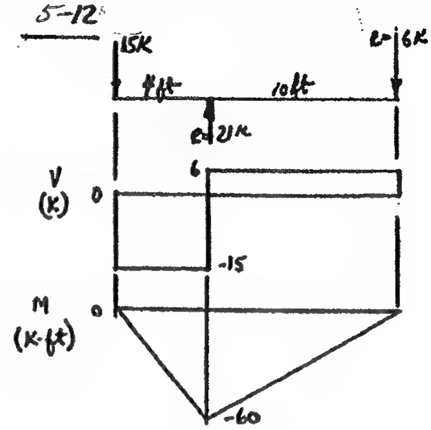

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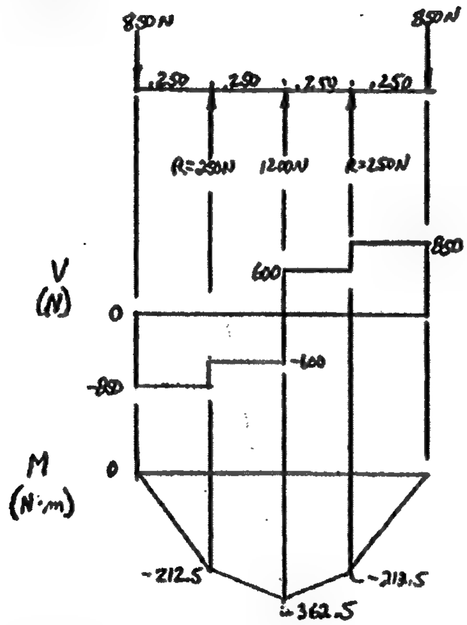

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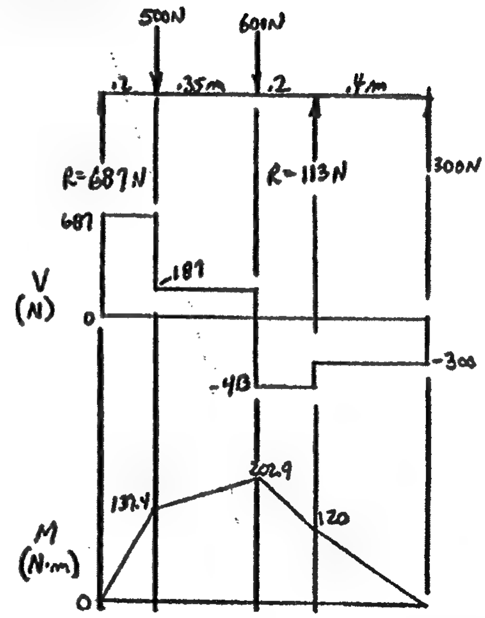

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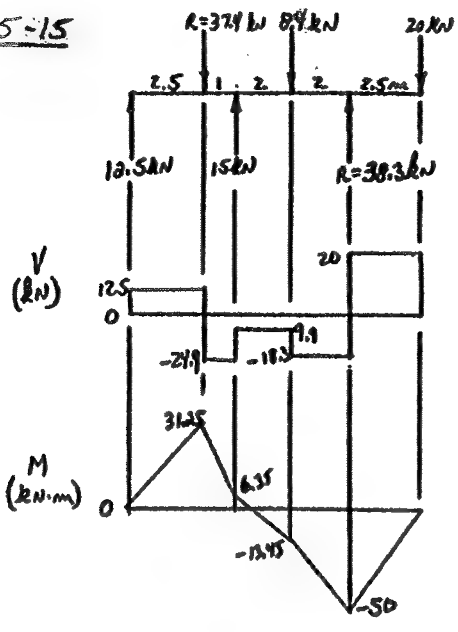

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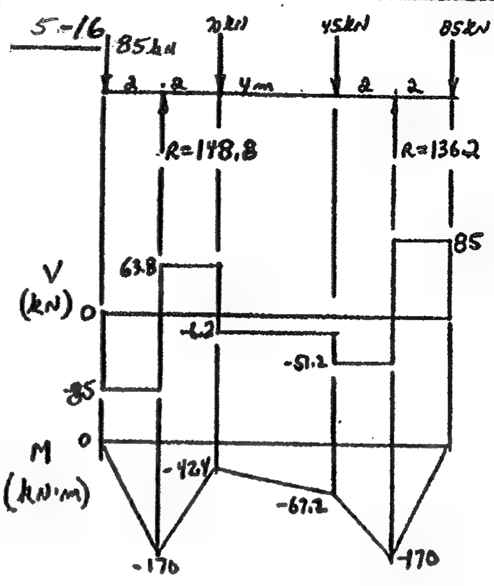

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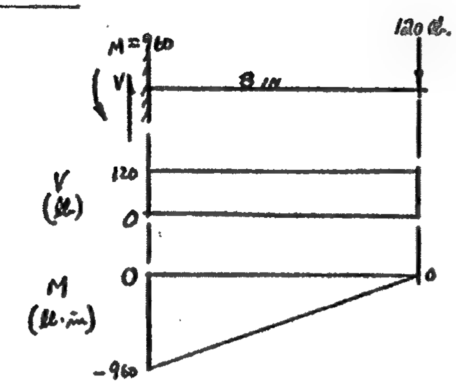

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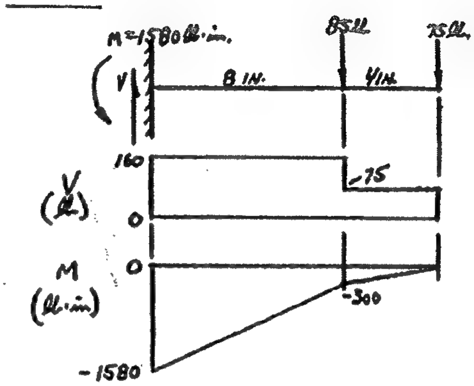

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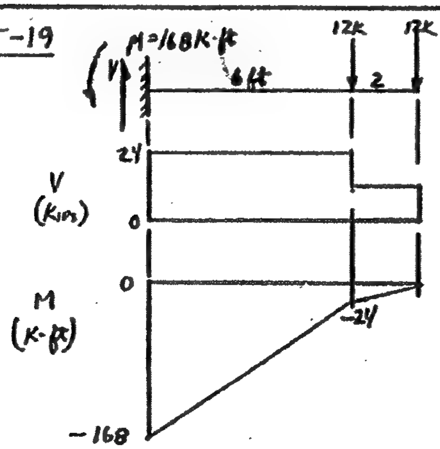

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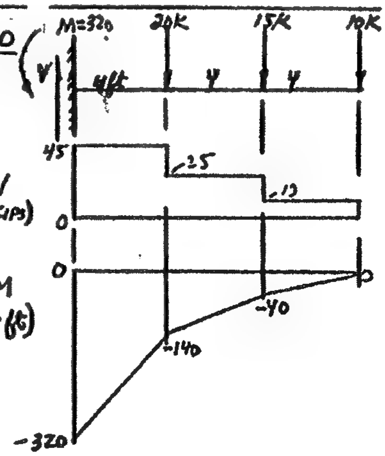

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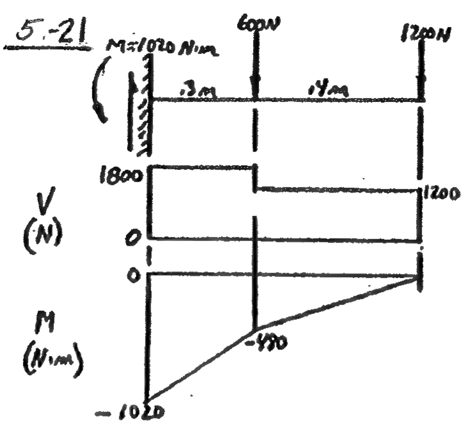

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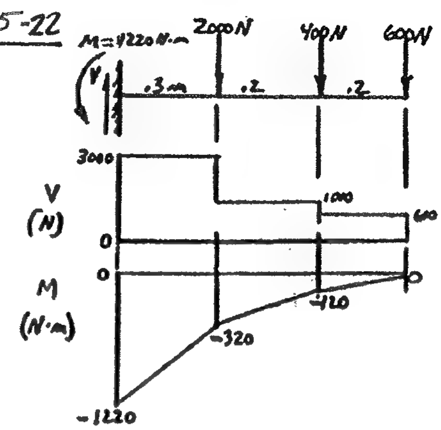

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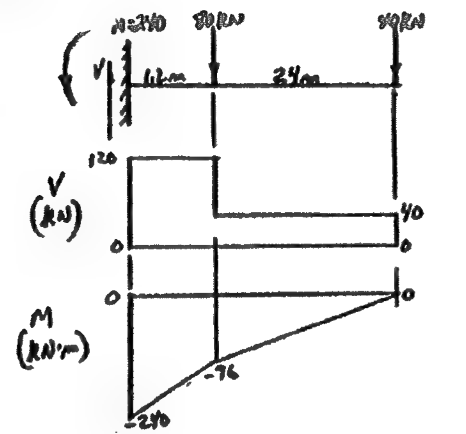

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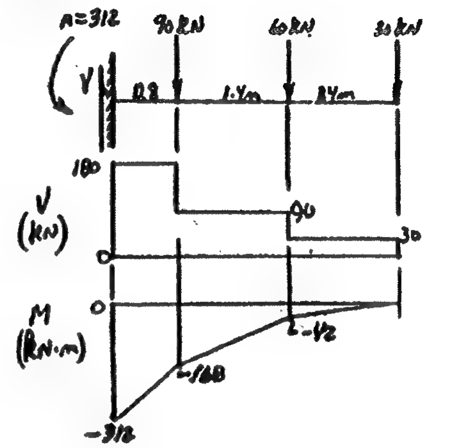

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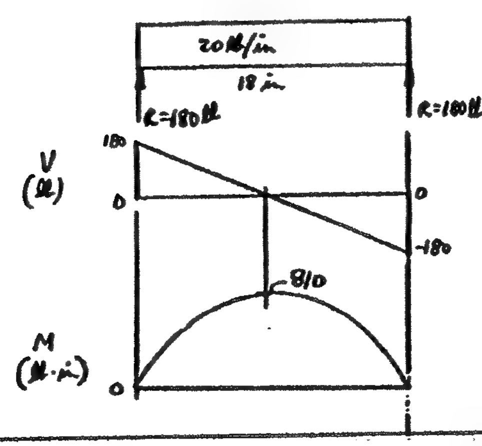

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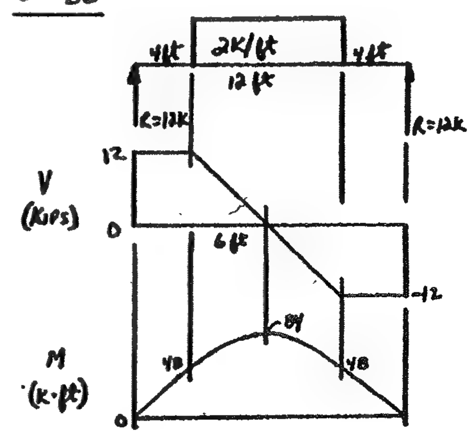

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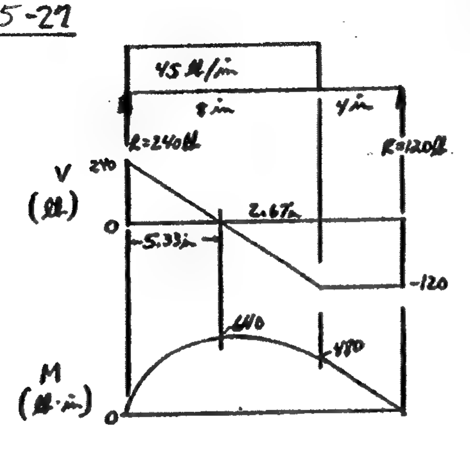

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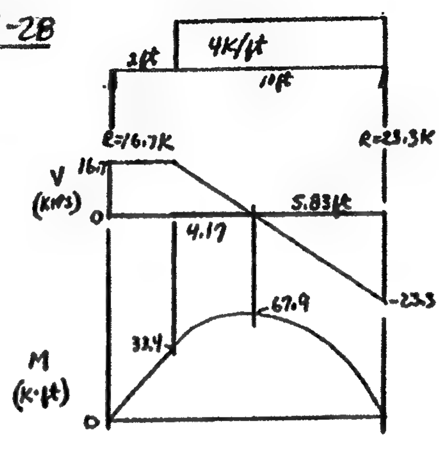

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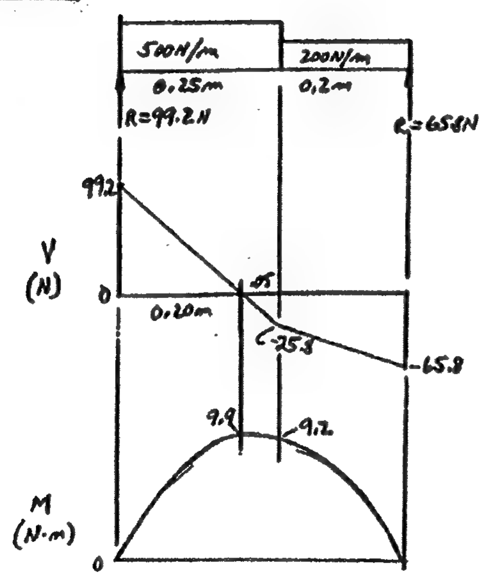

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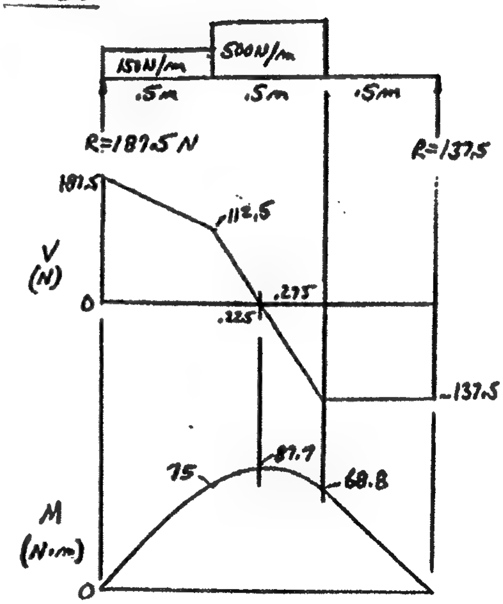

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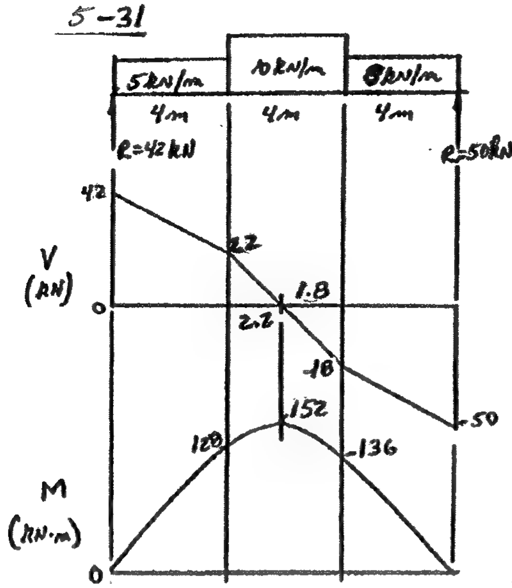

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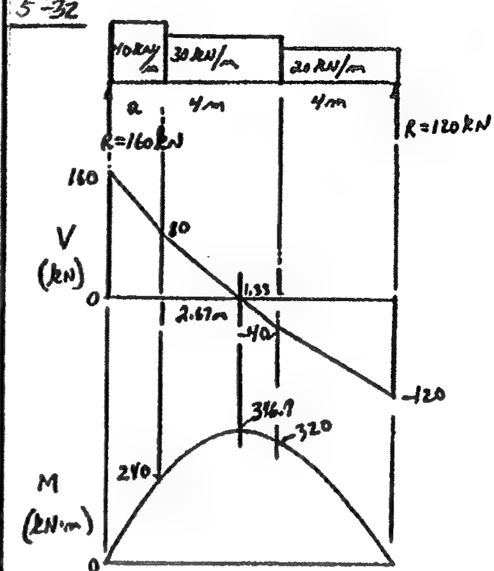

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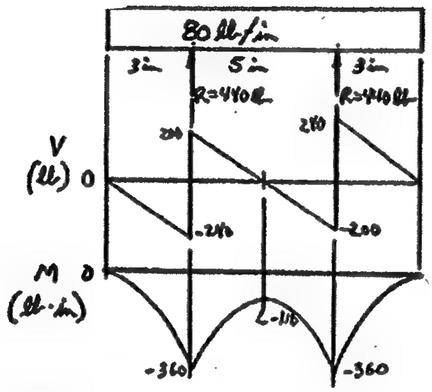

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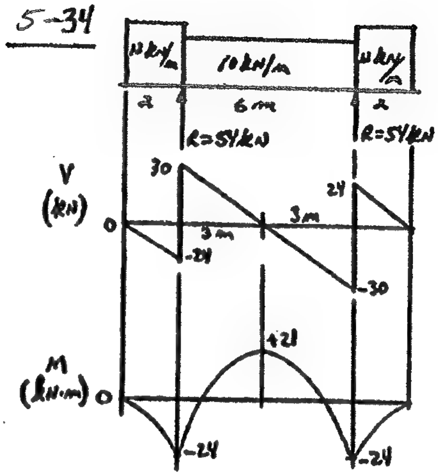

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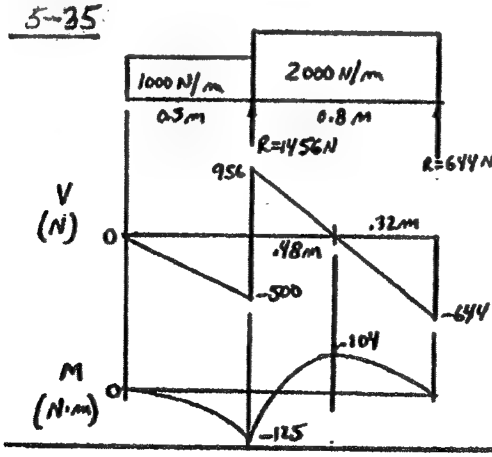

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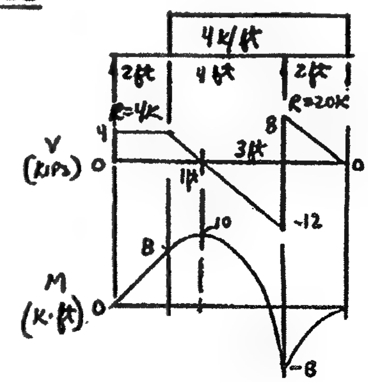

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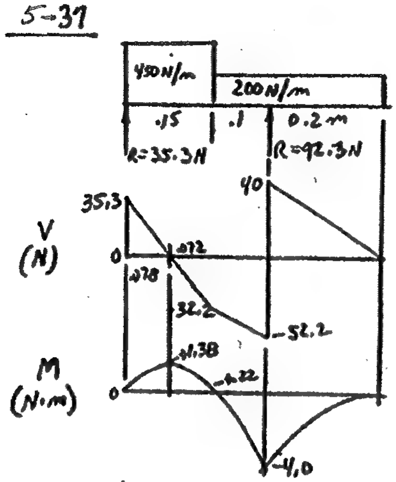

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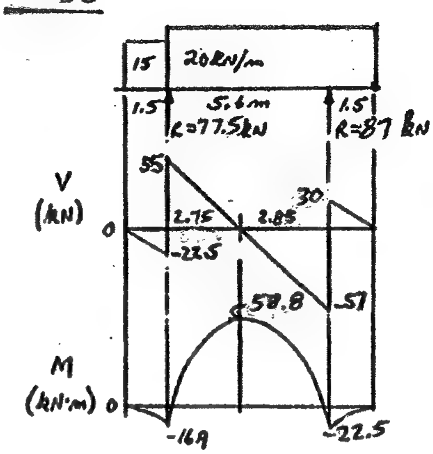

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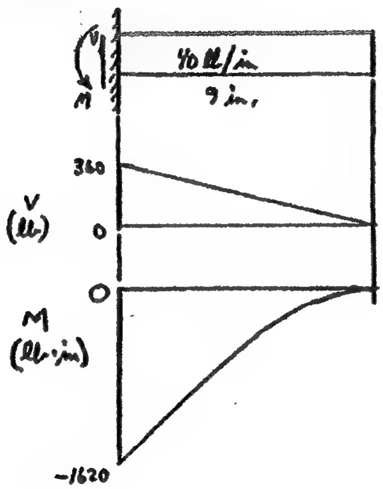

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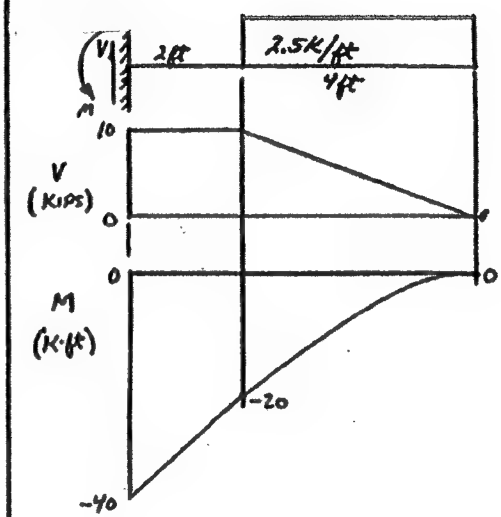

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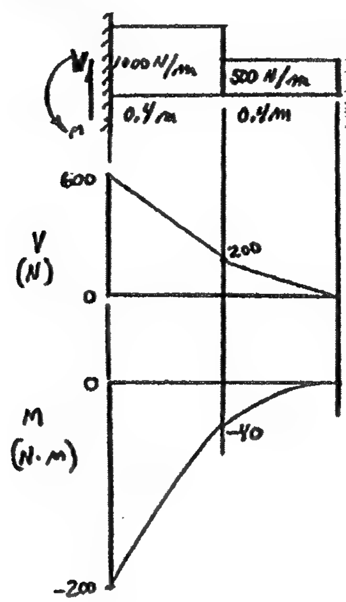

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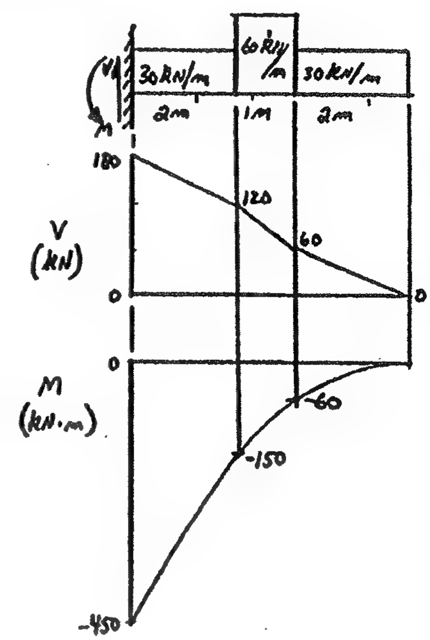

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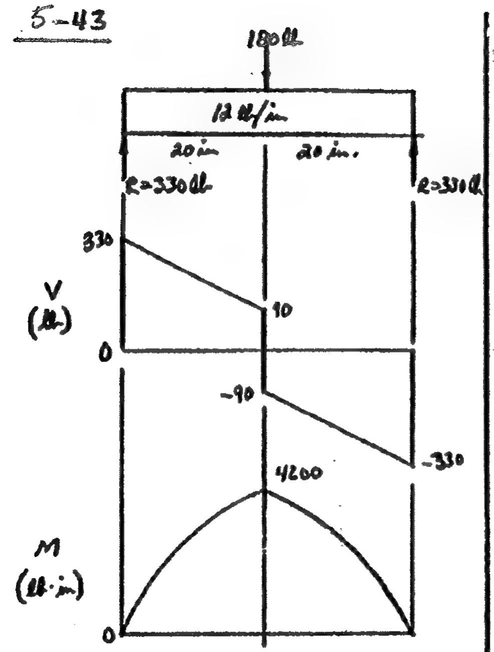

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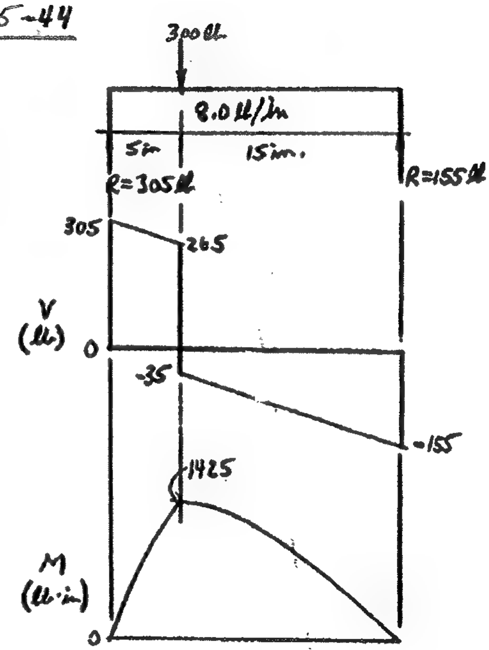

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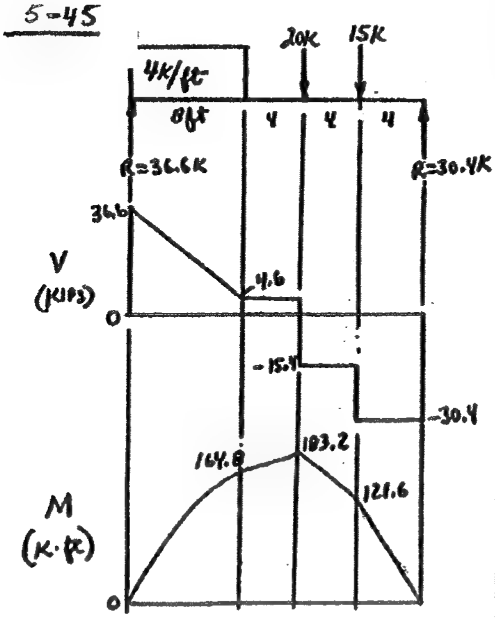

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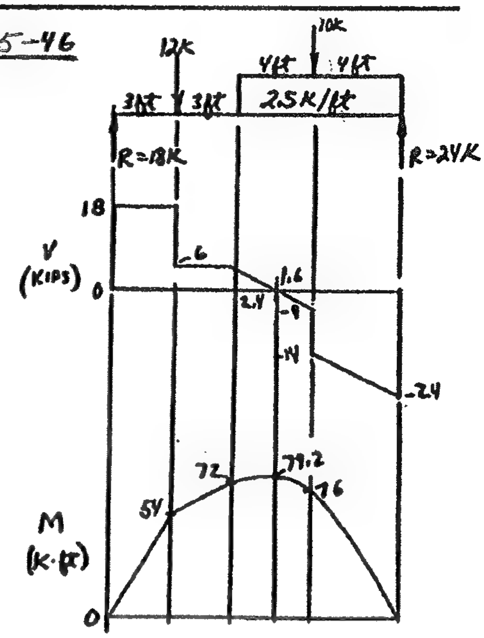

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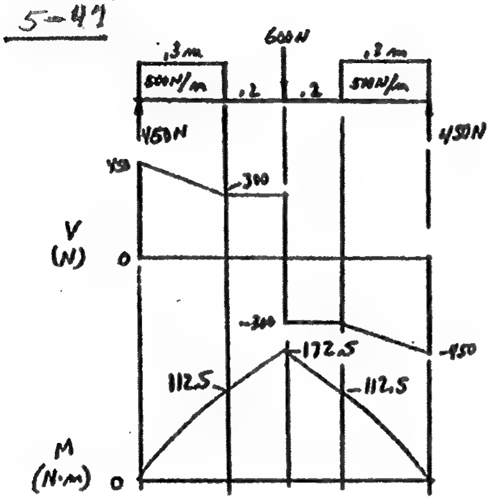

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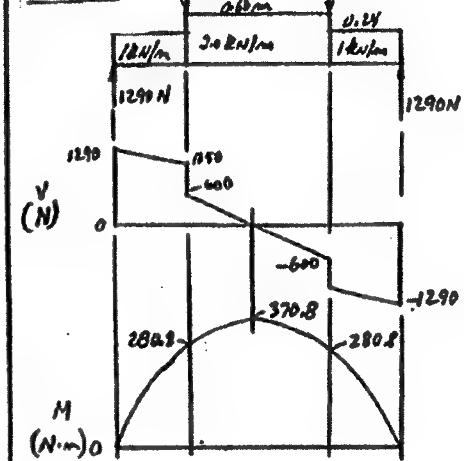

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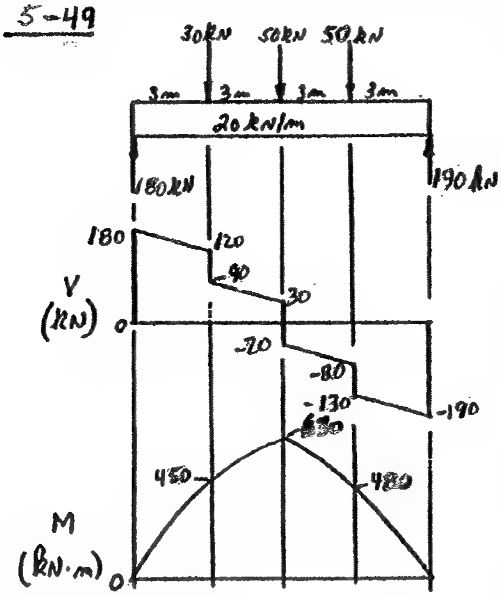

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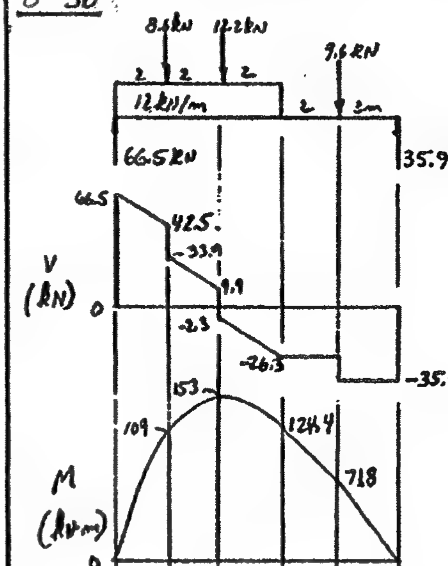

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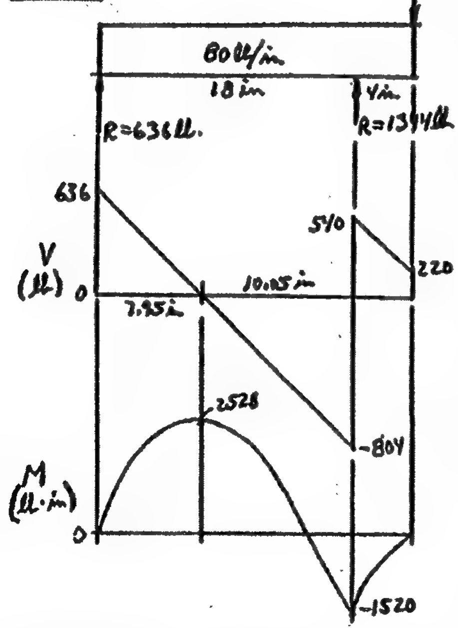

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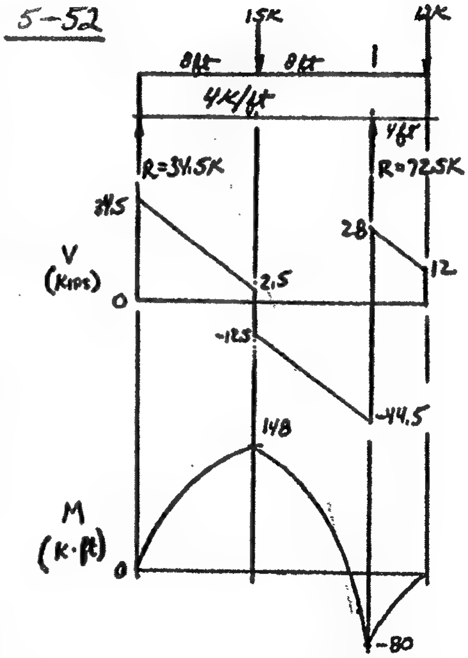

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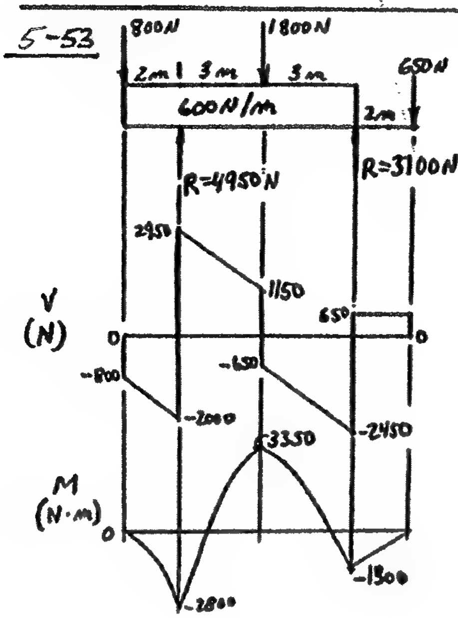

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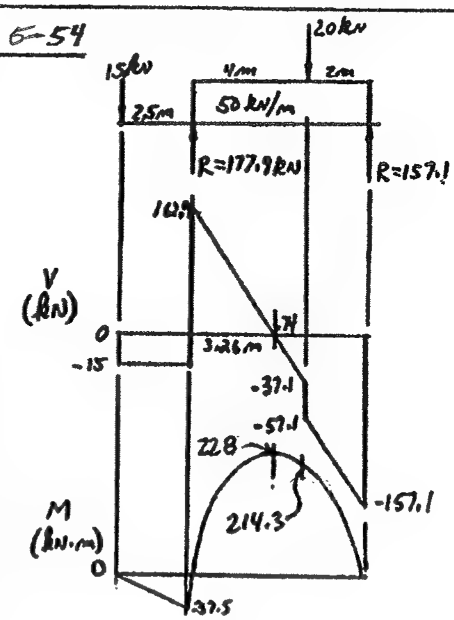

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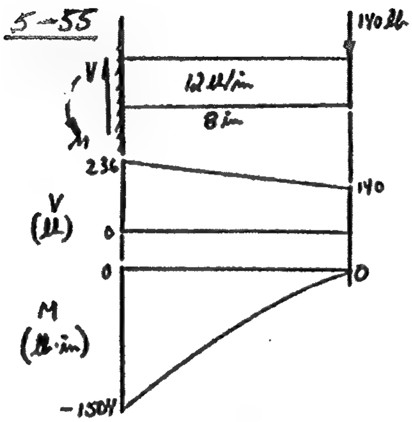

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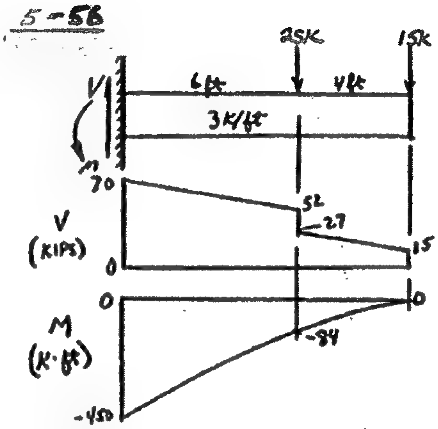

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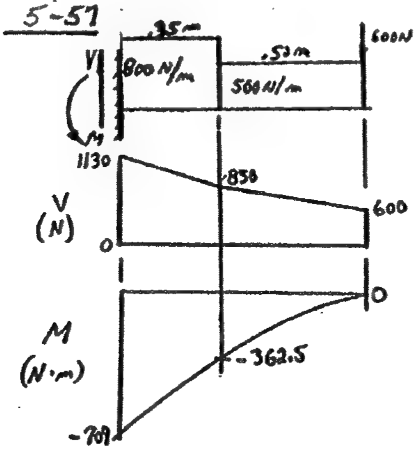

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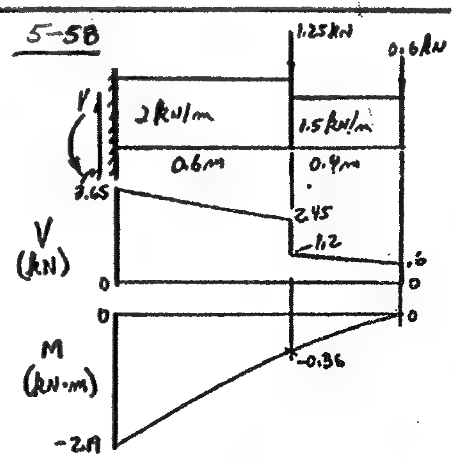

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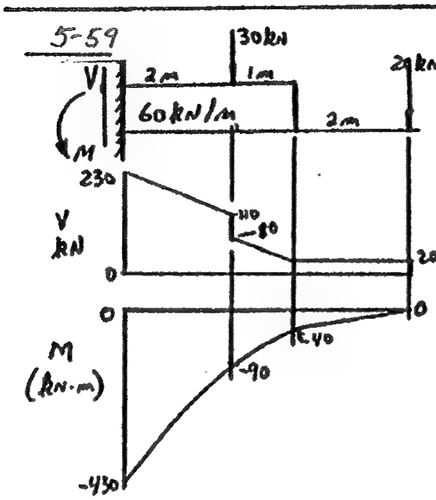

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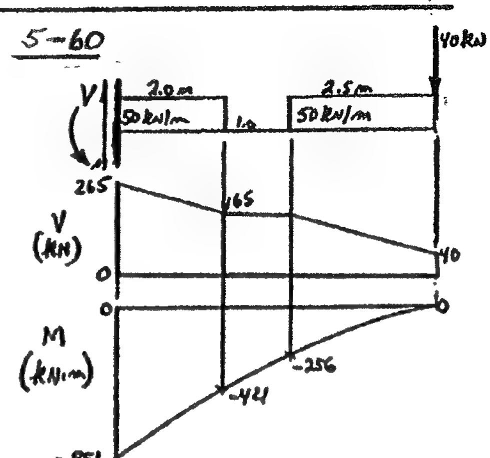

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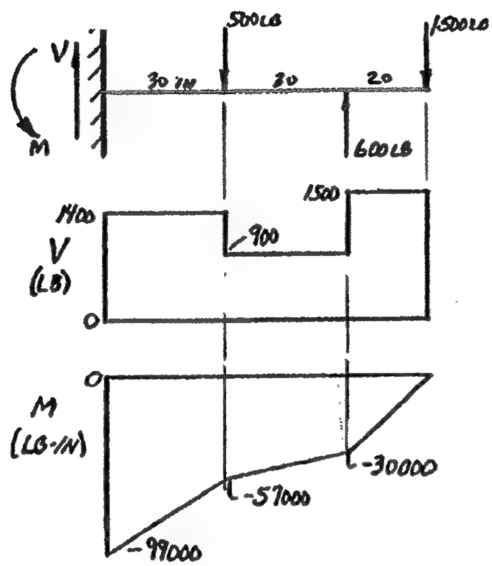

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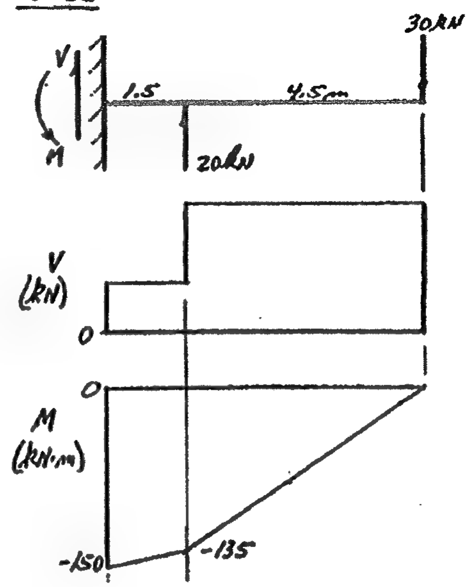
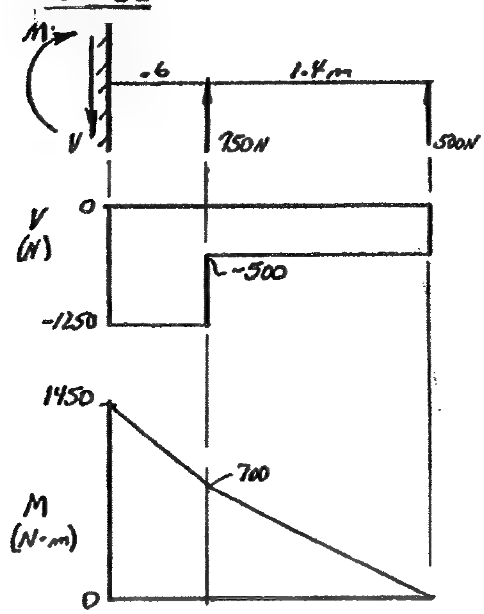
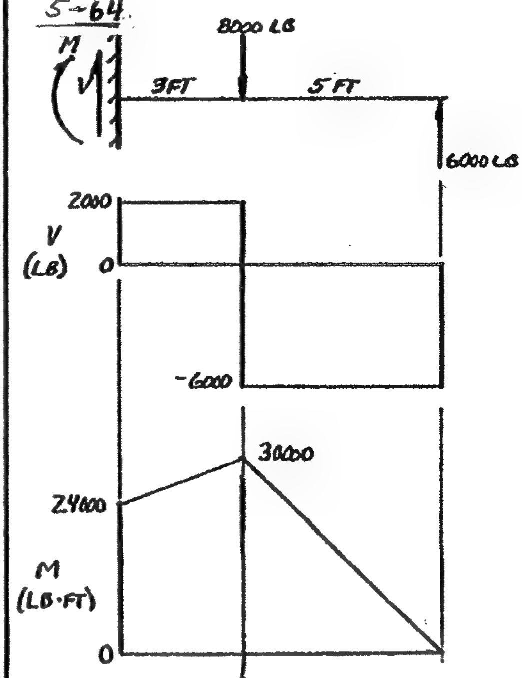


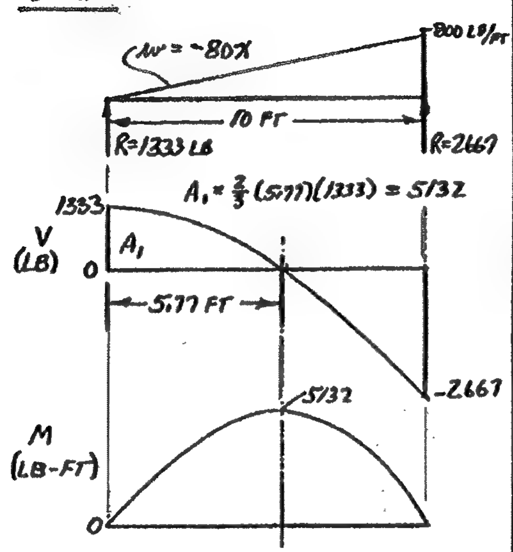
5-63



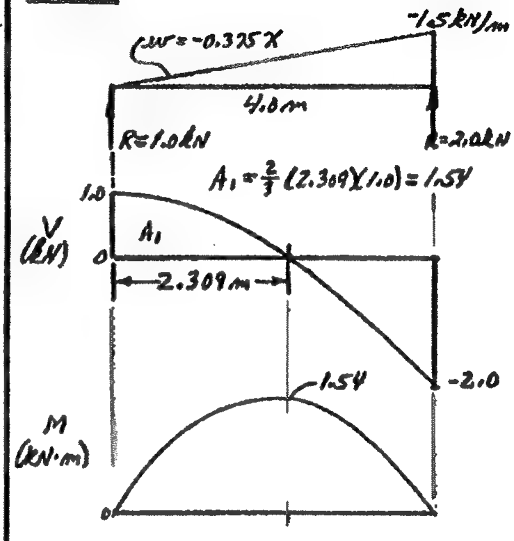
5-64



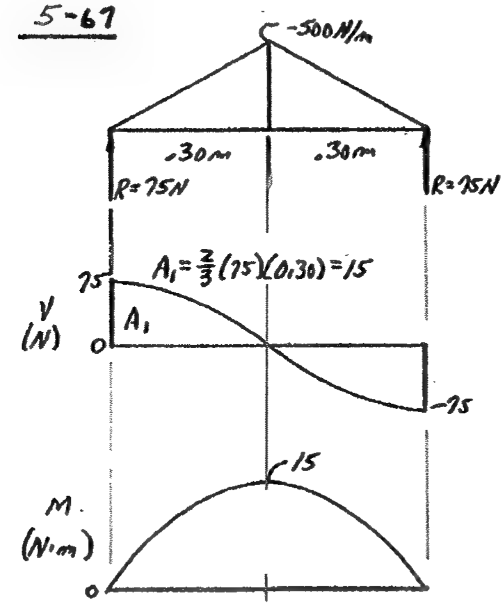
5-65



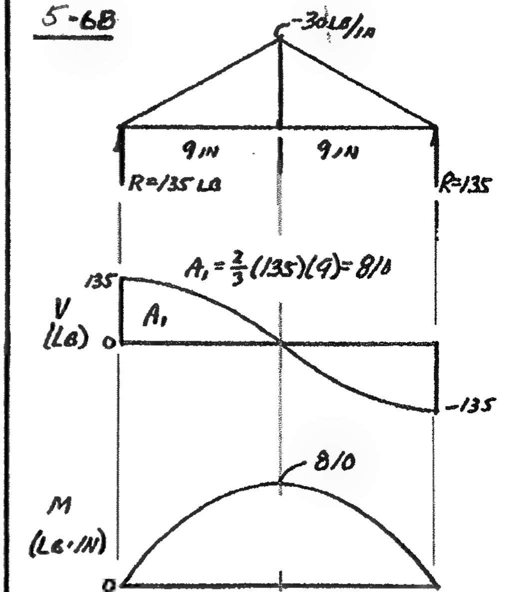
5-66



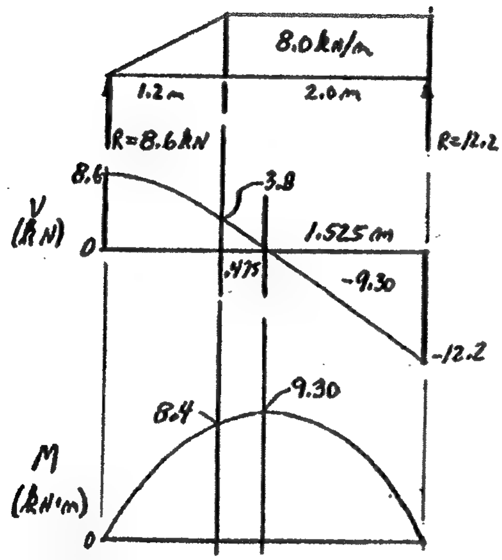
5-67



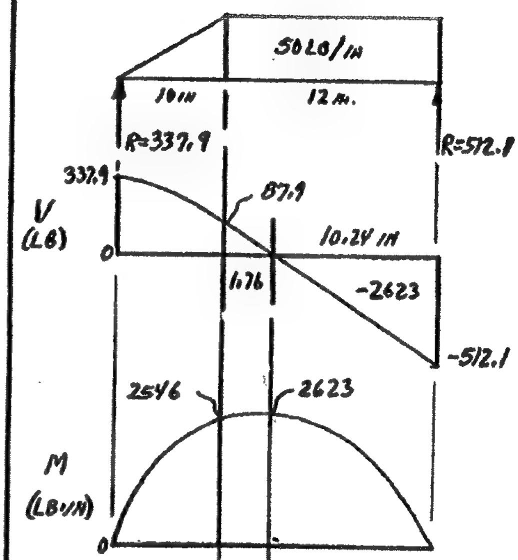
5-68



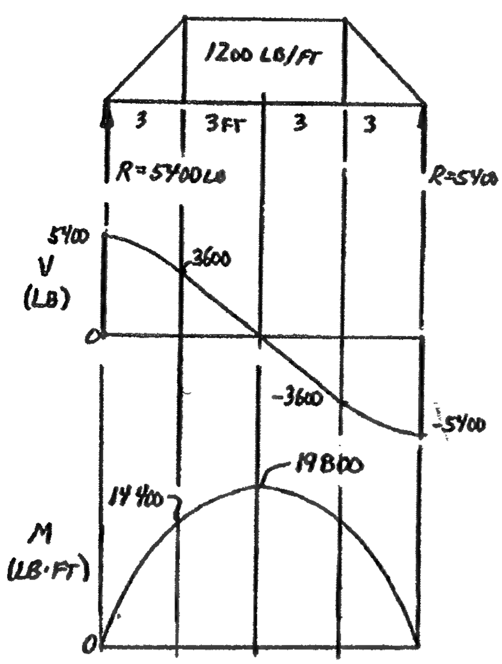
5-69



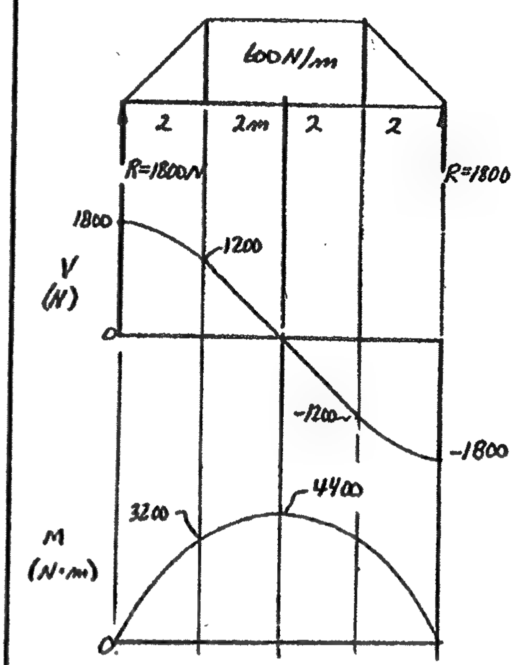
5-70



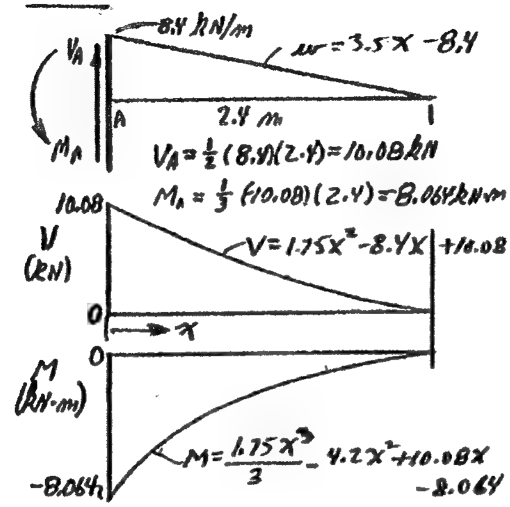
5-71



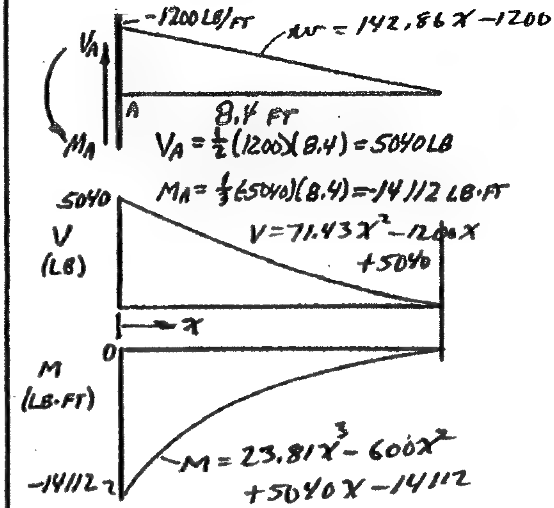
5-72



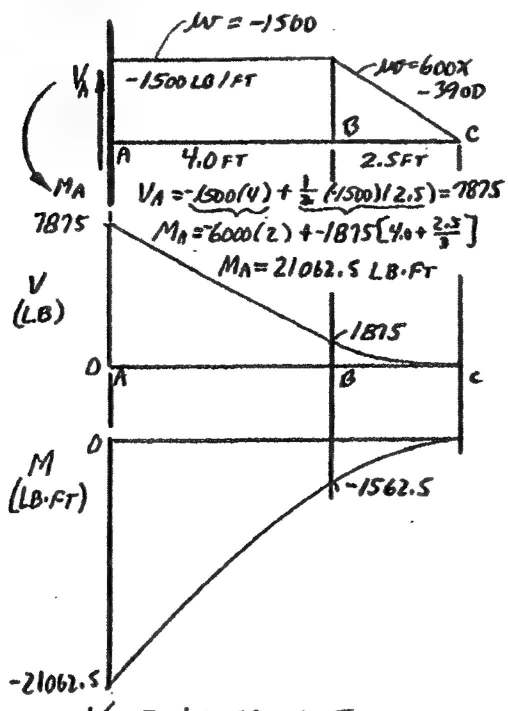
5-73



5-74



5-75



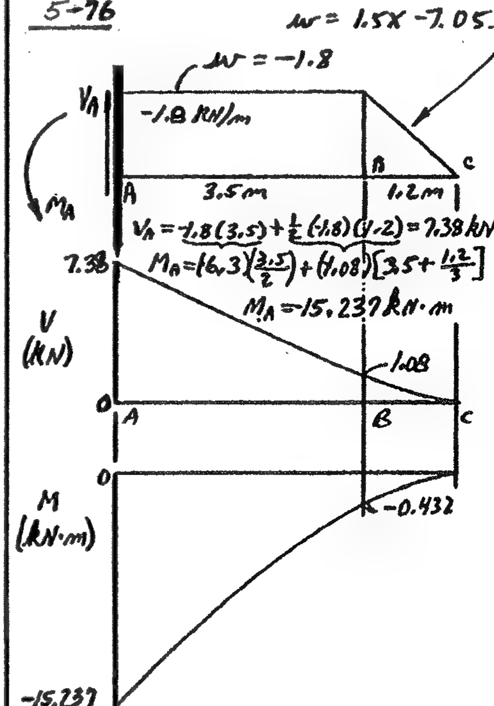
$V_{AB} = -1500x + 7875$

$M_{AB} = -750x^2 + 7875x - 21062$

$V_{BC} = 300x^2 - 3900x + 12675$

$M_{BC} = 100x^3 - 1950x^2 + 12675x - 27462.5$

5-76



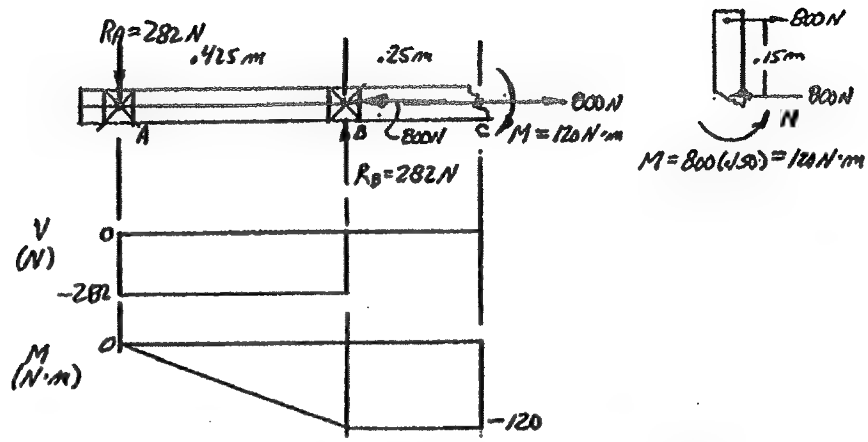
$V_{AB} = -1.8x + 7.38$

$M_{AB} = -0.9x^2 + 7.38x - 15.237$

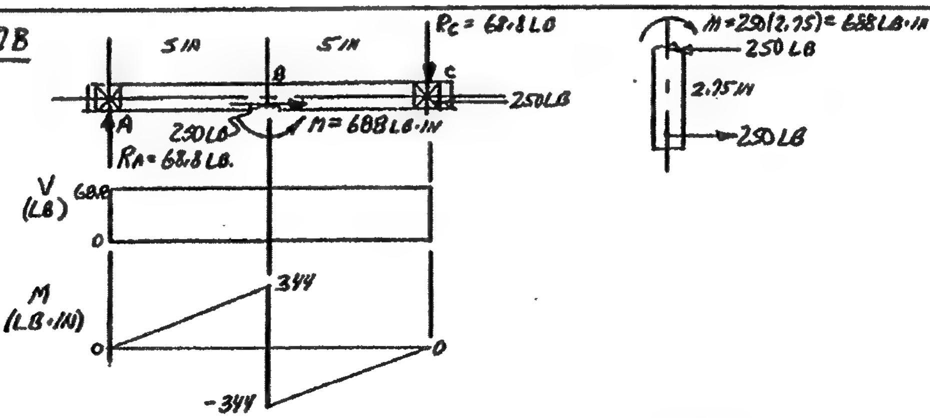
$V_{BC} = 0.75x^2 - 7.05x + 16.57$

$M_{BC} = 0.25x^3 - 3.525x^2 + 16.57x - 25.96$

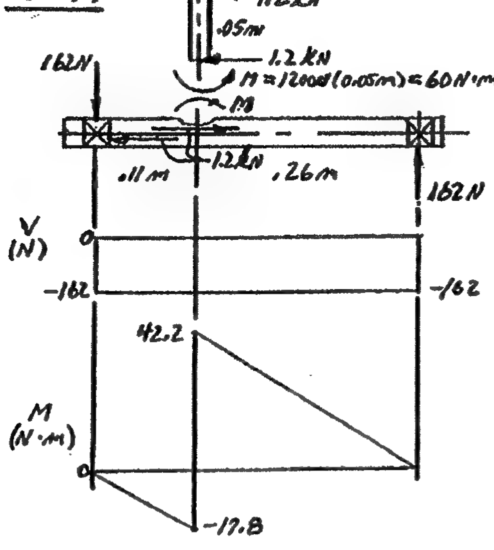
5-77



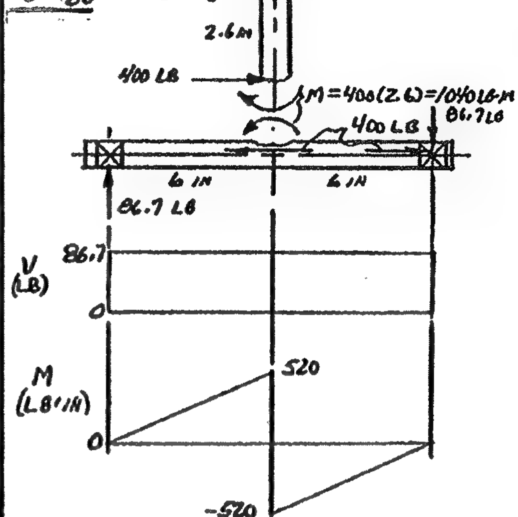
5-78



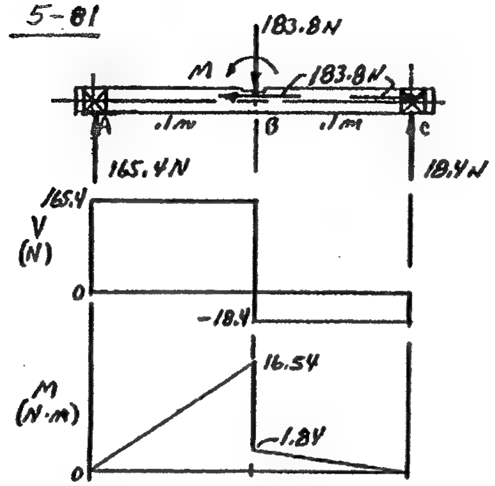
5-79

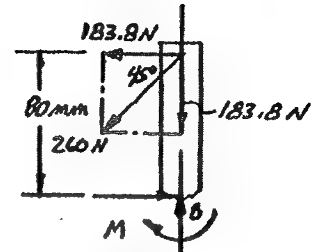


5-80



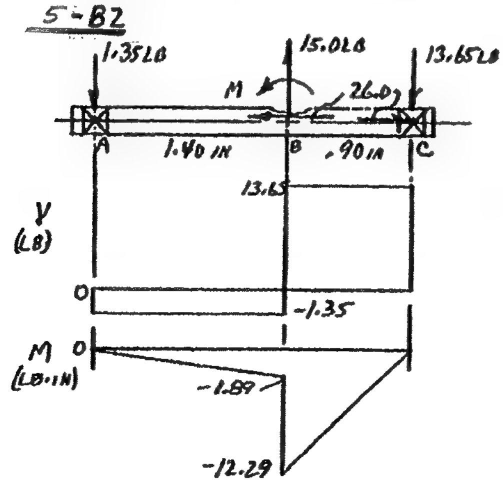
5-81

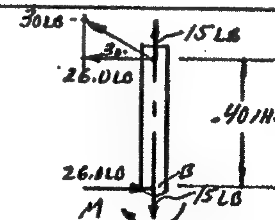




$M = 183.8N(0.08m) = 14.7 N\cdot m$

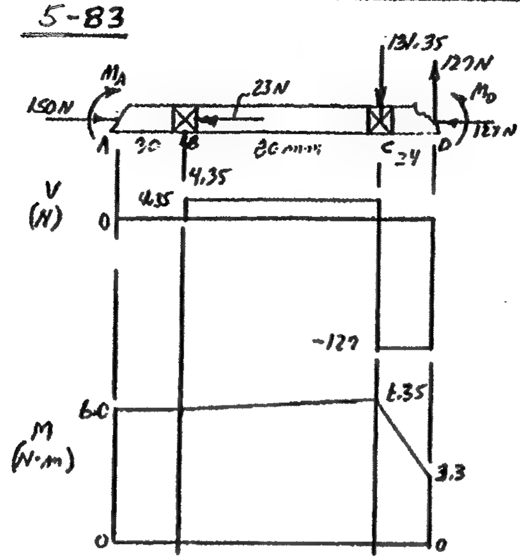
5-82

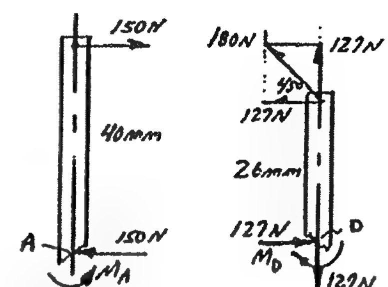




$M = (26.0)(0.40) = 10.4 LB\cdot IN$

5-83

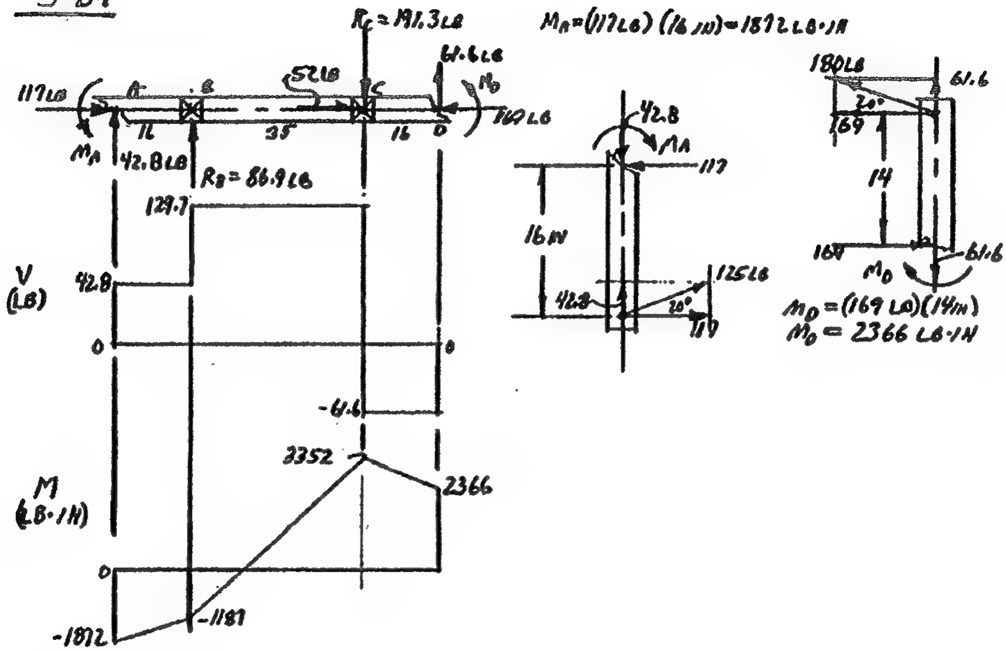




$M_A = 150N(.04m) = 6.0 N\cdot m$

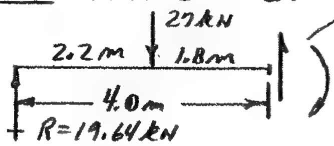
$M_D = 127N(.026m) = 3.3 N\cdot m$

5-84



$R_C = 191.3$ LB
$M_A = (117 LB)(16 IN) = 1872$ LB·IN
61.6 LB
117 LB
52 LB
160 LB
M_A
M_D
A
B
C
D
16
35
16
42.8 LB
$R_B = 86.9$ LB
129.7
V (LB)
42.8
0
0
-61.6
2352
2366
M (LB·IN)
0
-1187
-1872
42.8
117
16 IN
125 LB
20°
180 LB
61.6
169
14
61.6
$M_D = (169 LB)(14 IN)$
$M_D = 2366$ LB·IN

5-85 FIGURE 5-8.

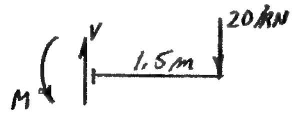


$V = 19.64 - 27.0 = \underline{-7.36\ kN}$

$M = (19.64\ kN)(4.0\ m) - (27.0\ kN)(1.8\ m)$

$\underline{M = 29.96\ kN \cdot m}$

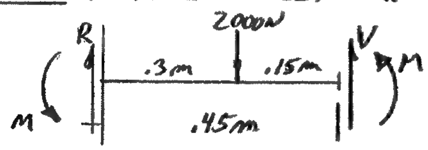
5-86 FIGURE 5-15. USE PART OF BEAM TO RIGHT OF CUT SECTION.



$\underline{V = 20\ kN}$

$M = (-20\ kN)(1.50\ m) = \underline{-30\ kN \cdot m}$

5-87 FIGURE 5-22. $R = 3000\ N$; $M = -1220\ N \cdot m$

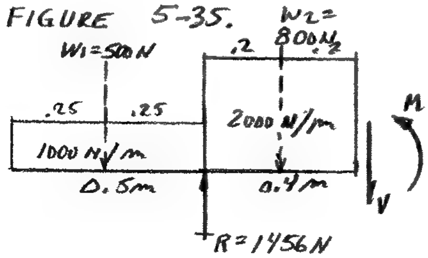


$V = 3000\ N - 2000\ N = \underline{1000\ N}$

$M = -1220\ N \cdot m + 3000\ N(0.45\ m) - 2000\ N(0.15\ m)$

$\underline{M = -170\ N \cdot m}$

5-88 FIGURE 5-35.

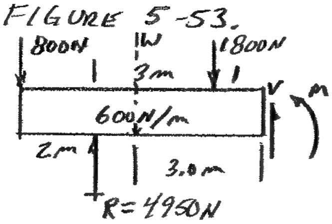


$V = 1456\ N - 500\ N - 800\ N = \underline{56\ N}$

$M = (1456\ N)(0.4\ m) - (500\ N)(.65\ m) - (800\ N)(0.2\ m)$

$\underline{M = 97.4\ N \cdot m}$

5-89 FIGURE 5-53.



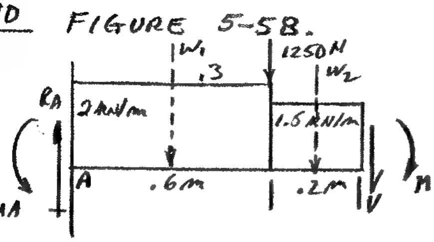
$W = (600\ N/m)(6.0\ m) = 3600\ N$

$V = 4950 - 800 - 3600 - 1800 = \underline{-1250\ N}$

$M = (4950)(4) - 800(6) - 1800(1) - 3600(3)$

$\underline{M = 2400\ N \cdot m}$

5-90 FIGURE 5-58.



$W_1 = 2\ kN/m(.6\ m) = 1.2\ kN$

$W_2 = 1.5\ kN/m(0.2\ m) = 0.30\ kN$

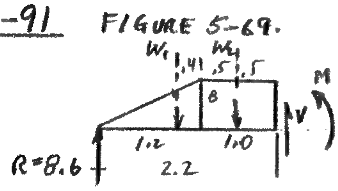
$R_A = 3.65\ kN$, $M_A = -2.19\ kN \cdot m$

$V = 3.65 - 1.2 - 1.25 - 0.30 = \underline{0.90\ kN}$

$M = -2.19 + 3.65(0.8) - 1.2(0.5) - 0.3(0.1) - 1.25(0.2)$

$\underline{M = -0.15\ kN \cdot m}$

5-91 FIGURE 5-69.

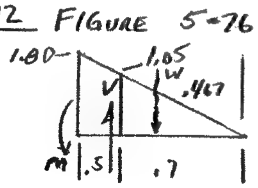


$W_1 = (.5)(8\ kN/m)(1.2\ m) = 4.8\ kN$; $W_2 = (8)(1) = 8\ kN$

$\underline{V} = 8.6 - 4.8 - 8.0 = \underline{-4.2\ kN}$

$\underline{M} = 8.6(2.2) - 4.8(1.4) - 8.0(0.5) = \underline{8.2\ kN \cdot m}$

P6-92 FIGURE 5-76, USE RIGHT PART FOR FBD.

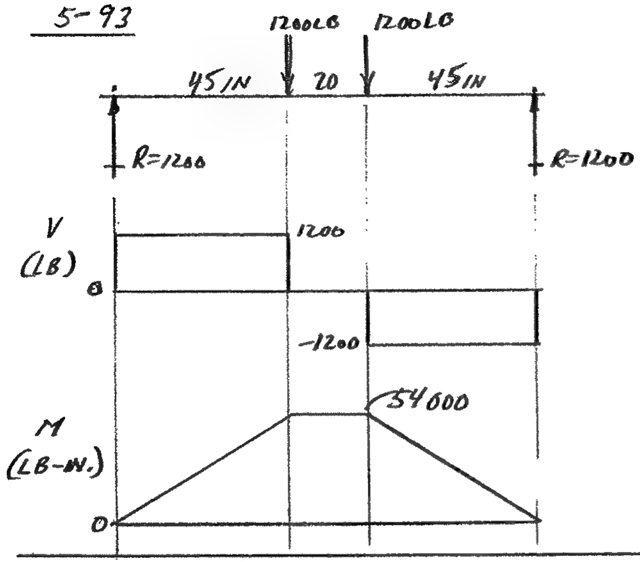


$W = (0.5)(1.05\ kN/m)(0.7\ m) = 0.37\ kN$

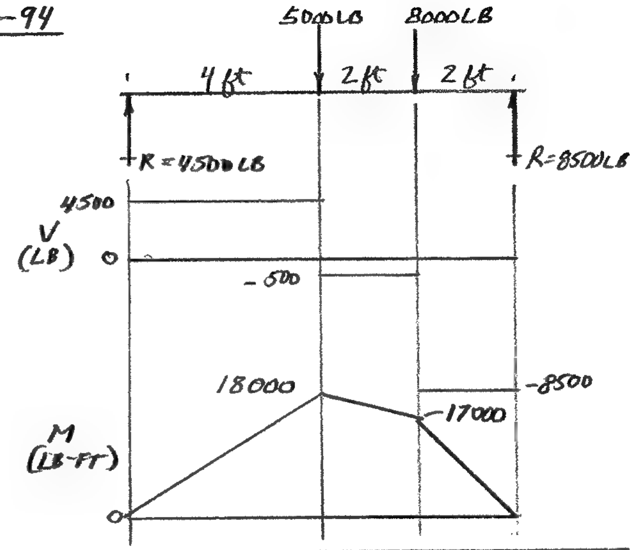
AT CUT: $V = W = \underline{0.37\ kN}$

$\underline{M} = -W(0.7 - 0.467) = (0.37\ kN)(.233) = \underline{-0.0858\ kN \cdot m}$

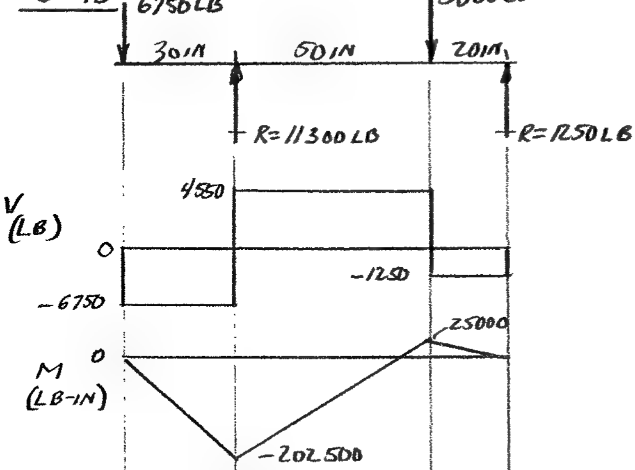
5-93



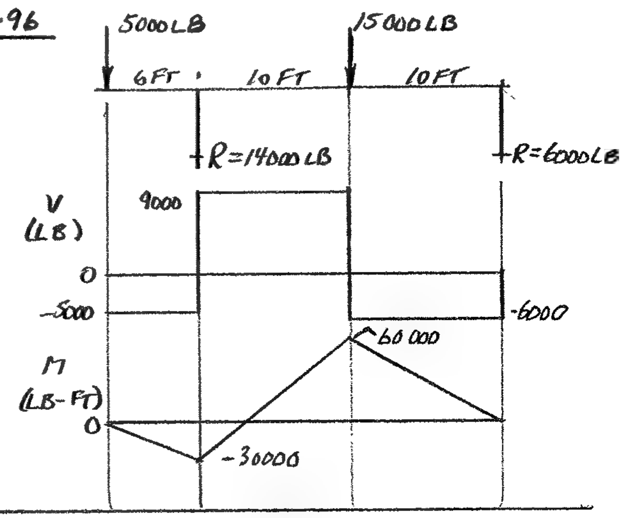
5-94



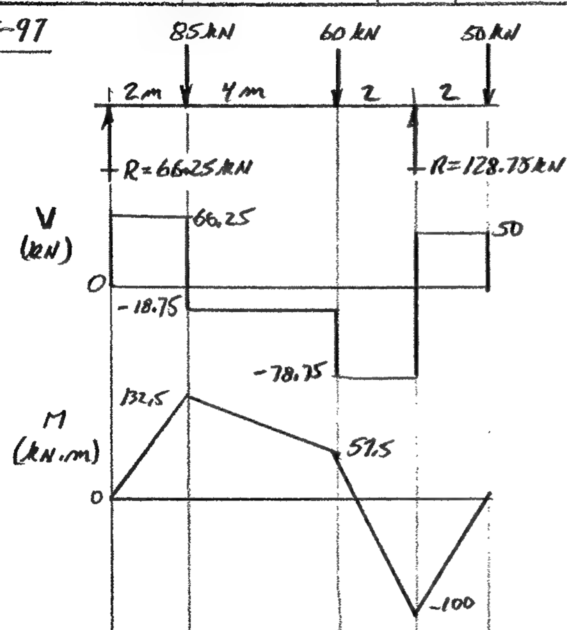
5-95



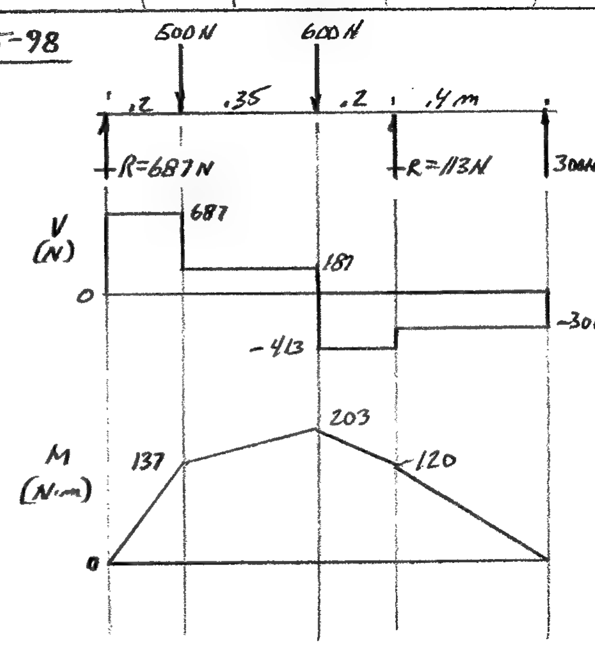
5-96



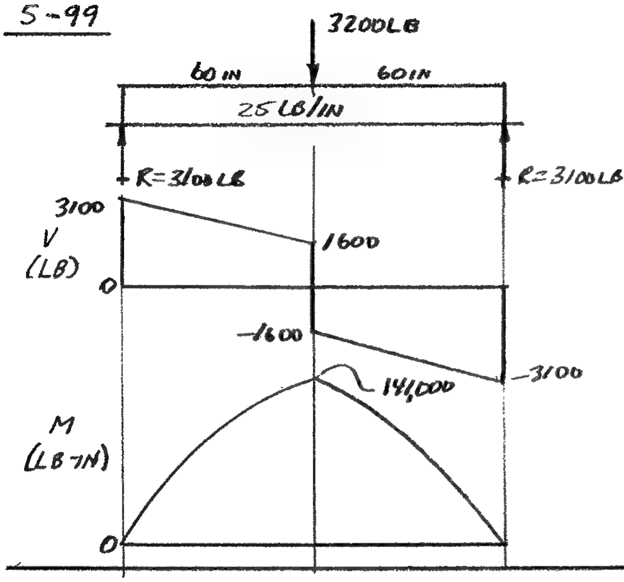
5-97



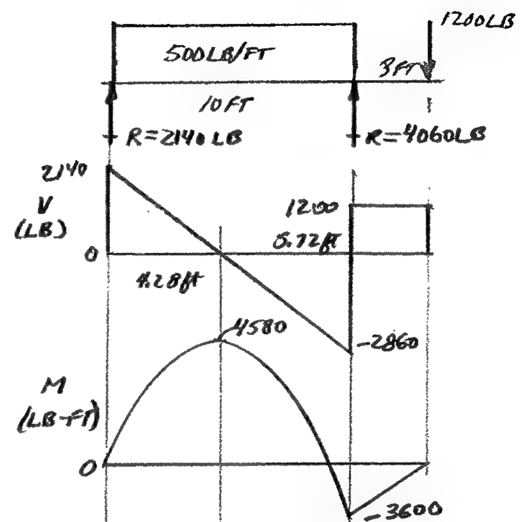
5-98



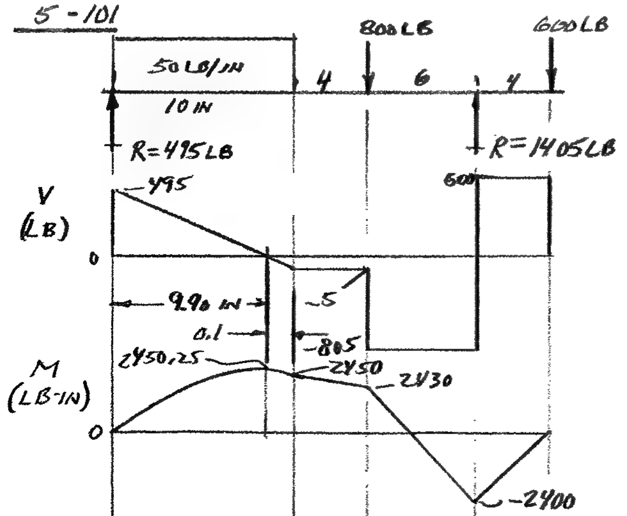
5-99



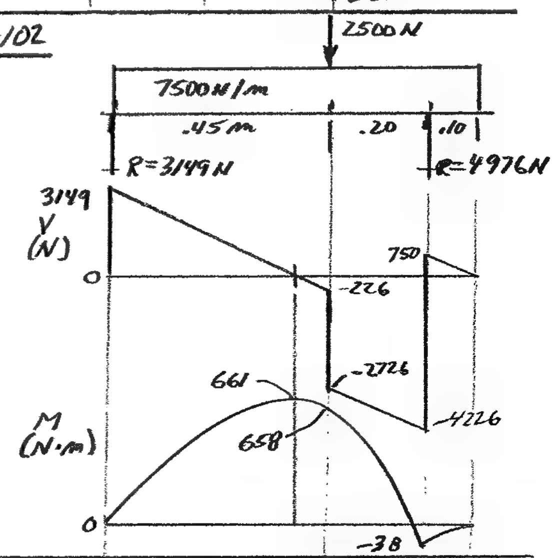
5-100



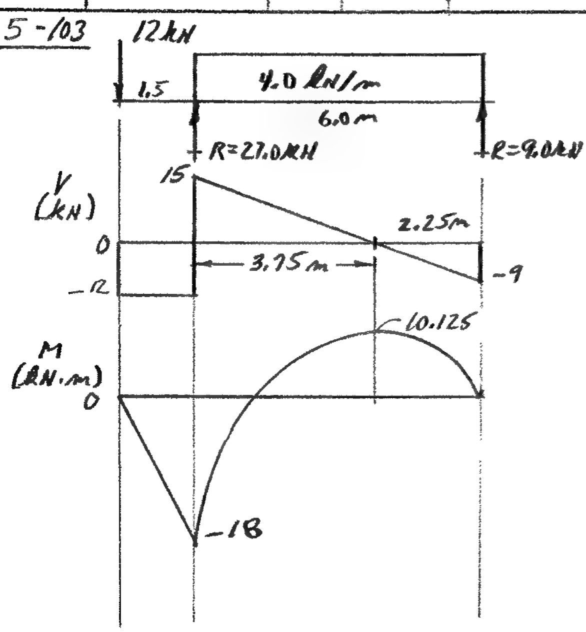
5-101



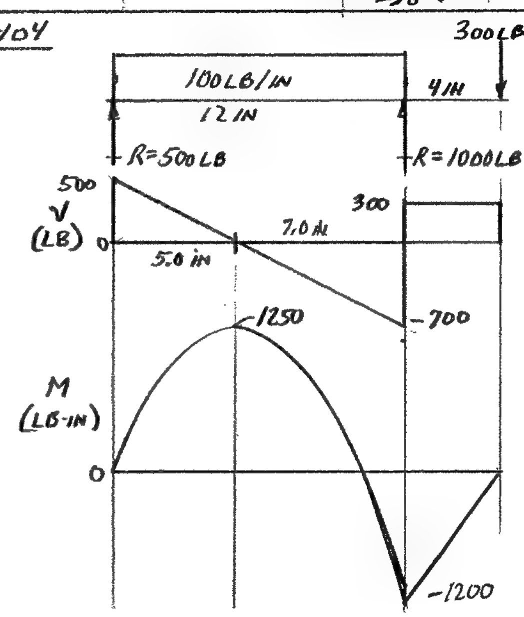
5-102



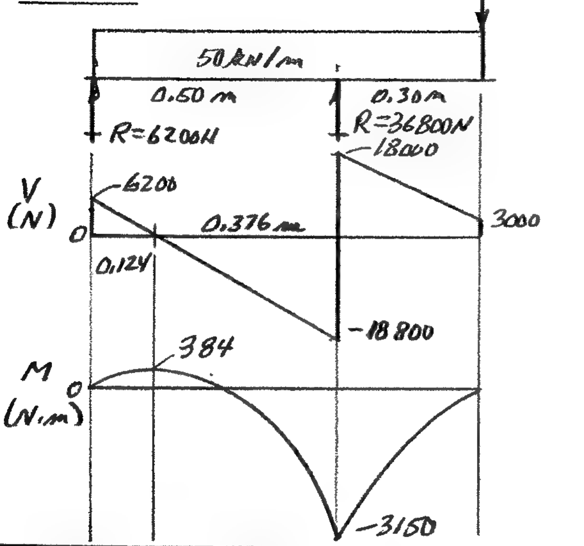
5-103



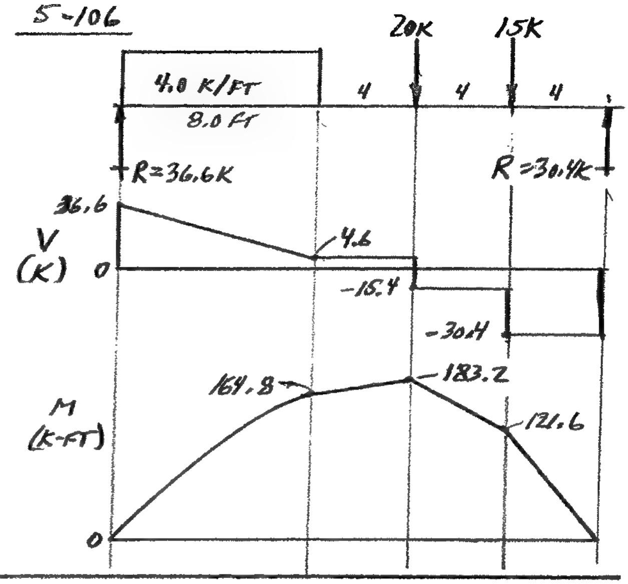
5-104



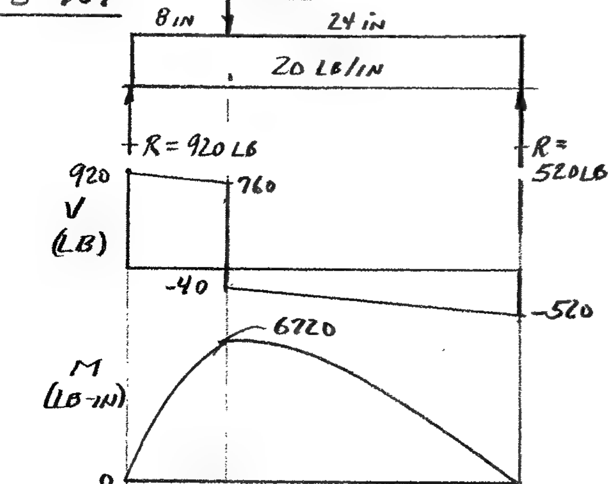
5-105



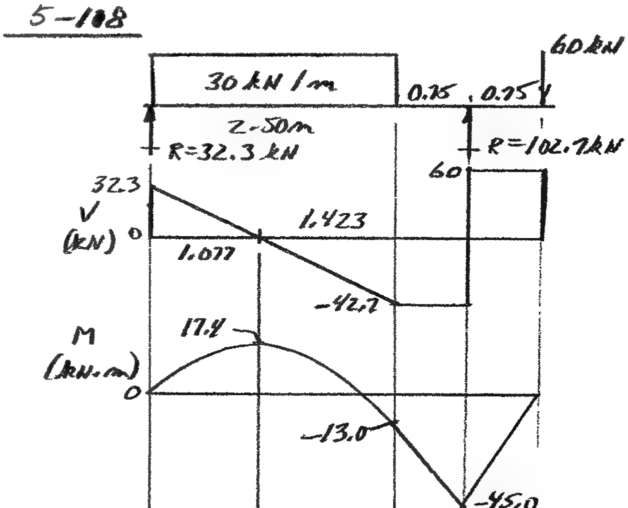
5-106



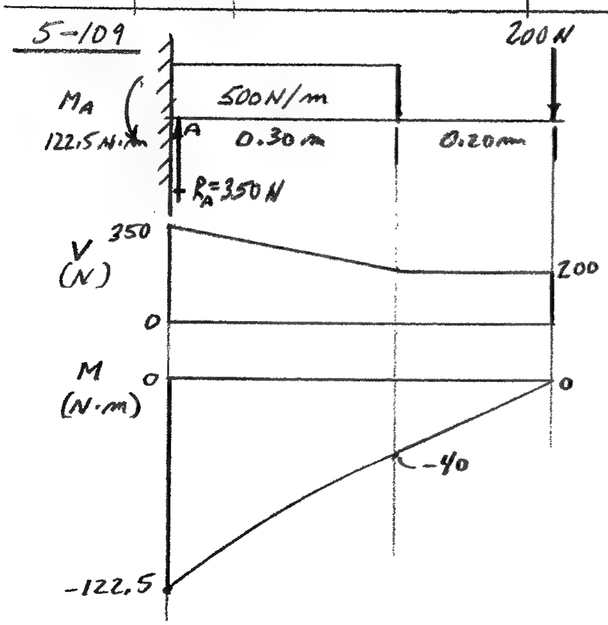
5-107



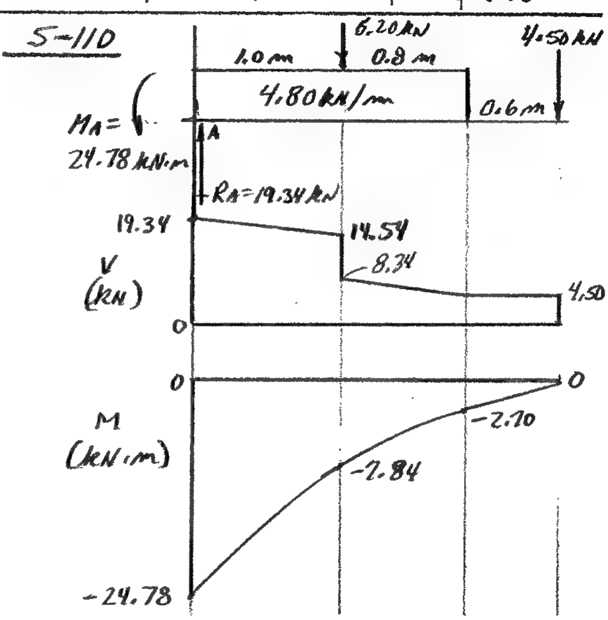
5-108



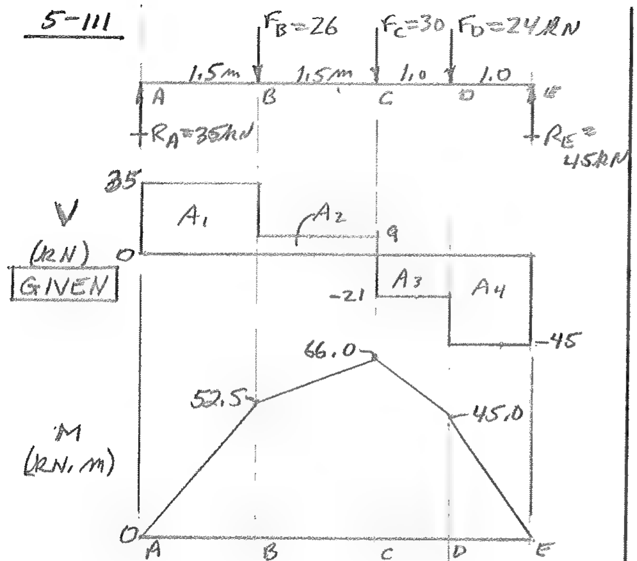
5-109



5-110



5-111



$R_A = V_A = 35\,kN\uparrow$

$F_B = V_{BL} - V_{BR} = 35-9 = \underline{26.0\,kN}\downarrow$

$F_C = V_{CL} - V_{CR} = 9-(-21) = \underline{30.0\,kN}\downarrow$

$F_D = V_{DL} - V_{DR} = -21-(-45) = \underline{24.0\,kN}\downarrow$

$R_E = V_E = -45\,kN\uparrow$

$A_1 = (35)(1.5) = \underline{52.5\,kN\cdot m} = \underline{M_B}$

$A_2 = (9)(1.5) = 13.5\,kN\cdot m$

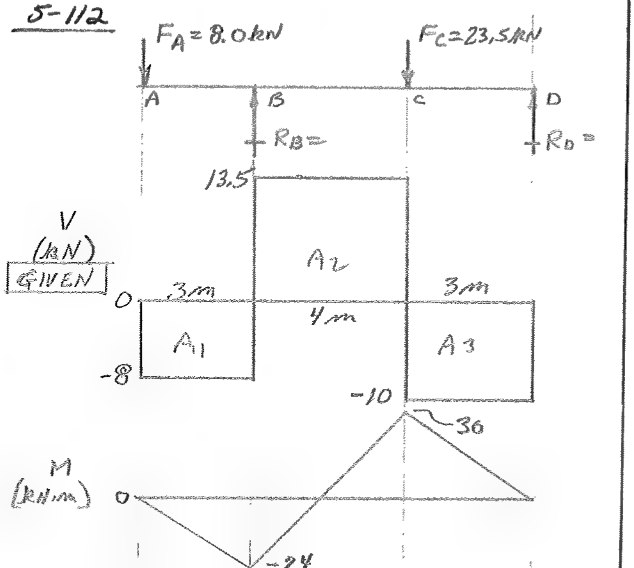
$\underline{M_C} = M_B + A_2 = 52.5 + 13.5 = \underline{66.0\,kN\cdot m}$

$A_3 = (-21)(1) = -21\,kN\cdot m$

$\underline{M_D} = M_C + A_3 = 66.0 - 21.0 = \underline{45.0\,kN\cdot m}$

$\underline{M_E} = M_D + A_4 = 45.0 - 45.0 = \underline{0\,kN\cdot m}$

5-112



$F_A = V_A = 8.0\,kN\downarrow$

$R_B = V_{BL} - V_{BR} = -8 - 13.5 = -21.5\,kN\uparrow$

$F_C = V_{CL} - V_{CR} = 13.5-(-10) = 23.5\,kN\downarrow$

$R_D = V_D = -10.0\,kN\uparrow$

$A_1 = -8(3) = \underline{-24\,kN\cdot m} = \underline{M_B}$

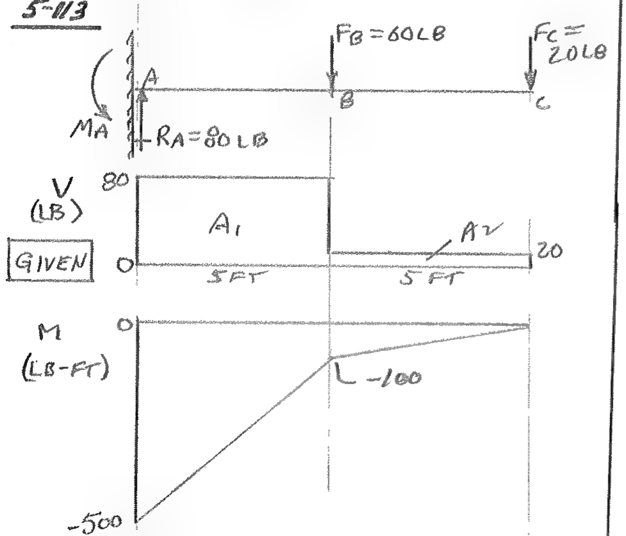
$A_2 = 13.5(4) = 54.0\,kN\cdot m$

$\underline{M_C} = M_B + A_2 = -24 + 54 = \underline{30.0\,kN\cdot m}$

$\underline{M_D} = M_C + A_3 = 30 - 30 = \underline{0\,kN\cdot m}$

$A_3 = -10(3) = -30\,kN\cdot m$

5-113



CANTILEVER

$R_A = V_A = \underline{80\,LB}\uparrow$

$F_B = V_{BL} - V_{BR} = 80 - 20 = \underline{60\,LB}\downarrow$

$F_C = V_C = \underline{20\,LB}\downarrow$

FORCES PRODUCE A NET CLOCKWISE MOMENT THAT MUST BE RESISTED AT A.

$\underline{M_A} = (60)(5) + (20)(10) = \underline{500\,LB\text{-}FT}$ NEG.

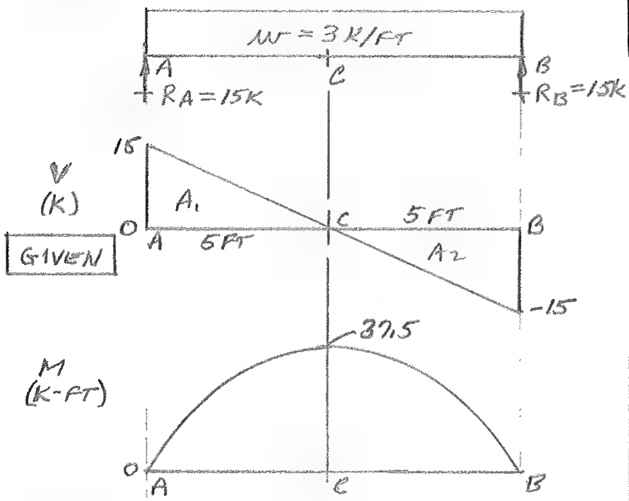
$A_1 = (80)(5) = 400\,LB\text{-}FT$

$A_2 = (20)(5) = 100\,LB\text{-}FT$

$\underline{M_B} = M_A + A_1 = -500 + 400 = \underline{-100\,LB\text{-}FT}$

$\underline{M_C} = M_B + A_2 = -100 + 100 = \underline{0\,LB\text{-}FT}$

5-114



$R_A = V_A = 15K \uparrow$

$R_B = V_B = 15K \uparrow$

V DROPS 15K IN 5 FT

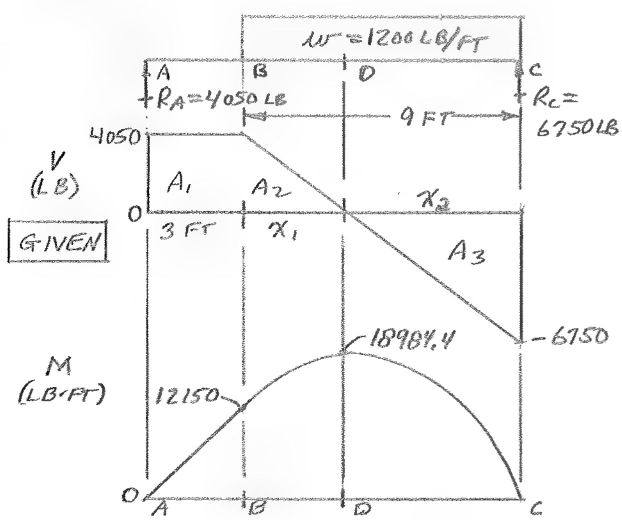
$w = \Delta V/\Delta x = 15K/5\,FT = 3.0\,K/FT$

$A_1 = \frac{1}{2}(15)(5) = 37.5\ K\text{-}FT = M_C$

$A_2 = -37.5\ K\text{-}FT$

$M_B = M_C + A_2 = 37.5 - 37.5 = \underline{0\ K\text{-}FT}$

5-115



$R_A = V_A = 4050\ LB \uparrow$ NO LOAD FROM A-B

$R_C = V_C = 6750\ LB \uparrow$

V DROPS: $4050 + 6750 = 10800\ LB$ IN 9 FT

$w = \dfrac{\Delta V}{\Delta x} = \dfrac{10800\ LB}{9.0\ FT} = 1200\ LB/FT$

x_1 WHERE V-CURVE CROSSES AXIS

$x_1 = \dfrac{\Delta V}{w} = \dfrac{4050\ LB}{1200\ LB/FT} = \underline{3.375\ FT}$

$x_2 = 9.0\ FT - x_1 = 9.0 - 3.375 = \underline{5.625\ FT}$

$A_1 = (4050\ LB)(3\ FT) = 12\,150\ LB\text{-}FT = M_B$

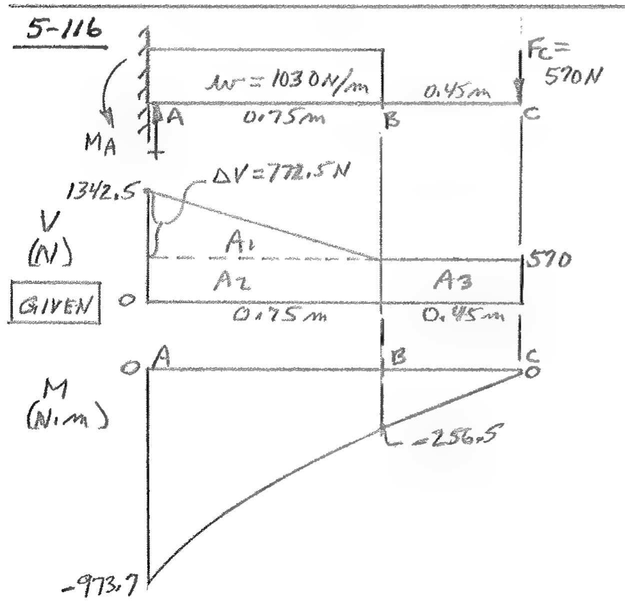
$A_2 = \frac{1}{2}(4050)(3.375) = 6834.4\ LB\cdot FT$

$A_3 = \frac{1}{2}(-6750)(5.625) = 18984.4\ LB\cdot FT$

$M_D = M_B + A_2 = 12150 + 6834.4 = \underline{18984.4\ LB\cdot FT}$

$M_C = M_D + A_3 = 18984.4 - 18984.4 = \underline{0\ LB\cdot FT}$

5-116



$R_A = V_A = 1342.5\ N \uparrow$

$F_C = V_C = 570\ N \downarrow$ NO LOAD FROM B-C.

V DROPS: $1342.5 - 570 = 772.5\ N$ IN 0.75 m

$w = \dfrac{\Delta V}{\Delta x} = \dfrac{772.5\ N}{0.75\ m} = \underline{1030\ N/m}$

REACTION MOMENT M_A REQUIRED

$M_A = (1030\ N/m)(0.75\ m)(0.75/2\ m) + (570\ N)(1.2\ m)$

$M_A = 973.7\ N\cdot m$

$A_1 = \frac{1}{2}(772.5)(0.75) = 289.7\ N\cdot m$

$A_2 = (570)(0.75) = 427.5\ N\cdot m$

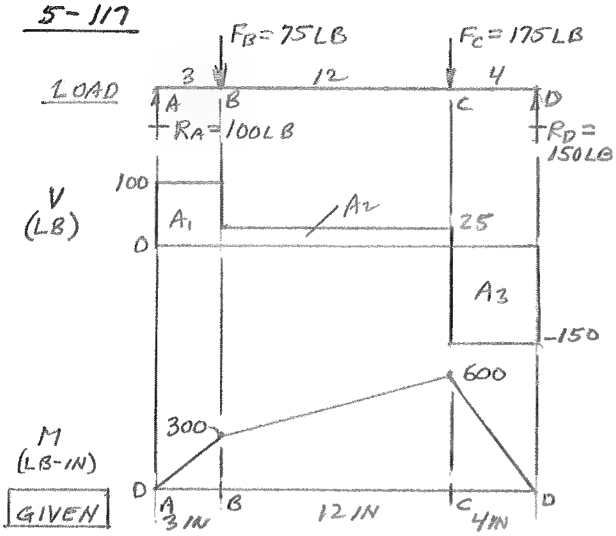
$A_3 = (570)(0.45) = 256.5\ N\cdot m$

$M_B = M_A + A_1 + A_2 = -973.7 + 289.7 + 427.5$

$M_B = -256.5\ N\cdot m$

$M_C = M_B + A_3 = -256.5 + 256.5 = \underline{0\ N\cdot m}$

5-117



TWO CONCENTRATED LOADS

$\Delta M_{A-B} = 300\ LB \cdot IN = A_1 = V_A(3\ IN)$

$V_A = 300\ LB \cdot IN / 3\ IN = 100\ LB$

$\Delta M_{B-C} = 600 - 300 = 300\ LB \cdot IN = A_2 = V_{BC}(12)$

$V_{BC} = 300\ LB \cdot IN / 12\ IN = 25\ LB$

$\Delta M_{CD} = -600\ LB \cdot IN = A_3 = V_{CD}(4)$

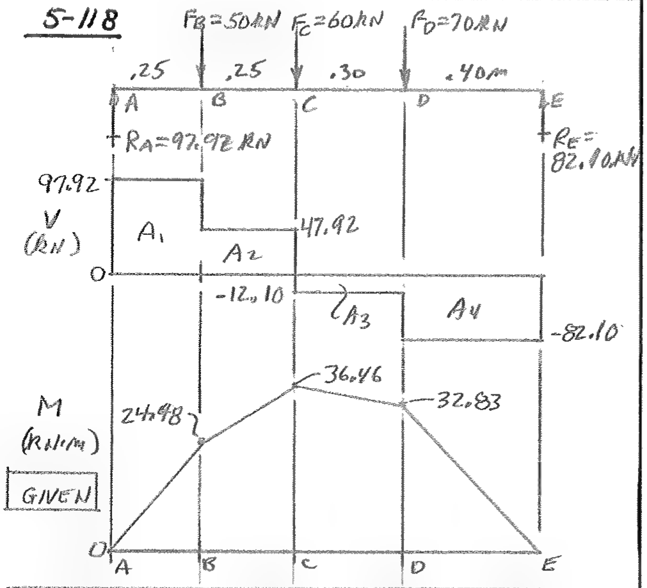
$V_{CD} = -600\ LB \cdot IN / 4\ IN = -150\ LB$

$R_A = V_A = 100\ LB \uparrow$

$F_B = V_{BL} - V_{BR} = 100 - 25 = 75\ LB \downarrow$

$F_C = V_{CL} - V_{CR} = 25 - (-150) = 175\ LB \downarrow$

5-118



THREE CONCENTRATED LOADS

$\Delta M_{A-B} = 24.48\ kN \cdot m = A_1 = V_{AB}(0.25\ m)$

$V_{AB} = 24.48/0.25 = 97.92\ kN$

$\Delta M_{B-C} = 36.46 - 24.48 = 11.98 = A_2 = V_{B-C}(0.25)$

$V_{B-C} = 11.98/0.25 = 47.92\ kN$

$\Delta M_{CD} = 32.83 - 36.46 = -3.63 = V_{CD}(0.3\ m) = A_3$

$V_{CD} = -3.63/0.3 = -12.10\ kN$

$\Delta M_{D-E} = -32.83 = V_{DE}(0.40\ m) = A_4$

$V_{DE} = -32.83/0.4 = -82.1\ kN$

$R_A = V_A = 97.92\ kN$

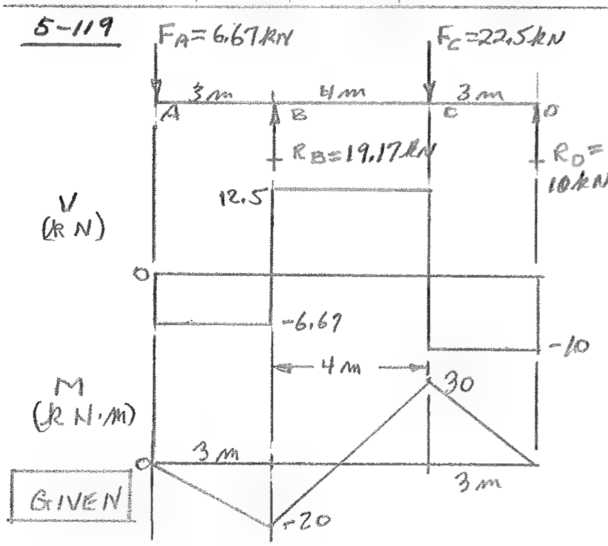
$F_B = V_{BL} - V_{BR} = 97.92 - 47.92 = 50.0\ kN$

$F_C = V_{CL} - V_{CR} = 47.92 - (-12.1) = 60.0\ kN$

$F_D = V_{DL} - V_{DR} = -12.1 - (-82.1) = 70.0\ kN$

$R_E = V_E = 82.10\ kN$

5-119



TWO CONCENTRATED LOADS - OVERHANG

$\Delta M_{AB} = -20\ kN \cdot m = V_{AB}(3\ m)$

$V_{AB} = 20\ kN \cdot m / 3\ m = -6.66\ kN$

$\Delta M_{BC} = 30 - (-20) = 50\ kN \cdot m = V_{BC}(4\ m)$

$V_{BC} = 50/4 = 12.5\ kN$

$\Delta M_{CD} = -30\ kN \cdot m = V_{CD}(3\ m)$

$V_{CD} = -30/3 = -10.0\ kN$

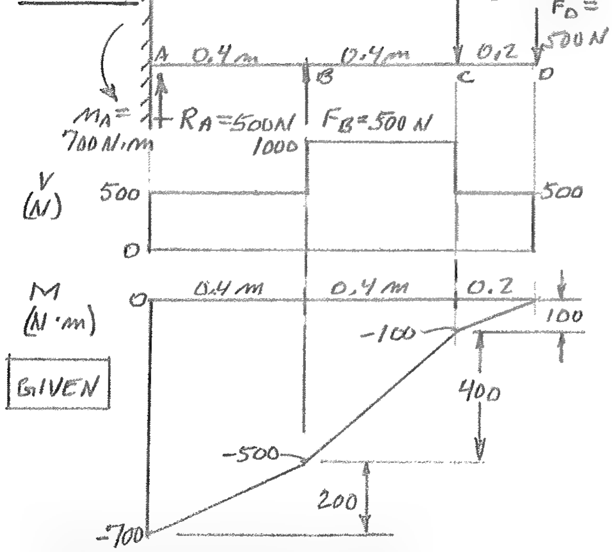
$F_A = V_A = -6.67\ kN \downarrow$

$R_B = V_{BR} - V_{BL} = 12.5 - (-6.67) = 19.17\ kN \uparrow$

$F_C = V_{CR} - V_{CL} = 12.5 - (-10) = 22.5\ kN \downarrow$

$R_D = V_D = 10\ kN \uparrow$

5-120



CANTILEVER – 3 CONCENTRATED LOADS

$M_A = -700\ N\cdot m$ = REACTION MOMENT AT A

$\Delta M_{A-B} = -500-(-700) = 200\ N\cdot m = V_{AB}(0.4\ m)$

$V_{AB} = \frac{200\ N\cdot m}{0.40\ m} = 500\ N$

$\Delta M_{B-C} = -100-(-500) = 400\ N\cdot m = V_{BC}(0.4\ m)$

$V_{BC} = \frac{400\ N\cdot m}{0.40\ m} = 1000\ N$

$\Delta M_{CD} = 100\ N\cdot m = V_{CD}(0.2\ m)$

$V_{CD} = \frac{100\ N\cdot m}{0.2\ m} = 500\ N$

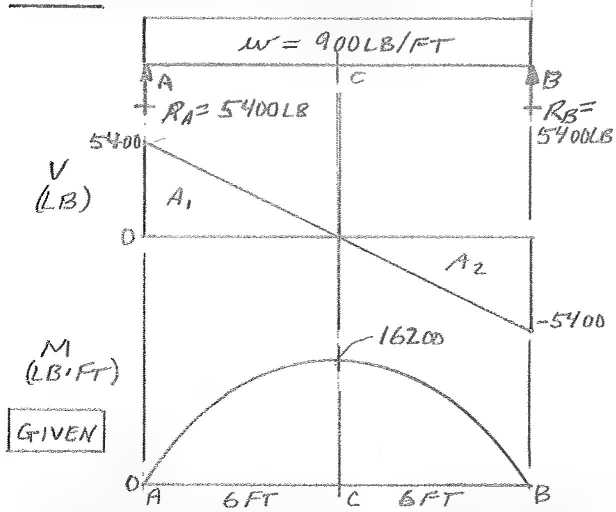
$R_A = V_A = 500\ N \uparrow$

$F_B = V_{BL} - V_{BR} = 500 - 1000 = -500\ N \uparrow$

$F_C = V_{CL} - V_{CR} = 1000 - 500 = 500\ N \downarrow$

$F_D = 500\ N \downarrow$

5-121



UNIFORMLY DISTRIBUTED LOAD

$\Delta M_{AC} = 16200\ LB\cdot FT = \frac{1}{2}(V_A)\,6\ FT = A_1$

$V_A = 16200\ LB\cdot FT / 3\ FT = 5400\ LB$

$V_B = -V_A = -5400\ LB$ (SYMMETRY)

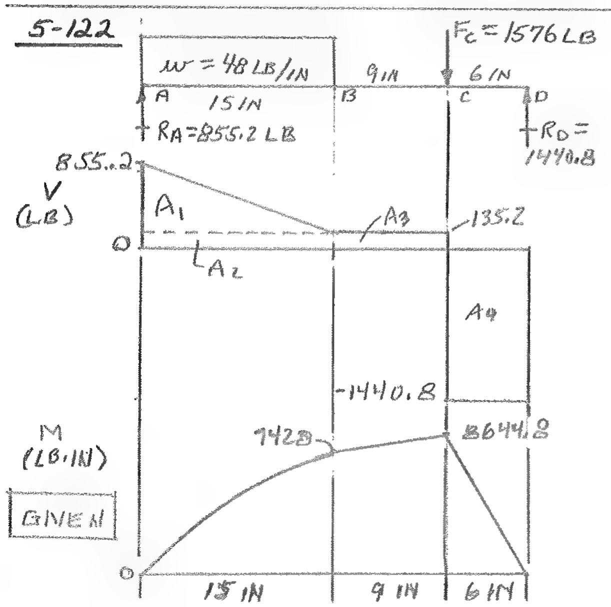
V DROPS 5400 LB IN 6.0 FT

$w = \frac{5400\ LB}{6.0\ FT} = 900\ LB/FT$

$R_A = V_A = 5400\ LB \uparrow$

$R_B = V_B = 5400\ LB \uparrow$

5-122



PARTIAL UNIFORMLY DISTR. LOAD + CONC. LOAD

FROM D TO LEFT:

$\Delta M_{CD} = -8644.8\ LB\cdot IN = V_{CD}(6\ IN) = A_4$

$V_{CD} = \frac{-8644.8\ LB\cdot IN}{6\ IN} = -1440.8\ LB$

$\Delta M_{BC} = 8644.8 - 7428.0 = 1216.8 = A_3 = V_{BC}(9\ IN)$

$V_{BC} = \frac{1216.8\ LB\cdot IN}{9\ IN} = 135.2\ LB$

$\Delta M_{AB} = 7428\ LB\cdot IN = A_1 + A_2$

$A_1 = (135.2\ LB)(15\ IN) = 2028\ LB\cdot IN$

$A_2 = 7428 - A_1 = 7428 - 2028 = 5400\ LB\cdot IN$

$A_2 = \frac{1}{2}(V_A - 135.2)(15) = 7.5V_A - 1014 = 5400$

$V_A = \frac{5400 + 1014}{7.5} = 855.2\ LB$

V DROPS 855.2 − 135.2 = 720 LB IN 15 IN.

$w = 720\ LB / 15\ IN = 48\ LB/IN$

$R_A = V_A = 855.2\ LB \uparrow$; $R_D = V_D = 1440.8\ LB \uparrow$

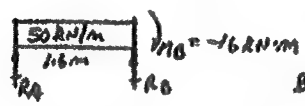
$F_C = V_{CL} - V_{CR} = 135.2 - (-1440.8) = 1576\ LB \downarrow$

5-123

USE EQ. 5-6 WITH $M_A = M_C = 0$

$0 + 4M_B + 0 = -50(1.6)^3/2 = -64$

$M_B = -64/4 = -16\ \text{kN·m}$

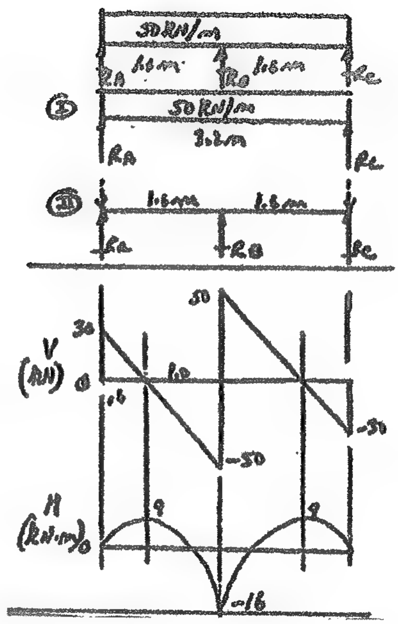


$\Sigma M_B = 0 = 80(.8) - R_A(1.6) - 16$

$R_A = 30\ \text{kN}$

BECAUSE OF SYMMETRY, $R_C = R_A = 30\ \text{kN}$

$R_B = 50(3.2) - 2(30) = 100\ \text{kN}$



5-124

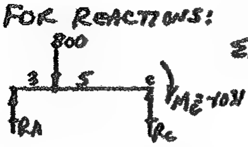
USE EQ(5-7) WITH $M_A = M_B = 0$

$$0 + 2M_C(8+8) + 0 = -\frac{800(3)}{8}(8^2-3^2) - \frac{800(3)}{8}(8^2-3^2)$$

$32M_C = -33000$; $M_C = -33000/32 = -1031\ \text{lb·ft}$

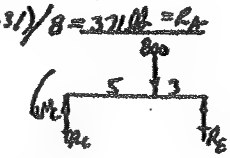
NOTE: M_C IS THE MOMENT AT THE MIDDLE SUPPORT. SUBSCRIPTS IN EQ(5-7) WERE ADJUSTED TO MATCH FIG. P5-124.

FOR REACTIONS:



$\Sigma M_C = 0 = 800(5) - R_A(8) - 1031$

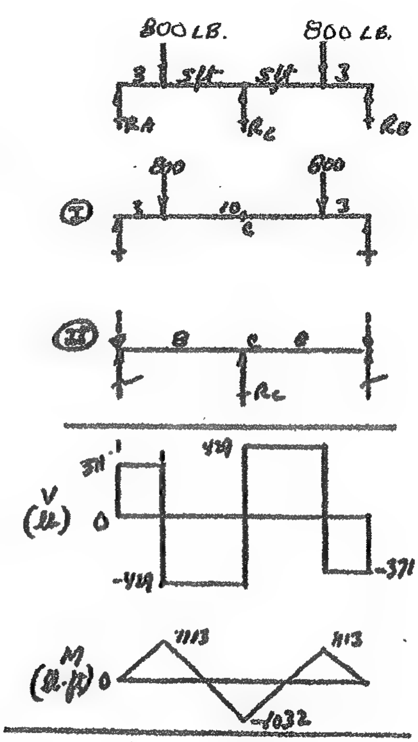
$R_A = (4000 - 1031)/8 = \underline{371\ \text{lb}} = R_A$



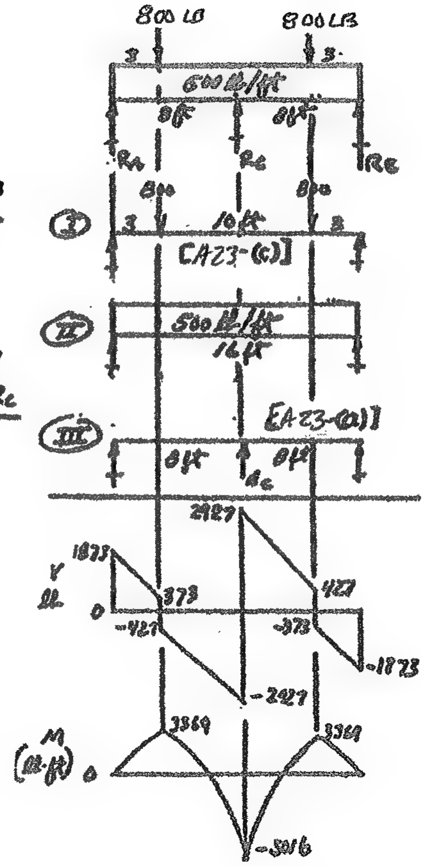
$\Sigma M_C = 0 = 800(5) - R_B(8) - 1031$

$\underline{R_B = 371\ \text{lb}}$

$R_C = 1600 - 2(371) = \underline{858\ \text{lb}} = R_C$



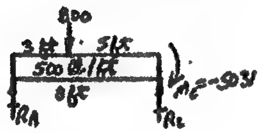
5-125



USE EQ.(5-9) WITH $M_A = M_E = 0$

$$0 + 2M_C(8+8) + 0 = \frac{-800(3)}{8}(8^2-3^2) - \frac{800(3)}{8}(8^2-3^2) - \frac{500(8)^3}{4} - \frac{500(8)^3}{4}$$

$32M_C = -161000$; $M_C = -161000/32 = -5031$ lb·ft

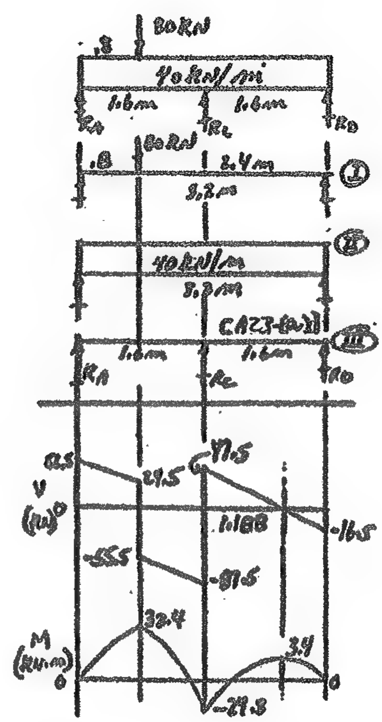


$\Sigma M_C = 0 = 800(5) + 4000(4) - R_A(8) - 5031$

$R_A = 1871$ lb $= R_E$ BECAUSE OF SYMMETRY

$R_C = 800 + 8000 + 800 - 2(1871) = 5858$ lb $= R_C$

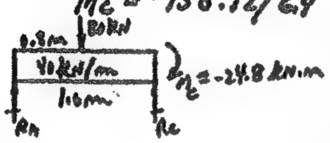
5-126



USE EQ.(5-9) WITH $M_A = M_D = 0$

$$0 + 2M_C(1.6+1.6) = \frac{-80(0.8)}{1.6}(1.6^2-0.8^2) - \frac{40(1.6)^3}{4} - \frac{40(1.6)^3}{4}$$

$M_C = -158.72/6.4 = -24.8$ kN·m



$\Sigma M_C = 0 = 80(0.8) + 64(0.8) - 24.8 - R_A(1.6)$

$R_A = 56.5$ kN

$\Sigma M_C = 0 = 64(0.8) - 24.8 - R_D(1.6)$

$R_D = 16.5$ kN

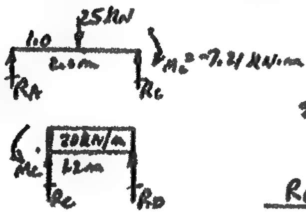
$R_C = 80 + 128 - 56.5 - 16.5 = 135$ kN $= R_C$

5-127

EQ.(13-6) WITH $M_A = M_D = 0$

$$0 + 2M_C(3.2) + 0 = \frac{-25(1.0)}{2.0}(2.0^2 - 1.0^2) - \frac{20(1.2)^3}{4} = -46.14$$

$$M_C = -46.14/6.4 = -7.21\ kN{\cdot}m$$



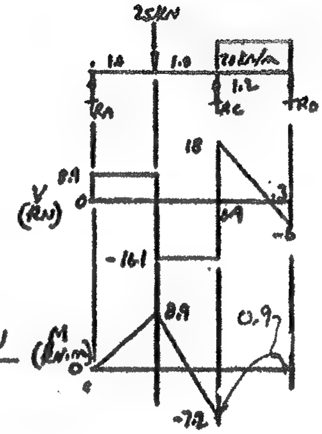
$\Sigma M_C = 0 = 25(1.0) - 7.21 - R_A(2)$

$\underline{R_A = 8.90\ kN}$

$\Sigma M_C = 0 = 24(0.6) - 7.21 - R_D(1.2)$

$\underline{R_D = 6.00\ kN}$

$\underline{R_C} = 25 + 24 - 8.90 - 6.00 = \underline{34.1\ kN}$



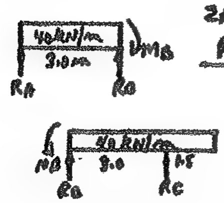
5-128

EQ(5-6) WITH $M_A = 0$

$$M_C = (40\ kN/m)(1.5\ m)\left(\frac{1.5\ m}{2}\right) = 45\ kN{\cdot}m$$

$$0 + 4M_B - 45 = -40(3)^2/2 = -180$$

$$M_B = (-180 + 45)/4 = -33.75\ kN{\cdot}m$$



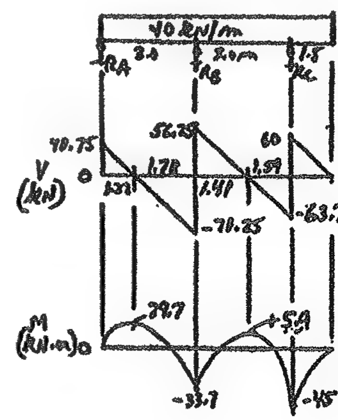
$\Sigma M_B = 0 = 120(1.5) - 33.75 - R_A(3)$

$\underline{R_A = 48.75\ kN}$

$\Sigma M_B = 0 = 180(2.25) - 33.75 - R_C(3)$

$\underline{R_C = 123.75\ kN}$

$\underline{R_B} = 300 - 48.75 - 123.75 = \underline{127.5\ kN}$

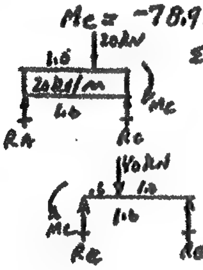


5-129

EQ.(5-9) WITH $M_A = M_E = 0$

$$0 + 2M_C(3.2) + 0 = \frac{-20(1)}{1.6}(1.6^2 - 1^2) - \frac{20(1.6)^3}{4} - \frac{40(1.0)}{1.6}(1.6^2 - 1^2)$$

$$M_C = -78.98/6.4 = -12.34\ kN{\cdot}m$$



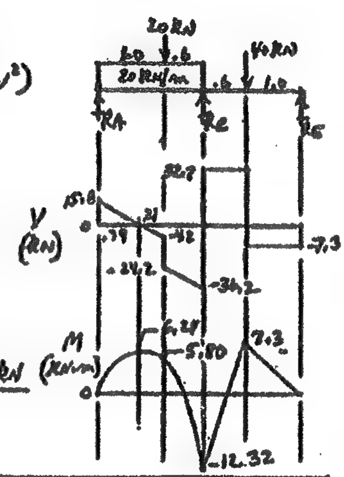
$\Sigma M_C = 0 = 20(.6) + 32(.8) - 12.34 - R_A(1.6)$

$\underline{R_A = 15.8\ kN}$

$\Sigma M_C = 0 = 40(0.6) - 12.34 - R_E(1.6)$

$\underline{R_E = 7.3\ kN}$

$\underline{R_C} = 20 + 32 + 40 - 15.8 - 7.3 = \underline{68.9\ kN}$



5-130

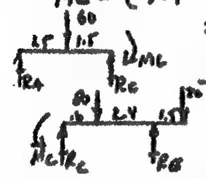
EQ. (5-7) WITH $M_A = 0$

$M_E = -20(1.5) = -30 \text{ kN}\cdot\text{m}$

$0 + 2M_C(6) - 30(3) = \dfrac{-60(1.5)}{3}(3^2 - 1.5^2) - \dfrac{80(2.4)}{3.0}(3^2 - 2.4^2)$

$12M_C - 90 = -409.86$

$M_C = (-409.86 + 90)/12 = -26.7 \text{ kN}\cdot\text{m}$



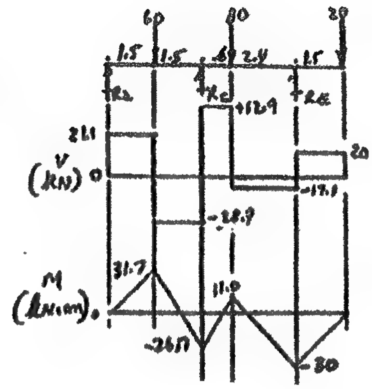
$\Sigma M_C = 0 = 60(1.5) - 26.7 - R_A(3)$

$\underline{R_A = 21.1 \text{ kN}}$

$\Sigma M_C = 0 = 80(.6) + 20(4.5) - 26.7 - R_E(3)$

$\underline{R_E = 37.1 \text{ kN}}$

$\underline{R_C} = 60 + 80 + 20 - 21.1 - 37.1 = \underline{101.8 \text{ kN}}$



CHAPTER 6 Centroids and Moments of Inertia of Areas

Notes concerning the format of solutions for Chapter 6 problems:

- Problem solutions for the moments of inertia of the shapes shown in Figures P6-1 through P6-48 are shown in the tabular format recommended in Section 6-6,
- Calculations were completed using a spreadsheet.
- The requested result includes the vertical *Y* distance to the centroidal axis from the reference axis and the moment of inertia *I* of the composite shape relative to the horizontal centroidal axis.
- In most problems, the reference axis for computing the location of the horizontal centroidal axis was taken as the base of the section. Exceptions are noted on the top or bottom lines of the solution. For example, in Figure P6-17 the reference axis is at the axis of symmetry at the mid-height of the shape, found by inspection.
- The left-most column of the solution gives a brief description of the part of the composite shape being analyzed.
- For some shapes, internal parts removed from the outer shape are shown to be negative.
- For composite shapes having parts that are commercially available structural shapes, pipes, or tubes, or wood beams, reference should be made to the Appendix tables for pertinent data.

FIGURE P6-1 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vertical	0.5000	1.0000	0.5000	0.16670	0.3365	0.0566	0.2233	
2-Horizontal	0.3125	0.1250	0.0391	0.00163	0.5384	0.0906	0.0922	
Total area =	0.8125	Sum Ay=	0.5391			Total I =	0.3156	in^4
Y=	0.6635	in						

FIGURE P6-2 Units: Inches NOTE: 6x8 rectangle with 5x6 rectangle removed

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Total 6x8	48.00	4.00	192.00	256.00	0.00	0.00	256.00	
2-Void 5x6	-30.00	4.00	-120.00	-90.00	0.00	0.00	-90.00	
Total area =	18.00	Sum Ay=	72.00			Total I =	166.00	in^4
Y=	4.00	in	NOTE: Reference axis is base of the shape					

FIGURE P6-3 Units: Inches NOTE: 6x8 rectangle with 4x6 rectangle removed

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Total 6x8	48.00	4.00	192.00	256.00	0.00	0.00	256.00	
2-Void 4x6	-24.00	4.00	-96.00	-72.00	0.00	0.00	-72.00	
Total area =	24.00	Sum Ay=	96.00			Total I =	184.00	in^4
Y=	4.00	in	NOTE: Reference axis is base of the shape					

FIGURE P6-4 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vertical	5000	100	5.000E+05	1.667E+07	52.50	1.38E+07	3.04E+07	
2-Horizontal	4375	213	9.297E+05	2.279E+05	60.00	1.58E+07	1.60E+07	
Total area =	9375	Sum Ay=	1.430E+06			Total I =	4.64E+07	mm^4
Y=	152.50	mm						

FIGURE P6-5 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vertical	250	30.0	7.500E+03	5.208E+04	5.00	6.25E+03	5.83E+04	
2-Horiz-bot	100	2.5	2.500E+02	2.080E+02	32.50	1.06E+05	1.06E+05	
3-Horiz-top	200	57.5	1.150E+04	4.170E+02	22.50	1.01E+05	1.02E+05	
Total area =	550	Sum Ay=	1.925E+04			Total I =	2.66E+05	mm^4
Y=	35.00	mm						

FIGURE P6-6 Units: mm NOTE: Both vertical rectangles (10x30) combined

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-20X30	600	15	9.000E+03	4.500E+04	2.50	3.75E+03	4.88E+04	
2-Hor-20X10	200	5	1.000E+03	1.667E+03	7.50	1.12E+04	1.29E+04	
Total area =	800	Sum Ay=	1.000E+04			Total I =	6.17E+04	mm^4
Y=	12.50	mm	NOTE: Reference axis is base of the shape					

FIGURE P6-7 Units: mm NOTE: Entire vertical stem; 2 horiz. flanges each 5x15

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04	
2-Horiz-bot	75	2.5	1.875E+02	1.562E+02	17.50	2.30E+04	2.31E+04	
3-Horiz-top	75	37.5	2.812E+03	1.562E+02	17.50	2.30E+04	2.31E+04	
Total area =	350	Sum Ay=	7.000E+03			Total I =	7.29E+04	mm^4
Y=	20.00	mm	NOTE: Reference axis is base of the shape					

FIGURE P6-8 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04	
2-Ver-5x40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04	
3-Hor-30x5	150	20.0	3.000E+03	3.125E+02	0.00	0E+00	3.12E+02	
Total area =	550	Sum Ay=	1.100E+04			Total I =	5.36E+04	mm^4
Y=	20.00	mm						

FIGURE P6-9 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X30	150	20.0	3.000E+03	1.125E+04	0.00	0E+00	1.12E+04	
2-Horiz-bot	200	2.5	5.000E+02	4.167E+02	17.50	6.12E+04	6.17E+04	
3-Horiz-top	200	37.5	7.500E+03	4.167E+02	17.50	6.12E+04	6.17E+04	
Total area =	550	Sum Ay=	1.100E+04			Total I =	1.35E+05	mm^4
Y=	20.00	mm						

FIGURE P6-10 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-5X50	250	30.0	7.500E+03	5.208E+04	0.00	0E+00	5.21E+04	
2-Horiz-bot	140	2.5	3.500E+02	2.917E+02	27.50	1.06E+05	1.06E+05	
3-Horiz-top	140	57.5	8.050E+03	2.917E+02	27.50	1.06E+05	1.06E+05	
Total area =	530	Sum Ay=	1.590E+04			Total I =	2.64E+05	mm^4
Y=	30.00	mm						

FIGURE P6-11 Units: mm NOTE: Both vert. combined 10x45; horiz. flanges combined 5x30

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-10X45	450	22.5	1.012E+04	7.594E+04	0.69	2.14E+02	7.62E+04	
2-Hor-5x30	150	2.5	3.750E+02	3.125E+02	19.31	5.59E+04	5.62E+04	
3-Hor-5x25	125	42.5	5.312E+03	2.604E+02	20.69	5.35E+04	5.38E+04	
Total area =	725	Sum Ay=	1.581E+04			Total I =	1.86E+05	mm^4
Y=	21.81	mm	NOTE: Reference axis is base of the shape					

FIGURE P6-12 Units: mm NOTE: All verticals massed together

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Ver-16X16	256	8	2.048E+03	5.461E+03	4.39	4.92E+03	1.04E+04	
2-Hor-4X50	200	18	3.600E+03	2.667E+02	5.61	6.30E+03	6.57E+03	
Total area =	456	Sum Ay=	5.648E+03			Total I =	1.70E+04	mm^4
Y=	12.39	mm	NOTE: Reference axis is base of the shape					

FIGURE P6-13 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Hor-5x10	50	2.5	1.250E+02	1.045E+02	20.83	2.17E+04	2.18E+04	
2-Ver-5X55	275	27.5	7.562E+03	6.932E+04	4.17	4.77E+03	7.41E+04	
3-Hor-5x20	100	27.5	2.750E+03	2.083E+02	4.17	1.74E+03	1.94E+03	
4-Ver-5x30	150	15.0	2.250E+03	1.125E+04	8.33	1.04E+04	2.17E+04	
5-Hor-5x5	25	52.5	1.312E+03	5.208E+01	29.17	2.13E+04	2.13E+04	
Total area =	600	Sum Ay=	1.400E+04			Total I =	1.41E+05	mm^4
Y=	23.33	mm						

FIGURE P6-14 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Bot plate	0.5200	0.1000	0.0520	0.00173	0.4330	0.0975	0.0992	
2-Bot flanges	0.1200	0.2500	0.0300	0.0001	0.2830	0.0096	0.0097	
3-2 Vert webs	0.3000	0.9500	0.2850	0.05625	0.4169	0.0522	0.1084	
4-Horiz-top	0.12	1.65	0.1980	0.0001	1.1170	0.1497	0.1498	
Total area =	1.06	Sum Ay=	0.5650			Total I =	0.3672	in^4
Y=	0.5330	in						

FIGURE P6-15 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Bot flange	0.1000	0.0500	0.0050	0.00008	1.0176	0.1036	0.1036	
2-2 Verticals	0.4800	1.2000	0.5760	0.23040	0.1323	0.0084	0.2388	
3-Mid-Horiz.	0.1	1.4500	0.1450	0.0000833	0.3823	0.0146	0.0147	
Total area =	0.68	Sum Ay=	0.7260			Total I =	0.3572	in^4
Y=	1.0676	in						

FIGURE P6-16 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Rectangle	1.1250	0.7500	0.8438	0.21094	0.1491	0.0250	0.2360	
2-Semicircle	0.2209	1.6590	0.3665	0.0022148	0.7598	0.1275	0.1297	
Total area =	1.34589	Sum Ay=	1.2102			Total I =	0.3657	in^4
Y=	0.8992	in						

FIGURE P6-17 Units: mm NOTE: Reference axis taken at y=125

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Rectangle	11400	0.0	0E+00	3.430E+07	0.00	0E+00	3.43E+07	
2-Semic-bot	1414	-107.7	-1.52E+05	9.072E+04	107.72	1.64E+07	1.65E+07	
3-Semic-top	1414	107.7	1.52E+05	9.072E+04	107.72	1.64E+07	1.65E+07	
Total area =	14227	Sum Ay=	0E+00			Total I =	6.73E+07	mm^4
Y=	0.00	mm						

FIGURE P6-18 Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Rectangle	1200	20.0	2.400E+04	1.60E+05	7.27	6.35E+04	2.23E+05	
2-Rect rem.	-400	20.0	-8.00E+03	-1.33E+04	7.27	-2.1E+04	-3.4E+04	
3-Triangle	300	46.7	1.40E+04	6.67E+03	19.39	1.13E+05	1.20E+05	
Total area =	1100	Sum Ay=	3.000E+04			Total I =	3.08E+05	mm^4
Y=	27.27	mm						

FIGURE P6-19 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Hor-.5x1.4	0.700	0.250	0.1750	0.0146	0.681	0.3242	0.3388	
2-Ver-.6x2.5	1.500	1.250	1.8750	0.7813	0.319	0.1531	0.9343	
3-2 Tr-.7x1.3	0.910	0.933	0.8493	0.0854	0.003	0.0000	0.0854	
4-Tri-rem	-0.460	0.807	-0.3711	-0.0216	0.124	-0.0071	-0.0287	
5-Hole-rem	-0.049	2.200	-0.1080	-0.0002	1.269	-0.0791	-0.0793	
Total area =	3	Sum Ay=	2.4202			Total I =	1.2506	in^4
Y=	0.9305	in						

FIGURE P6-20 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-2 Vert rect	1.2000	1.0000	1.2000	0.4000	0.2798	0.0940	0.4940	
2-2 Triangles	0.5100	1.1333	0.5780	0.0819	0.1465	0.0109	0.0928	
3-Top-.3x2.4	0.7200	1.8500	1.3320	0.0054	0.5701	0.2341	0.2395	
Total area =	2.4300	Sum Ay=	3.1100			Total I =	0.8263	in^4
Y=	1.2798	in						

FIGURE P6-21 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Vert rect	8.2500	4.2500	35.0625	20.7969	0	0.0000	20.7969	
2-Bot flange	5.2500	0.7500	3.9375	0.9844	3.5	64.3125	65.2969	
3-Top flange	5.2500	7.7500	40.6875	0.9844	3.5	64.3125	65.2969	
Total area =	18.7500	Sum Ay=	79.6875			Total I =	151.3906	in^4
Y=	4.2500	in						

FIGURE P6-22 Units: Inches NOTE: 7.25x7 rectangle with 4.25x5.5 rectangle removed

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Tot 7.25x7	50.75	3.50	177.63	207.23	0.00	0.00	207.23	
2-4.25x5.5 re	-23.38	3.50	-81.81	-58.92	0.00	0.00	-58.92	
Total area =	27.38	Sum Ay=	95.81			Total I =	148.30	in^4
Y=	3.50	in	NOTE: Reference axis is base of the shape					

FIGURE P6-23 Units: Inches NOTE: 24x4.5 rectangle with 21x3.5 rectangle removed

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Tot 24x4.5	108.00	2.25	243.00	182.25	0.00	0.00	182.25	
2-21x3.5 rem	-73.50	2.25	-165.38	-75.03	0.00	0.00	-75.03	
Total area =	34.50	Sum Ay=	77.63			Total I =	107.22	in^4
Y=	2.25	in	NOTE: Reference axis is base of the shape					

FIGURE P6-24 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-2 Verticals	33.75	5.63	189.84	355.96	2.13	152.40	508.36	
2-Top flange	16.88	12.00	202.50	3.16	4.25	304.80	307.97	
Total area =	50.63	Sum Ay=	392.34			Total I =	816.33	in^4
Y=	7.75	in						

FIGURE P6-25 Units: Inches NOTE: Beam depth = 13.7 in;Ref axis=centroid;7.35 in from bot

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W14x43	12.60	0.00	0.00	428.00	0.00	0.00	428.00	
2-bot plate	4.00	-7.10	-28.40	0.0833	7.10	201.64	201.72	
3-top plate	4.00	7.10	28.40	0.0833	7.10	201.64	201.72	
Total area =	20.60	Sum Ay =	0.00			**Total I =**	**831.45**	in^4
Y =	**0.00**	**Inches**						

FIGURE P6-26 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-S12x50	14.600	6.000	87.600	303.00	1.913	53.43	356.43	
2-C12x25	7.350	11.713	86.091	4.45	3.800	106.13	110.58	
Total area =	21.950	Sum Ay =	173.691			**Total I =**	**467.01**	in^4
Y =	**7.913**	**Inches**	NOTE: Web for C is 0.387 thick; Y-Y axis down 0.674 from top					
			NOTE: For Channel; y = 12.0 + 0.387 - 0.674 = 11.713 in					

FIGURE P6-27 Units: Inches NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-I12x14.292	12.15	6.00	72.92	317.33	1.40	23.73	341.06	
2-Top plate	3.50	12.25	42.88	0.07	4.85	82.41	82.49	
Total area =	15.65	Sum Ay=	115.79			Total I =	423.55	in^4
Y=	7.40	in						

FIGURE P6-28 Units: Inches NOTE: Depth of C12 is12.0; Ref. axis at centroid;y=6.50 from bot

Part	Area	y	Ay	Ic	d	Ad^2	Ic+Ad^2	
1-Two C12	14.07	0.00	0.00	319.520	0.00	0.00	319.52	
2-Bot plate	5.00	-6.25	-31.25	0.104	6.25	195.31	195.42	
3-Top plate	5.00	6.25	31.25	0.104	6.25	195.31	195.42	
Total area =	19.07	Sum Ay=	0.00			Total I =	710.35	in^4
Y=	0.00	in						

FIGURE P6-29 Units: Inches NOTE: Overall depth=7.0 in; Ref axis=centroid; 3.5 in from bot

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1 Vert plate	3.00	0.000	0.00	9.00	0.000	0.000	9.000	
2-Bot angles (2)	2.74	-2.368	-6.49	0.9520	2.368	15.364	16.316	
3-Top angles (2)	2.74	2.368	6.49	0.9520	2.368	15.364	16.316	
4-Top plate	2.25	3.250	7.31	0.0469	3.250	23.766	23.813	
5-Bot plate	2.25	-3.250	-7.31	0.0469	3.250	23.766	23.813	
Total area =	12.980	Sum Ay =	0.00			**Total I =**	**89.258**	in^4
Y =	**0.00**	**Inches**	NOTE: For Angle; y = 3.50 - 0.50 - 0.632 = 2.638 in					

FIGURE P6-30 Units: Inches NOTE: Overall depth = 6.0 in; Ref axis=centroid; 3.0 in from bot

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Vert plates (2)	3.000	0.000	0.000	9.000	0.000	0.000	9.000	
2-bot channel	1.760	-2.545	-4.479	0.300	2.545	11.3996	11.6996	
3-top channel	1.760	2.545	4.479	0.300	2.545	11.3996	11.6996	
Total area =	6.520	Sum Ay =	0.000			**Total I =**	**32.3991**	in^4
Y =	**0.00**	**Inches**						

FIGURE P6-31 Units: Inches NOTE: Ref axis=centroid; 3.0 in from center of either pipe

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Vert plate	2.050	0.000	0.000	2.8717	0.000	0.000	2.872	
2-bot pipe	0.799	-3.000	-2.397	0.3099	3.000	7.1910	7.5009	
3-top pipe	0.799	3.000	2.397	0.3099	3.000	7.1910	7.5009	
Total area =	3.648	Sum Ay =	0.000			**Total I =**	**17.8735**	in^4
Y =	**0.00**	**Inches**	NOTE: Length of plate = 6.00 - pipe dia = 6.00 - 1.90 = 4.10 in					

FIGURE P6-32 Units: Inches NOTE: Reference axis=centroid= 12 in from CL of pipes

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Top pipes (2)	4.4560	12.0000	53.4720	6.0340	12.00	641.66	647.70	
2-Bot pipes (2)	4.4560	-12.0000	-53.4720	6.0340	12.00	641.66	647.70	
Total area =	8.9120	Sum Ay =	0.0000			**Total I =**	**1295.40**	in^4
Y =	**0.0000**	**Inches**						

FIGURE P6-33 Units: Inches NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Base plate	2.500	0.125	0.313	0.0130	2.592	16.800	16.813	
2-Angles (2)	3.380	1.470	4.969	5.5000	1.247	5.2582	10.7582	
3-top plate	6.000	4.500	27.000	0.1250	1.783	19.0689	19.1939	
Total area =	11.880	Sum Ay =	32.281			**Total I =**	**46.7646**	in^4
Y =	**2.717**	**Inches**						

FIGURE P6-34 Units: Inches NOTE: Reference axis=centroid= 3.00 in from bottom

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Channel	3.830	0.000	0.000	17.300	0.000	0.000	17.300	
2-Channel	2.830	0.000	0.000	17.300	0.000	0.000	17.300	
Total area =	6.660	Sum Ay =	0.000			**Total I =**	**34.600**	in^4
Y =	**0.0000**	**Inches**						

FIGURE P6-35 Units: Inches NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Angles (2)	1.888	0.586	1.106	0.6920	2.023	7.730	8.422	
2-Bot channel	2.640	0.478	1.262	0.6240	2.131	11.9931	12.6171	
3-Vert webs (2)	4.500	3.000	13.500	13.5000	0.391	0.6866	14.1866	
4-Top channel	2.640	5.522	14.578	0.6240	2.913	22.3959	23.0199	
Total area =	11.668	Sum Ay =	30.446			**Total I =**	**58.2452**	in^4
Y =	**2.609**	**Inches**						

FIGURE P6-36 Units: Inches NOTE: Reference axis is bases of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W6x15	4.430	2.995	13.268	29.100	0.642	1.827	30.927	
Angles (2)	1.888	0.846	1.597	0.692	1.507	4.287	4.979	
Total area =	6.318	Sum Ay =	14.865			**Total I =**	**35.906**	in^4
Y =	**2.3528**	**Inches**						

FIGURE P6-37 Units: Inches NOTE: Reference axis is bases of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Bot tube	2.440	2.000	4.880	4.490	1.500	5.490	9.980	
2-Top tube	2.440	5.000	12.200	1.480	1.500	5.490	6.970	
Total area =	4.880	Sum Ay =	17.080			**Total I =**	**16.950**	in^4
Y =	**3.5000**	**Inches**						

FIGURE P6-38 Units: Inches NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-6x6x1/2	9.740	3.000	29.220	48.3000	0.051	0.025	48.325	
2-4x2x1/4	2.440	1.465	3.575	1.4800	1.484	5.3736	6.8536	
3-3x2x1/4	1.970	4.535	8.934	1.1100	1.586	4.9552	6.0652	
Total area =	14.150	Sum Ay =	41.729			**Total I =**	**61.2442**	in^4
Y =	**2.949**	**Inches**						

NOTE: Use design wall thickness for 6x6x1/2; t_w = 0.465 in

FIGURE P6-39 Units: Inches NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Bot channel	1.590	0.457	0.727	0.3120	2.904	13.411	13.723	
2-6x2x1/4	3.370	3.184	10.730	13.1000	0.177	0.1058	13.2058	
3-Toop channel	1.590	6.641	10.559	0.3120	3.280	17.1037	17.4157	
Total area =	6.550	Sum Ay =	22.016			**Total I =**	**44.3442**	**in^4**
Y =	**3.361**	**Inches**						

FIGURE P6-40 Units: Inches NOTE: Reference axis is bases of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Bot channel	1.881	0.730	1.373	0.980	0.321	0.193	1.173	
2-Top I-beam	1.726	1.400	2.416	0.680	0.349	0.211	0.891	
Total area =	3.607	Sum Ay =	3.790			**Total I =**	**2.064**	**in^4**
Y =	**1.0506**	**Inches**						

FIGURE P6-41 Units: Inches NOTE: Reference axis is bases of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W12x30	8.790	6.150	54.059	238.000	2.022	35.953	273.953	
2-C6x13	3.830	12.814	49.078	1.050	4.642	82.514	83.564	
Total area =	12.620	Sum Ay =	103.136			**Total I =**	**357.517**	**in^4**
Y =	**8.1724**	**Inches**	NOTE: depth of W-beam = 12.3 in					

FIGURE P6-42 Units: Inches NOTE: Ref axis=centroid of W-shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W4x13	3.830	0.000	0.000	11.3000	0.000	0.000	11.300	
2-bot tube	2.440	-3.080	-7.515	1.4800	3.080	23.1468	24.6268	
3-top tube	2.440	3.080	7.515	1.4800	3.080	23.1468	24.6268	
Total area =	8.710	Sum Ay =	0.000			**Total I =**	**60.5536**	**in^4**
Y =	**0.00**	**Inches**	NOTE: depth of W-beam = 4.16 in					

FIGURE P6-43 Units: Inches NOTE: Ref axis is bottom of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W12x30	8.790	6.150	54.059	238.0	2.047	36.828	274.828	
2-L4x3x1/4	1.690	13.520	22.849	2.750	5.323	47.8871	50.6371	
3-L4X3X1/4	1.690	13.520	22.849	2.750	5.323	47.8871	50.6371	
Total area =	12.170	Sum Ay =	99.756			**Total I =**	**376.1020**	**in^4**
Y =	**8.20**	**Inches**	NOTE: depth of W-beam = 12.3 in					

FIGURE P6-44 Units: Inches NOTE: Ref axis=centroid of tube

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-6x2x1/4	3.370	0.000	0.000	13.1000	0.000	0.000	13.100	
2-bot plate	1.000	-3.250	-3.250	0.0208	3.250	10.5625	10.5833	
3-top plate	1.000	3.250	3.250	0.0208	3.250	10.5625	10.5833	
Total area =	5.370	Sum Ay =	0.000			**Total I =**	**34.2667**	**in^4**
Y =	**0.00**	**Inches**						

FIGURE P6-45 Units: Inches NOTE: Ref axis is bottom of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-C8x4.147	3.526	4.000	14.104	37.40	1.023	3.692	41.092	
2-C8x4.147	3.526	4.000	14.104	37.40	1.023	3.692	41.092	
3-C8x4.147-Top	3.526	7.070	24.929	3.250	2.047	14.770	18.020	
Total area =	10.578	Sum Ay =	53.137			**Total I =**	**100.205**	in^4
Y =	**5.023**	**Inches**						

FIGURE P6-46 Units: Inches NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-1/2x18 plate	9.000	0.250	2.250	0.1875	1.857	31.036	31.223	
2-3/4x10 plate	7.500	5.500	41.250	62.500	3.393	86.344	148.844	
3-L8x4x1/2	5.800	1.354	7.853	6.750	0.753	3.289	10.039	
4-L8x4x1/2	5.800	1.354	7.853	6.750	0.753	3.289	10.039	
Total area =	28.100	Sum Ay =	59.206			**Total I =**	**200.144**	in^4
Y =	**2.107**	**Inches**						

FIGURE P6-47 Units: mm NOTE: Reference axis is bottom of the shape

Part	Area, A_i		y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-Flanges(2)	72.000		1.500	108.0	54.0	13.1	12434.2	12488.2
2-Vert. webs(2)	150.000		12.500	1875.0	7812.5	2.1	687.9	8500.4
3-Semicircle(+)	157.080		29.240	4593.0	1120.0	14.6	33476.6	34596.6
4-Semicircle(-)	-76.970		27.968	-2152.7	-268.9	13.3	-13669.7	-13938.6
Total area =	302.110 mm^2	Sum Ay =		4423.3 mm^3			**Total I_c =**	**41646.6 mm^4**
		Y =	**14.641 mm**					

FIGURE P6-48 Units: Inches NOTE: Reference axis is base of the shape

Part	Area, A_i		y_i	A_iy_i	I_c	d_i	$A_id_i^2$	I_c+Ad^2
1-Rect. (2)	1.250		0.625	0.781	0.163	0.391	0.191	0.353
2-Semicircle	0.884		1.568	1.385	0.035	0.552	0.270	0.305
Total area =	2.134 in^2	Sum Ay =		2.167 in^3			**Total I_c =**	**0.659 in^4**
		Y =	**1.016 in**					

SOLUTIONS FOR PROBLEMS 7-49 THROUGH 7-66

Each problem requires the computation of the radius of gyration

$r_X = (I_X/A)^{1/2}$ with respect to the horizontal centroidal axis.

Data for *I* and *A* are taken from the solution for the given figure number.

Prob. No.	Fig. No.	I_X	A	r_X
6-49	P6-2	166.0 in^4	18.00 in^2	3.04 in
6-50	P6-3	184.0 in^4	24.00 in^2	2.77 in
6-51	P6-4	4.64E+07 mm^4	9375 mm^2	70.35 mm
6-52	P6-5	2.66E+05 mm^4	550 mm^2	21.99 mm
6-53	P6-6	6.17E+04 mm^4	800 mm^2	8.78 mm
6-54	P6-8	5.36E+04 mm^4	550 mm^2	9.87 mm
6-55	P6-9	1.35E+05 mm^4	550 mm^2	15.67 mm
6-56	P6-11	1.86E+05 mm^4	725 mm^2	16.02 mm
6-57	P6-12	1.70E+04 mm^4	456 mm^2	6.11 mm
6-58	P6-14	0.3672 in^4	1.06 in^2	0.59 in
6-59	P6-15	0.3572 in^4	0.68 in^2	0.72 in
6-60	P6-16	0.3657 in^4	1.35 in^2	0.52 in
6-61	P6-17	6.73E+07 mm^4	14227 mm^2	68.78 mm
6-62	P6-21	151.4 in^4	18.75 in^2	2.84 in
6-63	P6-22	148.3 in^4	27.38 in^2	2.33 in
6-64	P6-23	107.2 in^4	34.50 in^2	1.76 in
6-65	P6-24	816.3 in^4	50.63 in^2	4.02 in
6-66	P6-25	831.45 in^4	20.6 in^2	6.35 in

SOLUTIONS FOR PROBLEMS 6-67 THROUGH 6-81

Each problem requires computation of the radius of gyration:

$r_y = (I/A)$^0.5 with respect to the vertical *Y-Y centroidal* axis

Data for A are taken from the solutions for I_x *for the given figure number.*

All sections have a vertical axis of symmetry and they can be broken into parts that all have their centroidal axis on the axis of symmetry.

Therefore, the total I is the algebraic sum of the I values for all parts.

		I_1	I_2	I_3	Total I_y	A	r_y
6-67	**P6-2**	18.00	0.50	18.00	36.500	18.00	1.424
6-68	**P6-3***	144.00	-32.00	0.00	112.000	24.00	2.160
6-69M	**P6-4**	2.60E+05	1.12E+07	0.00	1.14E+07	9375	34.911
6-70M	**P6-5**	3333	521	26667	3.05E+04	550	7.449
6-71	**P6-16**	0.0527	0.00775	0.00	0.0605	1.35	0.212
6-72M	**P6-17**	3.42E+06	3.18E+05	3.18E+05	4.06E+06	14227	16.883
6-73	**P6-21**	5.359	1.547	5.359	12.266	18.75	0.809
6-74	**P6-22***	222.30	-35.18	0.00	187.11	27.38	2.614
6-75	**P6-23***	5184.00	-2701.13	0.00	2482.88	34.50	8.483
6-76	**P6-24**	25.31	177.98	0.00	203.29	50.63	2.004
6-77	**P6-25**	45.20	21.33	21.33	87.867	20.60	2.065
6-78	**P6-26**	15.60	144.00	0.00	159.60	21.95	2.696
6-79	**P6-27**	35.48	14.29	0.00	49.772	15.65	1.783
6-80	**P6-42**	3.860	4.490	4.490	12.840	8.71	1.214
6-81	**P6-44**	2.210	0.333	0.333	2.877	5.37	0.732

*I_1 is for the large outside rectangle; I_2 is for the internal rectangle and is negative

For Problems 6-69M, 6-70M, and 6-72M: I in mm^4, A in mm^2, r in mm

For all other problems: I in in^4, A in in^2, r in inches

SOLUTIONS TO PROBLEMS 6-21M TO 6-46M
METRIC VERSIONS OF FIGURES P6-21 TO P6-46

FIGURE P6-21M Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Bot 2x4	3382	19	64258	4.070E+05	89	2.679E+07	2.72E+07	
2-Web-2x6	5320	108	574560	8.689E+06	0	0.0000	8.69E+06	
3-Top 2x4	3382	197	666254	4.070E+05	89	2.679E+07	2.72E+07	
Total area =	12084	Sum Ay =	1305072			**Total I =**	**6.31E+07**	**mm^4**
Y =	**108**	**mm**	Used 2x4 = 38 mm x 89 mm; 2x6 = 38 mm x 140 mm					

FIGURE P6-22M Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Outside rect.	32752	89.0	2.915E+06	8.648E+07	0.00	0.000E+00	8.648E+07	
2-Inside rect.	-15120	89.0	-1.346E+06	-2.470E+07	0.00	0.000E+00	-2.470E+07	
3-	0	0.0	0.000E+00	0.000E+00	0.00	0.000E+00	0.000E+00	
Total area =	17632	Sum Ay =	1.569E+06			**Total I =**	**6.18E+07**	**mm^4**
Y =	**89.00**	**mm**	Used 1x8 = 19 mm x 184 mm; 2x6 = 38 mm x 140 mm Outside rectangle - Inside rectangle					

FIGURE P6-23M Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Outside rect.	69784	57.2	3.992E+06	7.611E+07	0.00	0.000E+00	7.611E+07	
2-Inside rect.	-47526	57.2	-2.718E+06	-3.137E+07	0.00	0.000E+00	-3.137E+07	
3-	0	0.0	0.000E+00	0.000E+00	0.00	0.000E+00	0.000E+00	
Total area =	22258	Sum Ay =	1.273E+06			**Total I =**	**4.47E+07**	**mm^4**
Y =	**57.20**	**mm**	Used 2x4 = 38 mm x 89 mm; 1/2x24 = 12.7 mm x 610 mm Outside rectangle - Inside rectangle					

FIGURE P6-24M Units: mm NOTE: Reference axis is base of the shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Verticals (2)	21736	143.0	3.108E+06	1.482E+08	54.00	6.338E+07	2.115E+08	
2-Top flange	10868	305.0	3.315E+06	1.308E+06	108.00	1.268E+08	1.281E+08	
3-	0	0.0	0.000E+00	0.000E+00	0.00	0.000E+00	0.000E+00	
Total area =	32604	Sum Ay =	6.423E+06			**Total I =**	**3.40E+08**	**mm^4**
Y =	**197.00**	**mm**	Used 2x12 = 38 mm x 286 mm					

FIGURE P6-25M Units: mm NOTE: Beam depth = 348 mm;Ref axis = centroid;186 mm from bot

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W360x64	8130	0.00	0.00	1.780E+08	0.00	0.00	1.780E+08	
2-bot plate	2400	-186	-446400	2.880E+04	186	8.303E+07	8.306E+07	
3-top plate	2400	186	446400	2.880E+04	186	8.303E+07	8.306E+07	
Total area =	12930	Sum Ay =	0.00			**Total I =**	**3.44E+08**	**mm^4**
Y =	**0.00**	**Inches**						

FIGURE P6-26M Units: mm NOTE: Reference axis is base of the shape; Depth of S = 305 mm

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-S300x74	9420	152.500	1.437E+06	1.260E+08	48.615	2.226E+07	1.483E+08	
2-C300x37	4740	297.730	1.411E+06	1.850E+06	96.615	4.425E+07	4.610E+07	
Total area =	14160	Sum Ay =	2.848E+06			**Total I =**	**1.94E+08**	**mm^4**
Y =	**201.12**	**mm**	NOTE: Web for C is 9.83 mm thick; Y-Y axis down 17.1 mm from top NOTE: For Channel; y = 305+9.83-17.1 = 297.73 mm					

FIGURE P6-27M Units: mm NOTE: Reference axis is base of the shape; Depth of S = 305 mm

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-I305x23.80 Al	7841	152.5	1.196E+06	1.320E+08	34.233	9.189E+06	1.412E+08
2-12x180 Plate	2160	311.0	6.718E+05	2.592E+04	124.267	3.336E+07	3.338E+07
Total area =	10001	Sum Ay =	1.868E+06			**Total I =**	**1.75E+08** **mm^4**
Y =	**186.73**	**mm**					

FIGURE P6-28M Units: mm NOTE: Channel depth=305 mm;Ref axis = centroid;164.5 mm from bot

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-C305x12.31 (2)	9080	0.00	0.00	1.330E+08	0.00	0.00	1.330E+08
2-bot plate	3000	-158.5	-475500	3.600E+04	158.5	7.537E+07	7.540E+07
3-top plate	3000	158.5	475500	3.600E+04	158.5	7.537E+07	7.540E+07
Total area =	15080	Sum Ay =	0.00			**Total I =**	**2.84E+08** **mm^4**
Y =	**0.00**	**Inches**	NOTE: Top and Bottom plates are 12 mm x 250 mm				

Figure P6-29M Units: mm NOTE: Reference axis = centroid = 87 mm from bottom

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1 Vert plate	1800	0.0	0.00	3.375E+06	0.0	0.000E+00	3.375E+06
2-Bot angles (2)	1768	-58.9	-104135	3.960E+05	58.9	6.134E+06	6.530E+06
3-Top angles (2)	1768	58.9	104135	3.960E+05	58.9	6.134E+06	6.530E+06
4-Top plate	1320	81.0	106920	1.584E+04	81.0	8.661E+06	8.676E+06
5-Bot plate	1320	-81.0	-106920	1.584E+04	81.0	8.661E+06	8.676E+06
Total area =	7976	Sum Ay =	0.00			**Total I =**	**3.38E+07** **mm^4**
Y =	**0.00**	**mm**	NOTE: For Angle; y = 87 - 12 - 16.1 = 58.9 mm from centroid				

FIGURE P6-30M Units: mm NOTE: Overall depth = 150 mm; Ref axis=centroid; 75 mm from bot

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Vert plates (2)	3600	0.00	0.00	3.375E+06	0.000	0.00	3.375E+06
2-bot channel	1140	-63.4	-72276	1.250E+05	63.4	4.582E+06	4.707E+06
3-top channel	1140	63.4	72276	1.250E+05	63.4	4.582E+06	4.707E+06
Total area =	5880	Sum Ay =	0.00			**Total I =**	**1.28E+07** **mm^4**
Y =	**0.00**	**mm**					

FIGURE P6-31M Units: mm NOTE: Ref axis=centroid; 75 mm from center of either pipe

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Vert plate	1220.88	0.00	0.000	1.053E+06	0.00	0.000E+00	1.053E+06
2-bot pipe	515.80	-75.00	-38685	1.290E+05	75.00	2.901E+06	3.030E+06
3-top pipe	515.80	75.00	38685	1.290E+05	75.00	2.901E+06	3.030E+06
Total area =	2252.48	Sum Ay =	0.000			**Total I =**	**7.11E+06** **mm^4**
Y =	**0.00**	**mm**	NOTE: Web length: 150 mm - pipe OD = 150 mm - 48.26 = 101.74 mm				

FIGURE P6-32M Units: mm NOTE: Reference axis=centroid= 300 mm from CL of pipes

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Top pipes (2)	2876	300	8.628E+05	2.520E+06	300	2.588E+08	2.614E+08
2-Bot pipes (2)	2876	-300	-8.628E+05	2.520E+06	300	2.588E+08	2.614E+08
Total area =	5752	Sum Ay =	0.0000			**Total I =**	**5.23E+08** **mm^4**
Y =	**0.00**	**mm**					

FIGURE P6-33M Units: mm NOTE: Ref axis at base of shape; Overall height = 120 mm

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Base plate	1500	3.0	4500	4.500E+03	65.071	6.351E+06	6.356E+06
2-Angles (2)	2180	37.0	80660	2.280E+06	31.071	2.105E+06	4.385E+06
3-top plate	3600	114.0	410400	4.320E+04	45.929	7.594E+06	7.637E+06

Total area = 7280 Sum Ay = 495560 **Total I = 1.84E+07 mm^4**

Y = 68.071 mm

FIGURE P6-34M Units: mm NOTE: Reference axis = centroid = 76 mm from bottom

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Channel	2470	0.00	0.00	7.200E+06	0.00	0.00	7.200E+06
2-Channel	2470	0.00	0.00	7.200E+06	0.00	0.00	7.200E+06

Total area = 4940 Sum Ay = 0.00 **Total I = 1.44E+07 mm^4**

Y = 0.00 mm NOTE: Depth of channel = 152 mm

FIGURE P6-35M Units: mm NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Angles (2)	1218	14.9	18148	2.880E+05	50.491	3.105E+06	3.393E+06
2-Bot channel	1700	12.1	20570	2.600E+05	53.291	4.828E+06	5.088E+06
3-Vert webs (2)	3000	75.0	225000	5.625E+06	9.609	2.770E+05	5.902E+06
4-Top channel	1700	137.9	234430	2.600E+05	72.509	8.938E+06	9.198E+06

Total area = 7618 Sum Ay = 498148 **Total I = 2.36E+07 mm^4**

Y = 65.391 mm

FIGURE P6-36M Units: mm NOTE: Reference axis is bases of shape; Overall height = 152 mm

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-W150x22.5	2860	76.000	217360	1.210E+07	16.278	7.578E+05	1.286E+07
2-Angles (2)	1218	21.500	26187	2.880E+05	38.222	1.779E+06	2.067E+06

Total area = 4078 Sum Ay = 243547 **Total I = 1.49E+07 mm^4**

Y = 59.722 mm NOTE: y for angles = 14.9 + 6.60 = 21.50 mm; t_f = 6.60 mm

FIGURE P6-37M Units: mm NOTE: Reference axis is bases of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Bot tube	1570	51.00	80070	1.870E+06	38.25	2.297E+06	4.167E+06
2-Top tube	1570	127.50	200175	6.160E+05	38.25	2.297E+06	2.913E+06

Total area = 3140 Sum Ay = 280245 **Total I = 7.08E+06 mm^4**

Y = 89.25 mm

FIGURE P6-38M Units: mm NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Outside tube	6280	76.00	477280	2.010E+07	1.269	1.011E+04	2.011E+07
2-Bot inner tube	1570	37.30	58561	6.160E+05	37.431	2.200E+06	2.816E+06
3-Top inner tube	1271	114.70	145784	4.620E+05	39.969	2.030E+06	2.492E+06

Total area = 9121 Sum Ay = 681625 **Total I = 2.542E+07 mm^4**

Y = 74.731 mm NOTE: Use design wall thickness for HSS 152x152x12.7; t_w = 11.8 mm

FIGURE P6-39M Units: mm NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$
1-Bot channel	1020	11.60	11832.0	1.300E+05	73.559	5.519E+06	5.649E+06
2-152x51x6.4	2170	80.67	175053.9	5.450E+06	4.489	4.374E+04	5.494E+06
3-Toop channel	1020	168.27	171635.4	1.300E+05	83.111	7.046E+06	7.176E+06

Total area = 4210 Sum Ay = 358521.300 **Total I = 1.83E+07 mm^4**

Y = 85.16 mm

FIGURE P6-40M Units: mm NOTE: Reference axis is bases of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-Bot channel	1214	18.500	22459	4.080E+05	8.278	8.320E+04	4.912E+05	
2-Top I-beam	1114	35.800	39881	2.830E+05	9.022	9.067E+04	3.737E+05	
Total area =	2328	Sum Ay =	62340			**Total I =**	**8.65E+05**	**mm^4**
Y =	**26.78**	**mm**						

FIGURE P6-41M Units: mm NOTE: Reference axis is bases of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W310x44.5	5670	156.000	884520	9.910E+07	51.312	1.493E+07	1.140E+08	
2-C150x19.3	2470	325.100	802997	4.370E+05	117.788	3.427E+07	3.471E+07	
Total area =	8140	Sum Ay =	1687517			**Total I =**	**1.49E+08**	**mm^4**
Y =	**207.31**	**mm**	NOTE: depth of W-beam = 312 mm					

FIGURE P6-42M Units: mm NOTE: Ref axis=centroid of W-shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W100x19.3	2470	0.000	0	4.700E+06	0.0	0.000E+00	4.700E+06	
2-bot tube	1570	-78.5	-123245	6.160E+05	78.5	9.675E+06	1.029E+07	
3-top tube	1570	78.5	123245	6.160E+05	78.5	9.675E+06	1.029E+07	
Total area =	5610	Sum Ay =	0.000			**Total I =**	**2.53E+07**	**mm^4**
Y =	**0.00**	**mm**	NOTE: depth of W-beam = 106 mm					

FIGURE P6-43M Units: mm NOTE: Ref axis is bottom of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-W310x44.5	5670	156	884520	9.910E+07	51.931	1.529E+07	1.144E+08	
2-L4x3x1/4	1090	343	373870	1.140E+06	135.069	1.989E+07	2.103E+07	
3-L4X3X1/4	1090	343	373870	1.140E+06	135.069	1.989E+07	2.103E+07	
Total area =	7850	Sum Ay =	1632260			**Total I =**	**1.56E+08**	**mm^4**
Y =	**207.93**	**mm**	NOTE: depth of W-beam = 312 mm					

FIGURE P6-44M Units: mm NOTE: Ref axis=centroid of tube

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-152x51x6.4	2170	0.000	0.000	5.450E+06	0.000	0.000E+00	5.450E+06	
2-bot plate	600	-82	-49200	7.200E+03	82	4.034E+06	4.042E+06	
3-top plate	600	82	49200	7.200E+03	82	4.034E+06	4.042E+06	
Total area =	3370	Sum Ay =	0.000			**Total I =**	**1.35E+07**	**mm^4**
Y =	**0.00**	**mm**	Note: Plates are 12 mm x 50 mm					

FIGURE P6-45M Units: mm NOTE: Ref axis is bottom of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-C203x6.17	2275	101.5	230912.5	1.560E+07	25.967	1.534E+06	1.713E+07	
2-C203x6.17	2275	101.5	230912.5	1.560E+07	25.967	1.534E+06	1.713E+07	
3-C203x6.17-Top	2275	179.4	408135.0	1.350E+06	51.933	6.136E+06	7.486E+06	
Total area =	6825	Sum Ay =	869960.0			**Total I =**	**4.18E+07**	**mm^4**
Y =	**127.47**	**mm**						

FIGURE P6-46M Units: mm NOTE: Ref axis at base of shape

Part	Area	y	Ay	I_c	d	Ad^2	$I_c + Ad^2$	
1-12x450 plate	5400	6.0	32400	6.480E+04	48.221	1.256E+07	1.262E+07	
2-20x250 plate	5000	137.0	685000	2.604E+07	82.779	3.426E+07	6.030E+07	
3-L203x102x12.7	3740	33.7	126038	2.810E+06	20.521	1.575E+06	4.385E+06	
4-L203x102x12.7	3740	33.7	126038	2.810E+06	20.521	1.575E+06	4.385E+06	
Total area =	17880	Sum Ay =	969476			**Total I =**	**8.17E+07**	**mm^4**
Y =	**54.22**	**mm**						

ANALYSIS OF BENDING STRESSES

7-1 $\sigma = Mc/I = ((425\ N\cdot m)(15\ mm)/67500\ mm^4)\ \frac{10^3\ mm}{m} = \underline{94.4\ MPa}$

$I = (30)^4/12 = 67\,500\ mm^4$

7-2 $I = \pi D^4/64 = \pi(20)^4/64 = 7854\ mm^4$

$\sigma = \frac{Mc}{I} = \frac{(120\ N\cdot m)(10\ mm)}{7854\ mm^4} \times \frac{10^3\ mm}{m} = \underline{152.8\ MPa}$

7-3 (a) $I = bh^3/12 = 0.75(1.5)^3/12 = 0.211\ in^4$

$\sigma = \frac{Mc}{I} = \frac{(5800\ lb\cdot in)(0.75\ in)}{0.211\ in^4} = \underline{20\,620\ psi}$

(b) $I = 1.5(0.75)^3/12 = 0.0527\ in^4$

$\sigma = \frac{(5800\ lb\cdot in)(0.375\ in)}{0.0527\ in^4} = \underline{41\,240\ psi}$

7-4 $I = 1.5(7.25)^3/12 = 47.63\ in^4$; $c = 7.25/2 = 3.625\ in.$

$\sigma = \frac{Mc}{I} = \frac{(15\,500\ lb\cdot in)(3.625)\ in}{47.63\ in^4} = \underline{1180\ psi}$

7-5 $M = 30\ k\cdot ft$; $S = 17.1\ in^3$

$\sigma = \frac{M}{S} = \frac{30\,000\ lb\cdot ft}{17.1\ in^3} \times \frac{12\ in}{ft} = \underline{21\,050\ psi}$

7-6 $M = 60\ k\cdot ft$; $S = 38.1\ in^3$

$\sigma = \frac{M}{S} = \frac{(60\,000\ lb\cdot ft)}{38.1\ in^3} \times \frac{12\ in}{ft} = \underline{18\,900\ psi}$

7-7

ALUM. C4X2.331; $I = 1.02\ in^4$; $c_t = 0.78\ in.$; $c_b = 1.47\ in$

BEAM IS IN NEGATIVE BENDING.

TENSILE - TOP

$\sigma = \frac{Mc_t}{I} = \frac{(9000\ lb\cdot in)(0.78\ in)}{1.02\ in^4} = \underline{6882\ psi}$

COMP. - BOTTOM

$\sigma = \frac{-Mc_b}{I} = \frac{-(9000\ lb\cdot in)(1.47\ in)}{1.02\ in^4} = \underline{-12\,970\ psi}$

7-8 $\sigma = \frac{M}{S} = \frac{4550\ lb\cdot in}{0.326\ in^3} = \underline{13\,960\ psi}$

7-9

$M = 71.5\ kN\cdot m = 71500\ N\cdot m \times 8.851\ lb\cdot in/N\cdot m = 6.33\times10^5\ lb\cdot in$

$I = 710.4\ in^4$; $C = \bar{Y} = 6.50\ in$

$$\sigma = \frac{MC}{I} = \frac{6.33\times10^5\ lb\cdot in\,(6.50\ in)}{710.4\ in^4} = \underline{5794\ psi\ (39.9\ MPa)}$$

7-10

$S = 22.67\ in^3 \times 1.639\times10^4\ mm^3/in^3 = 3.716\times10^5\ mm^3$

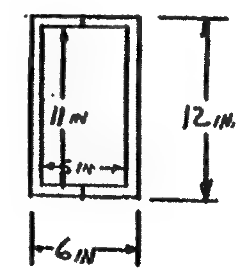
$$\sigma = \frac{M}{S} = \frac{(43.2\ kN\cdot m)}{3.716\times10^5\ mm^3}\times\frac{10^3\ N}{kN}\times\frac{10^3\ mm}{m} = \underline{116\ MPa}\ \underline{(16\ 800\ psi)}$$

7-11

$$I = \frac{6(12)^3}{12} - \frac{5(11)^3}{12} = 309.4\ in^4$$

$$\sigma = \frac{Mc}{I} = \frac{(60000\ lb\cdot ft)(6\ in)}{309.4\ in^4}\times\frac{12\ in}{ft}$$

$\sigma = 13963\ psi$



DESIGN OF BEAMS

7-12 $\sigma = Mc/I = M/S$; $S = \pi D^3/32$; $D = \sqrt[3]{32S/\pi}$

$$\text{REQ'D } S = \frac{M}{S_d} = \frac{240\ N\cdot m}{125\ N/mm^2}\times\frac{10^3\ mm}{m} = 1920\ mm^3$$

$$D = \sqrt[3]{32(1920\ mm^3)/\pi} = \underline{26.9\ mm}$$

7-13 $S = bh^2/6 = b(3b)^2/6 = 9b^3/6 = 1.5b^3$

$$b = \sqrt[3]{S/1.5} = \sqrt[3]{2636\ mm^3/1.5} = 12.1\ mm$$

$$\text{REQD } S = \frac{M}{S_d} = \frac{145\ N\cdot m}{55\ N/mm^2}\times\frac{10^3\ mm}{m} = 2636\ mm^3$$

$\underline{b = 12.1\ mm\ ;\ h = 3b = 36.3\ mm}$

7-14 $C_b = \bar{Y} = 152.5\ mm$, $C_t = 225 - 152.5 = 72.5\ mm$

$$\sigma_{BOTT} = \frac{MC_b}{I} = \frac{(28\ 000\ N\cdot m)(152.5\ mm)}{46.4\times10^6\ mm^4}\times\frac{10^3\ mm}{m} = \underline{92.0\ MPa}$$

FOR AISI 1020 HR , $S_y = 331\ MPa$

$\sigma_d = S_y/2 = 331\ MPa/2 = 165\ MPa$: SINCE $< S_d$, $\underline{OK}$

7-15 $I = 2.66\times10^5\ mm^4$; $c_b = \bar{Y} = 35.0\ mm$

$$\sigma = \frac{Mc_b}{I} = \frac{(275\ N\cdot m)(35.0\ mm)}{2.66\times10^5\ mm}\times\frac{10^3\ mm}{m} = \underline{36.2\ MPa}$$

LET $\sigma = \sigma_d = S_u/8$

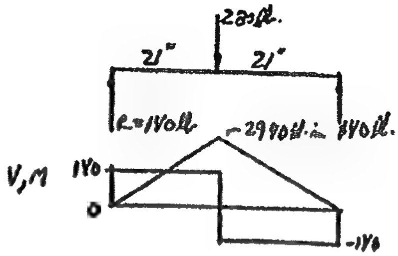
REQ'D $S_u = 8S = 8(36.2\ MPa) = \underline{290\ MPa}$

COULD USE $\underline{6061-T6}$, $S_u = 310\ MPa$; 17% ELONGATION

8-16

$$\text{REQ'D } S = \frac{M}{\sigma_d} = \frac{2940 \text{ lb}\cdot\text{in}}{10000 \text{ lb/in}^2} = 0.294 \text{ in}^3$$

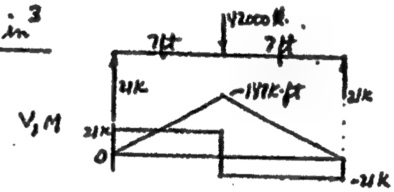
USE $1\frac{1}{2}$ IN SCH 40 PIPE, $= 0.3262 \text{ in}^3$



8-17

$$S = \frac{M}{\sigma_d} = \frac{147000 \text{ lb}\cdot\text{ft}}{20000 \text{ lb/in}^2} \times \frac{12 \text{ in}}{\text{ft}} = 88.2 \text{ in}^3$$

USE A W20X66 ; $S = 119 \text{ in}^3$



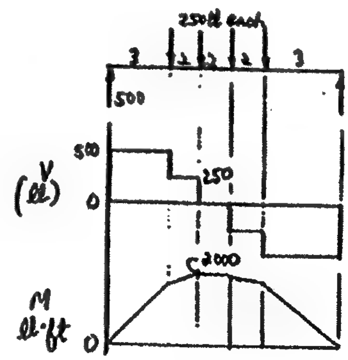
8-18

$I = 107.2 \text{ in}^4$; $C = 4.50 \text{ in}/2 = 2.25 \text{ in}$

$$\sigma = \frac{Mc}{I} = \frac{(2000 \text{ lb}\cdot\text{ft})(2.25 \text{ in})(12 \text{ in})}{(107.2 \text{ in}^4) \quad \text{ft}}$$

$\sigma = 504 \text{ psi}$

FROM TABLE A-19, MINIMUM ALLOWABLE BENDING STRESS = 625 psi - OK



8-19

$I = 30(3)^3/12 - 2[14(2)^3/12] = 48.83 \text{ in}^4$; $C = 1.50 \text{ in}$

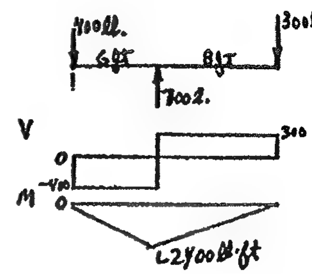
$$\sigma = \frac{Mc}{I} = \frac{2400 \text{ lb}\cdot\text{ft}(1.50 \text{ in})}{48.83 \text{ in}^4} \times \frac{12 \text{ in}}{\text{ft}}$$

$\sigma = 885 \text{ psi}$

FOR IMPACT;

REQUIRED $S_u = 12\sigma = 12(885) = 10620$ PSI

FOR 6061-T4 $S_u = 35000$ PSI - OK



8-20

$I = 1.86 \times 10^5 \text{ mm}^4$; $C_b = \bar{Y} = 21.8 \text{ mm}$; $C_t = 45 - 21.8 = 23.2 \text{ mm}$

MAX STRESS AT TOP

$$\sigma = \frac{M C_t}{I} = \frac{(318 \text{ N}\cdot\text{m})(23.2 \text{ mm})}{1.86 \times 10^5 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = 39.7 \text{ MPa (COMP.)}$$

REQ'D $S_y = 2\sigma = 2(39.7 \text{ MPa}) = 79.4 \text{ MPa}$ OK FOR 6061-T4 $S_y = 145 \text{ MPa}$

8-21

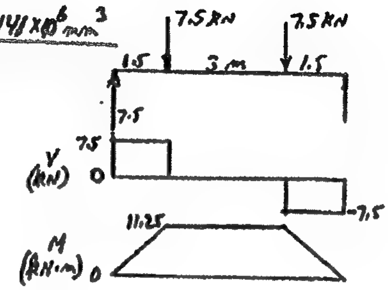
$I = 1.7 \times 10^4 \text{ mm}^4$; $C_b = \bar{Y} = 12.39 \text{ mm}$

$$\sigma = \frac{Mc}{I} = \frac{(195 \text{ N}\cdot\text{m})(12.39 \text{ mm})}{1.7 \times 10^4 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = 142 \text{ MPa}$$

REQ'D $S_y = 2\sigma = 284 \text{ MPa}$ — 2014-T4 HAS $S_y = 290 \text{ MPa}$

7-22

REQ'D $S = \frac{M}{\sigma_d} = \frac{11250\ N\cdot m}{80\ N/mm^2} \times \frac{10^3 mm}{m} = \underline{0.141 \times 10^6\ mm^3}$



(a) ROUND : $S = \pi D^3/32$

$D = \sqrt[3]{32(0.141 \times 10^6)/\pi} = \underline{112.8\ mm}$

$A = \frac{\pi D^2}{4} = \underline{9998\ mm^2}$

(b) SQUARE : $S = b^3/6$

$b = \sqrt[3]{6(0.141 \times 10^6)} = \underline{94.6\ mm}$

$A = b^2 = \underline{8945\ mm^2}$

(c) RECT. ; $h = 4b$; $= \frac{bh^2}{6} = \frac{b(4b)^2}{6} = \frac{16b^3}{6} = 8b^3/3$

$b = \sqrt[3]{3S/8} = \sqrt[3]{3(0.141 \times 10^6)/8} = \underline{37.5\ mm}$

$h = 4b = \underline{150\ mm}$

$A = bh = \underline{5625\ mm^2}$

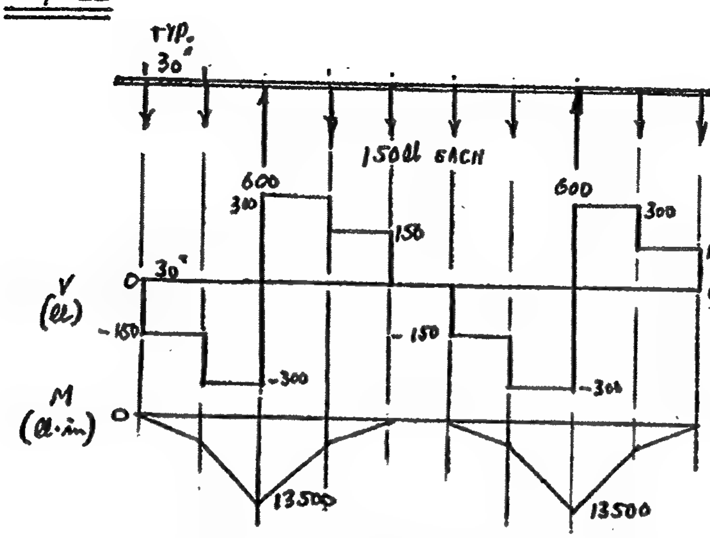
(d) S-BEAM ; $S = 0.141 \times 10^6\ mm^3 = 1.41 \times 10^5\ mm^3$

S150 X25.7 , $S = 1.43 \times 10^5\ mm^3$

[S6 × 17.25] $A = 3260\ mm^2$

LIGHTEST

7-23



$M_{MAX} = 13500\ lb\cdot in$

REQ'D $S = \frac{M}{\sigma_d} = \frac{13500\ lb\cdot in}{10\,000\ lb/in^2} = 1.35\ in^3$

USE $\underline{\text{3 IN. SCH 40 PIPE}}$, $\underline{S = 1.724\ IN.^3}$

7-24

$\sigma_d = S_y/4 = 46\,000\ psi/4 = 11500\ psi$

$M = (4800\ LB)(14\ IN) = 67200\ LB\cdot IN$

REQ'D. $S = M/\sigma_d = (67200\ LB\cdot IN)/(11500\ LB/IN^2) = 5.84\ IN^3$

USE EITHER 6x4x1/4 OR 8x2x1/4 : EACH WEIGHS 15.6 LB/FT.

7-25

LET $\sigma_d = S_y/4 = 40000\ psi/4 = 10000\ psi$

REQ'D. $S = M/\sigma_d = 67200/10000 = 6.72\ IN^3 \Rightarrow$ 6I x 4.030

7-26

$\sigma_d = S_y/4 = 50\,000\ psi/4 = 12500\ psi$

REQ'D. $S = M/\sigma_d = 67200/12500 = 5.38\ IN^3 \Rightarrow$ W 8 x 10

7-27

$\sigma_d = S_y/4 = 36000\ psi/4 = 9000\ psi$

$M/\sigma_d = S = 7.47\ IN^3$ FOR Y-AXIS : NO SUITABLE CHANNEL

7-28

$S_y = 36\ ksi$ - THEN REQ'D. $S = 7.47\ IN^3$ (PROB 7-27) : 6-IN SCH 40 PIPE

$A = 5.581\ IN^2$ - HEAVIER THAN OTHER STEEL DESIGNS

7-29

DESIGN PROBLEM - MULTIPLE SOLUTIONS POSSIBLE - WT < 4.030 LB/FT

FOR ALUM 6I x 4.030. USE ALUM 2014-T6, $S_y = 60\ ksi$

ALLOWS USE OF 5I x 3.700.

7-30

FROM FIG P 6-15 ; $I = 0.3572\ IN^4$; $c_b = 1.068\ IN.$; $c_t = 1.332\ IN$

$S_{MIN} = \frac{I}{c_t} = \frac{0.3572\ IN^4}{1.332\ IN} = 0.268\ IN^3$

$\sigma = \frac{M}{S} = \frac{117\ LB\cdot FT}{0.268\ IN^3} \times \frac{12\ IN}{FT} = 5239\ psi$

REQ'D $\sigma_d = 4\sigma = 20\,960\ psi$

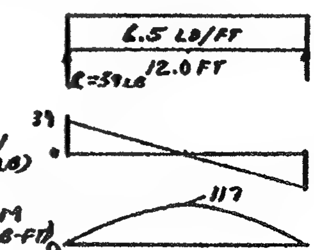
SEVERAL POSSIBLE CHOICES : APP. A-20

EXAMPLE: NYLON, POLYESTER

HIGH MODULUS OF ELASTICITY

GOOD ELECTRICAL PROPERTIES (TABLE 2-12)

MUST CHECK EXTRUDABILITY.



7-31

$M_{MAX} = 24\ kN\cdot m$ (SEE PROB P5-34)

$S = \frac{M}{\sigma_d} = \frac{24000\ N\cdot m}{150\ N/mm^2} \times \frac{10^3\ mm}{m} = 160\,000\ mm^3 \times \frac{6.102 \times 10^{-5}\ in^3}{mm^3}$

$S = 9.76\ in^3$; USE W10x12 (APP A-7(U.S)), $S = 10.9\ in^3$ | W 250 x 17.9 A-7(SI) $S = 1.79 \times 10^5\ mm^3$

7-32

FROM PROB. P5-35, $M_{MAX} = 125\ N\cdot m$

$\sigma_d = S_u/8 = 648\ MPa/8 = 81\ MPa$

$S = \frac{M}{\sigma_d} = \frac{125\ N\cdot m}{81\ N/mm^2} \times \frac{10^3\ mm}{m} = 1543\ mm^3 = \pi D^3/32$

$D = \sqrt[3]{32S/\pi} = 25.1\ mm^3$

7-33 Specify the lightest wide-flange beam. ASTM A992 structural steel. Sy = 50 ksi
Design stress: $\sigma_d = 0.66\ s_y = (0.66)(50\ 000\ \text{psi}) = 33\ 000\ \text{psi}$
From Figure P5-3: M_{max} = (45.7 K-ft)(1000 lb/K)(12 in/ft) = 548 400 lb-in
Required section modulus: $S = M/\sigma_d$ = (548 400 lb-in)/(33 000 lb/in^2) = 16.6 in^3
Specify: W12x16 steel beam from Appendix A-7(US); S = 17.1 in^3

7-34 Specify the lightest wide-flange beam. ASTM A992 structural steel. Sy = 345 MPa
Design stress: $\sigma_d = 0.66\ s_y = (0.66)(345\ \text{MPa}) = 227.7\ \text{MPa} = 227.7\ \text{N/mm}^2$
From Figure P5-7: M_{max} = (71.5 kN-m)(1000 N/kN)(1000 mm/m) = 71.5x10^6 N-mm
Required section modulus: $S = M/\sigma_d$ = (71.5x10^6 N-mm)/(227.7 N/mm^2) = 3.14x10^5 mm^3
Specify: W360x39 steel beam from Appendix A-7(SI); US designation: W14x26
S = 5.79x10^5 mm^3

Problems 7-35 to 7-42 are similar to 7-33 and 7-34 above with the same design stress. Maximum bending moment varies with the beam loading shown in the indicated figures from Chapter 5.

Problems 7-43 to 7-52 use the same set of beam loadings as 7-33 and 7-34 but the objective is to specify the lightest American Standard S-beam. The required section modulus is the same.

Problems 7-53 to 7-62 use the same set of beam loadings as 7-33 and 7-42 but the material is ASTM A572 Grade 60 with s_y = 60 ksi (414 MPa). Then,

Design stress: $\sigma_d = 0.66\ s_y = (0.66)(60\ 000\ \text{psi}) = 39\ 600\ \text{psi}\ (273\ \text{MPa})$

The objective is to specify the lightest wide flange beam.

The results for these three sets of problems are summarized below.

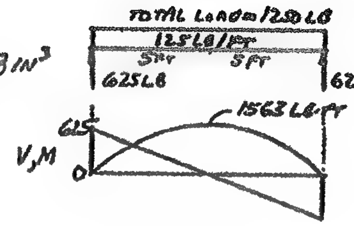
ASTM A992; s_y = 50 ksi (345 MPa)							ASTM A5762 Gr. 60 s_y = 60 ksi (414 MPa)		
Prob. No.	Fig. No.	M_{max}	Req'd. S	Lightest W-beam	Prob. No.	Lightest S-beam	Prob. No.	Req'd. S	Lightest W-beam
7-33	P5-3	45.7 K-ft	16.6 in^3	W12x16	7-43	S10x25.4	7-53	13.8 in^3	W12x16
7-34	P5-7	71.5 kN-m	3.14x10^5 mm^3	W360x39	7-44	S250x37.8	7-54	2.62x10^5 mm^3	W310x23.8
7-35	P5-8	43.2 kN-m	1.89x10^5 mm^3	W310x23.8	7-45	S200x27.4	7-55	1.58x10^5 mm^3	W250x17.9
7-36	P5-11	60.0 K-ft	21.8 in^3	W14x26	7-46	S10x25.4	7-56	18.2 in^3	W8x21
7-37	P5-16	170 kN-m	7.47x10^5 mm^3	W460x60	7-47	S380x64	7-57	6.23x10^5 mm^3	W310x44.5
7-38	P5-36	10.0 K-ft	3.64 in^3	W8x10	7-48	S5x10	7-58	3.03 in^3	W8x10
7-39	P5-40	40.0 K-ft	14.5 in^3	W12x16	7-49	S8x23	7-59	12.1 in^3	W12x16
7-40	P5-52	148 K-ft	53.8 in^3	W18x40	7-50	S15x42.9	7-60	44.8 in^3	W18x40
7-41	P5-63	1450 N-m	6.37x10^3 mm^3	W200x15	7-51	S80x8.5	7-61	5.31x10^3 mm^3	W200x15
7-42	P5-64	30.0 K-ft	10.9 in^3	W10x12	7-52	S8x18.4	7-62	9.08 in^3	W10x12

7-63

$\sigma_d = 1000\ PSI$

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(1563\ LB\cdot FT)(12\ IN/FT)}{850\ LB/IN^2} = 18.8\ IN^3$

USE 2×10 WOOD BEAM



7-64

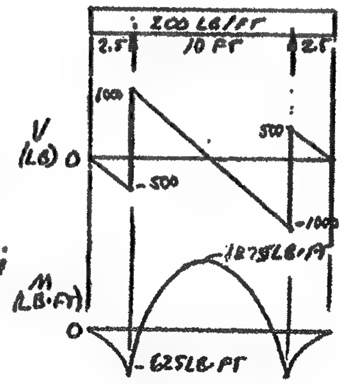
$\sigma_d = 1150\ PSI$

PART	A	$\bar{y}$	$A\bar{y}$	I	d	Ad^2	$I + Ad^2$
1	5.25	1.75	9.188	5.36	1.907	19.085	24.445
2	16.87	4.25	71.698	3.164	.593	5.939	9.103

$\Sigma A = 22.12$ $\Sigma A\bar{y} = 80.886$ $I_T = 33.55\ IN^4$

$\bar{Y} = 3.66\ IN = C$

$\sigma = \frac{MC}{I} = \frac{(1875\ LB\cdot FT)(12\ IN/FT)(3.66\ IN)}{33.55\ IN^4} = 2455\ PSI$ UNSAFE



7-65

PROCEDURE SAME AS 7-64 ; $\sigma_d = 1150\ PSI$

WITH 2×8 VERT. MEMBER, $\sigma = 798\ PSI$

WITH 2×8 VERT. MEMBER:

$\bar{Y} = C = 6.286\ IN$; $I = 177.3\ IN^4$

$\sigma = \frac{(1875)(12)(6.286)}{177.3} = 798\ PSI$ OK

7-66

PROCEDURE SAME AS 7-64 WITH DOUBLE-WIDTH WEB.

WITH 2-2×6 VERTICAL MEMBERS: $\bar{Y} = C = 4.519\ IN$; $I = 146.9\ IN^4$

$\sigma = \frac{(1875)(12)(4.519)}{146.9} = 692\ PSI$ OK

7-67

DESIGN PROBLEM — MULTIPLE SOLUTIONS POSSIBLE

7-68.

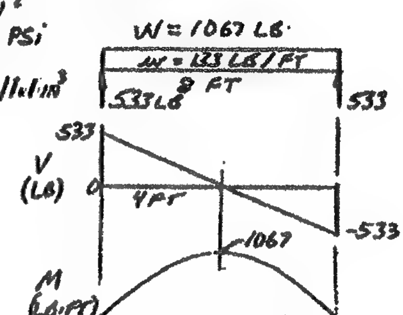
IN THE MIDDLE OF THE DECK, EACH FOOT OF JOIST LENGTH CARRIES A 16 IN WIDE PART OF THE DECK LOAD.

$w = \frac{100\ LB}{FT^2} \times \frac{(16\ IN)(12\ IN)}{FT} \times \frac{1\ FT^2}{144\ IN^2} = 133\ LB/FT$

SPECIFY NO. 2 HEMLOCK: $\sigma_d = 1150\ PSI$

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(1067\ LB\cdot FT)(12\ IN/FT)}{1150\ LB/IN^2} = 11.1\ IN^3$

USE 2×8 ; $S = 13.14\ IN^3$



7-69

JOISTS ARE 12 FT LONG BEAMS

AT ENDS: $w = 133\ LB/FT$:

$V_{MAX} = 800\ LB$ AT SUPPORTS :

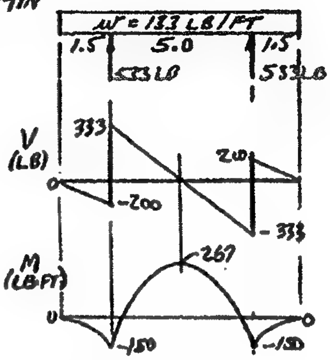
$M_{MAX} = 2400\ LB\cdot FT$ (28800 LB·IN)

REQ'D $S = \frac{M}{\sigma_d} = \frac{28800\ LB\cdot IN.}{1150\ LB/IN^2} = 25.0\ IN^3$ — USE 2×12 ; $S = 31.6\ IN^3$

7-70

REQ'D $S = \frac{M}{\sigma_d} = \frac{(267\ LB\cdot FT)(12\ IN/FT)}{1150\ LB/IN^2} = 2.79\ IN^3$

USE 2X4 JOISTS.: $S = 3.06\ IN^3$



7-71

JOISTS ARE 12 FT LONG; BEAMS 1.5 FT FROM EACH END: $w = 133\ LB/FT$:

$V_{MAX} = 600\ LB$ AT SUPPORTS:

$M_{MAX} = 1200\ LB\cdot FT$ (14400 LB·IN)

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(14400\ LB\cdot IN)}{1150\ LB/IN^2} = 12.5\ IN^3$

USE 2X8 ; $S = 13.1\ IN^3$

7-72

$M_{MAX} = 27733\ LB\cdot IN$

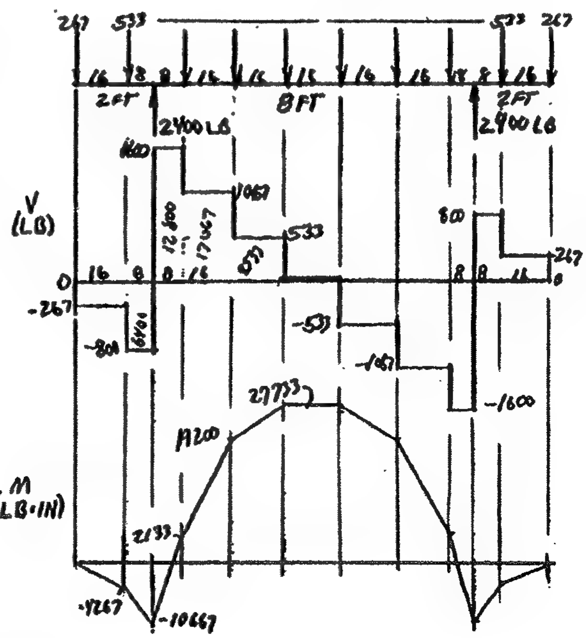
REQ'D $S = \frac{M}{\sigma_d} = \frac{27733\ LB\cdot IN}{1150\ LB/IN^2}$

$S = 24.1\ IN^3$

COULD USE:

2X12 : $S = 31.6\ IN^3$

4X8 : $S = 30.7\ IN^3$



DESIGN PROBLEM

7-73

TRY 2X6 DECK BOARDS WITH 3.0 FT SPAN

HEMLOCK: $\sigma_d = 1150\ LB/IN^2$

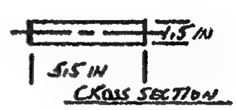
$S = bh^2/6 = (5.5)(1.5)^2/6 = 2.06\ IN^3$

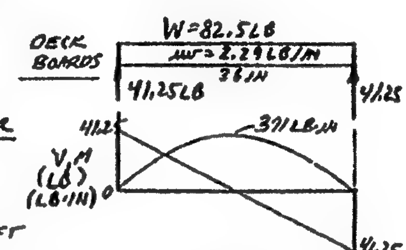
$W = \frac{60\ LB}{FT^2} \times \frac{(5.5)(36)\ IN^2}{BOARD} \times \frac{1\ FT^2}{144\ IN^2} = 82.5\ LB$ TOTAL

$w = 82.5\ LB/36\ IN = 2.29\ LB/IN$

$\sigma = \frac{M}{S} = \frac{371\ LB\cdot IN}{2.06\ IN^3} = 180\ PSI$ OK

MUST ALSO CHECK DEFLECTION AND SHEAR





BEAMS: TOTAL LOAD = $(60\ LB/FT^2)(30\ FT^2)$

$W_T = 1800\ LB$

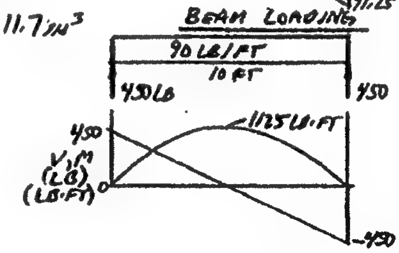
EACH BEAM CARRIES $900\ LB/10\ FT = 90\ LB/FT$

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(1125\ LB\cdot FT)(12\ IN/FT)}{1150\ LB/IN^2} = 11.7\ IN^3$

USE 4X6 BEAM: $S = 17.65\ IN^3$

NOTE: MANY OTHER DESIGNS ARE POSSIBLE AND PRACTICAL.

COULD USE 2X8 : $S = 13.14\ IN^3$

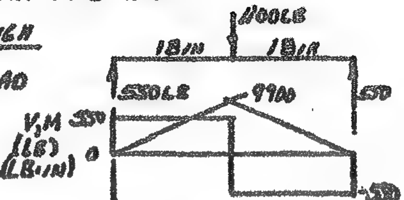


7-74

$\sigma_d = 1150$ PSI FROM PROB. 7-73.

ASSUME DECK BOARDS CARRY 1/2 OF TOTAL LOAD WHEN HORSE STEPS ON IT WITH ONE HOOF.

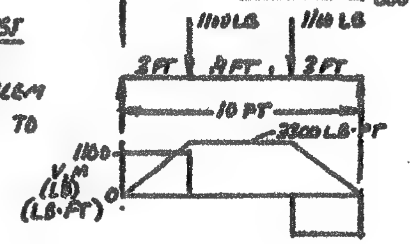
$\sigma = \frac{M}{S} = \frac{9900\ LB\cdot IN}{2.06\ IN^3} = \underline{4805\ PSI}$ TOO HIGH



ASSUME SIDE BEAMS CARRY FULL LOAD IF HORSE WALKS TOWARD ONE SIDE. LOAD SPLIT BETWEEN FRONT AND BACK; ASSUME 4FT SPREAD.

$\sigma = \frac{M}{S} = \frac{(3300\ LB\cdot FT)(12\ IN/FT)}{17.65\ IN^3} = \underline{2244\ PSI}$

TOO HIGH



CONCLUSION: BRIDGE DESIGNED IN PROBLEM 8-41 WOULD NOT BE SAFE FOR HORSE TO CROSS.

7-75

WOOD BEAM: ASSUME NO. 2 SOUTHERN PINE – $\sigma_d = 1000$ PSI

REQD $S = \frac{M}{\sigma_d} = \frac{(15750\ LB\cdot FT)(12\ IN/FT)}{1000\ LB/IN^2} = 189\ IN^3$

USE $\underline{10 \times 12\ \text{BEAM}}$: $\underline{S = 209\ IN^3}$

STEEL W-BEAM: ASTM A992 STEEL, $S_y = 50$ ksi

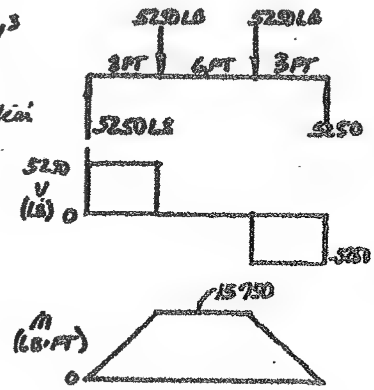
$\sigma_d = 0.66\ S_y = 0.66(50000\ PSI) = 33000$ PSI

REQ'D. $S = \frac{M}{\sigma_d} = \frac{(15750\ LB\cdot FT)(12\ IN/FT)}{33000\ LB/IN^2}$

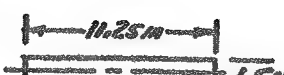
$S = 5.73\ IN^3$

USE: $\underline{W6 \times 12}$; $\underline{S = 7.31\ IN^3}$

OR $\underline{W8 \times 10}$; $\underline{S = 7.81\ IN^3}$



7-76



FOR 2X12: $S = \frac{bh^2}{6} = \frac{(11.25)(1.5)^2}{6}$

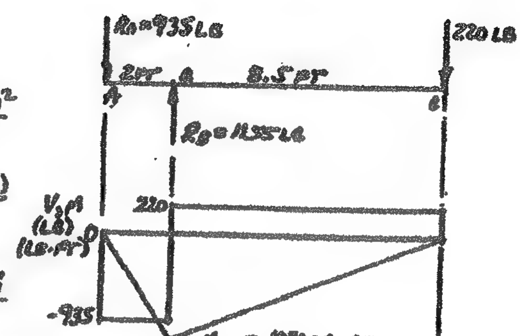
$S = 4.22\ IN^3$

$\sigma = \frac{M}{S} = \frac{(1870\ LB\cdot FT)(12\ IN/FT)}{4.22\ IN^3}$

$\underline{\sigma = 5319\ PSI}$

FOR NO. 2 SO. PINE: $\sigma_d = 1000$ PSI

UNSAFE



POSSIBLE REDESIGN APPROACHES:

SHORTER PLANK; STRONGER WOOD; THICKER PLANK; BUILT-UP BEAM.

7-77

$W = m \cdot g = 135\,kg \cdot 9.81\,m/s^2 = 1324\,N$

$w/2 = 662\,N$ ON EACH ROPE

FOR NO. 3 HEMLOCK: $\sigma_d = 4.3\,MPa$

AT A: $S = \pi D^3/32 = \pi(180)^3/32$

$S = 5.73 \times 10^5\,mm^3$

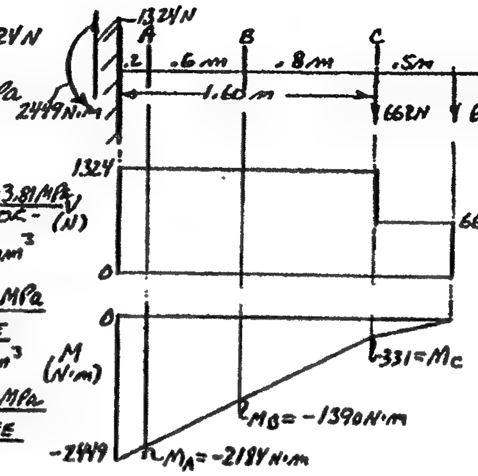
$\sigma = \frac{M}{S} = \frac{2184\,N\cdot m}{5.73\times10^5\,mm^3} \cdot \frac{10^3\,mm}{m} = 3.81\,MPa$ - OK -

AT B: $S = \pi(140)^3/32 = 2.69\times10^5\,mm^3$

$\sigma = \frac{M}{S} = \frac{1390\times10^3\,N\,mm}{2.69\times10^5\,mm^3} = 5.16\,MPa$ UNSAFE

AT C: $S = \pi(90)^3/32 = 7.16\times10^4\,mm^3$

$\sigma = \frac{M}{S} = \frac{(331\times10^3\,N\cdot mm)}{7.16\times10^4\,mm^3} = 4.62\,MPa$ UNSAFE



7-78

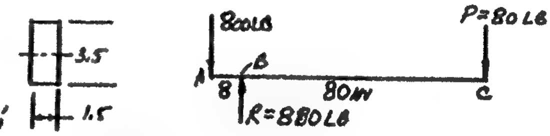
$M_{MAX} = 6400\,LB\cdot IN$ AT B

ASSUME 2×4 PLACED AS:

$S = 3.06\,IN^3$

$\sigma = \frac{M}{S} = \frac{6400\,LB\cdot IN}{3.06\,IN^3} = 2092\,psi$

$\sigma_d = 1000\,psi$ FOR NO. 2 SO. PINE - UNSAFE



7-79 FROM P 6-6; $I = 6.167\times10^4\,mm^4$

$C = 17.5\,mm$: $\sigma = \frac{Mc}{I}$

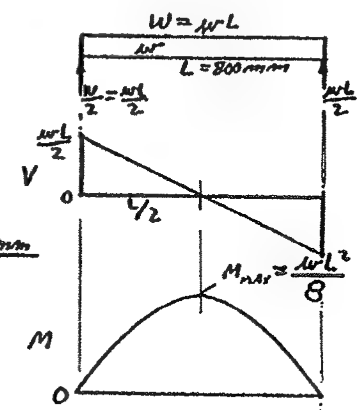
FOR NYLON 6/6: $S_f = 221\,MPa$

THEN $\sigma_d = S_f/2 = 110.5\,MPa$

$M_{MAX} = \frac{\sigma_d \cdot I}{C} = \frac{110.5\,N}{mm^2} \cdot \frac{6.167\times10^4\,mm^4}{17.5\,mm}$

$M_{MAX} = 3.89\times10^5\,N\cdot mm = wL^2/8$

$w_{MAX} = \frac{8M_{MAX}}{L^2} = \frac{(8)(3.89\times10^5\,N\cdot mm)}{(800\,mm)^2} = 4.86\,N/mm$



7-80 FROM P 6-9: $I = 1.35\times10^5\,mm^4$, $C = 20.0\,mm$

$M_{MAX} = (2.25\,kN)(0.20\,m) = 450\,N\cdot m$

$\sigma = \frac{Mc}{I} = \frac{(450\,N\cdot m)(20.0\,mm)}{1.35\times10^5\,mm^4} \cdot \frac{10^3\,mm}{m} = 66.7\,MPa$

REQ'D. FLEXURAL STRENGTH $= 3\sigma = 3(66.7) = 200\,MPa$

COULD USE: NYLON 6/6, POLYIMIDE

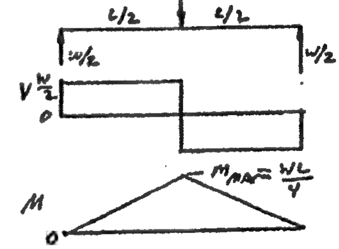
7-81

FROM P 6-5: $I = 2.66\times10^5\,mm^4$, $C = 35\,mm$

$\sigma_d = \frac{S_f}{2} = \frac{90\,MPa}{2} = 45.0\,MPa = \frac{M_{max}\,C}{I}$

$M_{MAX} = \frac{\sigma_d\,I}{C} = \frac{45.0\,N}{mm^2} \cdot \frac{2.66\times10^5\,mm^4}{35\,mm} = 3.42\times10^5\,N\cdot mm$

$W_{MAX} = \frac{4M_{MAX}}{L} = \frac{(4)(3.42\times10^5\,N\cdot mm)}{1250\,mm} = 1094\,N$

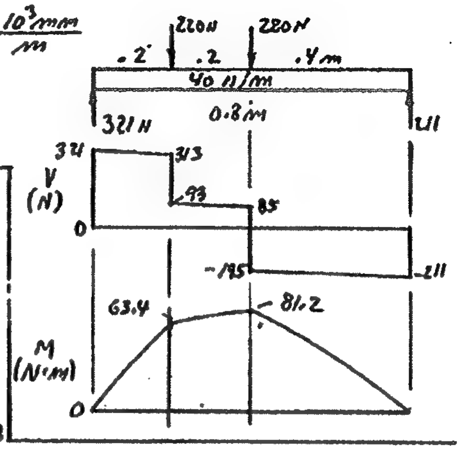


7-82 FROM P6-12: $I = 1.7\times10^4\ mm^4$, $c = 12.39\ mm$

$$\sigma = \frac{Mc}{I} = \frac{(81.2\ N\cdot m)(12.39\ mm)}{1.7\times10^4\ mm^4}\cdot\frac{10^3\ mm}{m}$$

$$\sigma = 59.3\ N/mm^2 = \underline{59.3\ MPa}$$

$$N = \frac{S_f}{\sigma} = \frac{76\ MPa}{59.3\ MPa} = \underline{1.28}$$



7-83 FROM P6-20: $I = 0.8263\ in^4$

$c = 1.28\ in$

$$\sigma_d = \frac{S_f}{3} = \frac{11000\ psi}{3} = 3667\ psi = \frac{M_{max}\,c}{I}$$

$$M_{MAX} = \frac{\sigma_d I}{c} = \frac{3667\ lb}{in^2}\cdot\frac{0.8263\ in^4}{1.28\ in}$$

$M_{MAX} = 2367\ lb\cdot in = WL/4$ (SEE 7-81)

$$W_{MAX} = \frac{4M_{MAX}}{L} = \frac{4(2367\ lb\cdot in)}{14.0\ in} = \underline{676\ lb}$$

7-84 FROM P6-15: $I = 0.3572\ in^4$; $c = 1.332\ in$; $L = 8\ ft \times 12\ in/ft = 96\ in$

$\sigma_d = S_y/2 = 21000\ psi/2 = 10500\ psi = M_{max}\,c/I$

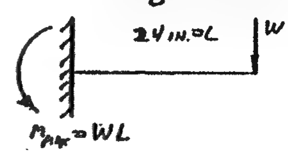
$$M_{MAX} = \frac{\sigma_d I}{c} = \frac{10500\ lb}{in^2}\cdot\frac{0.3572\ in^4}{1.332\ in} = 2816\ lb\cdot in = \frac{wL^2}{8}\ \text{(SEE 7-79)}$$

$$w_{MAX} = \frac{8M_{MAX}}{L^2} = \frac{8(2816\ lb\cdot in)}{(96\ in)^2} = 2.44\frac{lb}{in}\times\frac{12\ in}{ft} = \underline{29.3\ lb/ft}$$

7-85 FROM P6-14: $I = 0.3672\ in^4$; $c = 1.167\ in$; $\sigma_d = S_u/8 = \frac{62000\ psi}{8} = 7750\ psi$

$$M_{MAX} = \frac{\sigma_d I}{c} = \frac{7750\ lb}{in^2}\cdot\frac{0.3672\ in^4}{1.167\ in} = 2439\ lb\cdot in$$

$$W_{MAX} = \frac{M}{L} = \frac{2439\ lb\cdot in}{24\ in} = \underline{102\ lb}$$



7-86

HAT SECTION FROM P6-14 WITHOUT LOWER PLATE

PART	A	y	Ay	I	d	Ad^2	$I+Ad^2$	
1	.12	.05	.006					BOTTOM FLANGE (2 × .6 × .1)
2	.30	.75	.225					2 WEBS 2 × 1.5 × .1
3	.12	1.45	.174					TOP FLANGE (1.2 × 0.1)

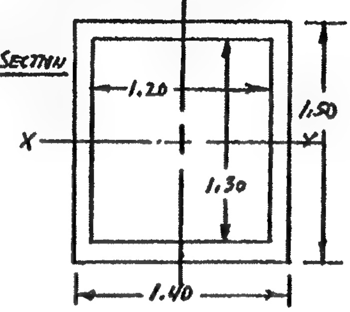
$\Sigma A = .54$ $\quad\Sigma Ay = .405$

$\bar{Y} = .405/.54 = \underline{0.75\ in}$ SYMMETRICAL → EQUIVALENT SECTION

$$I_x = \frac{1.40(1.50)^3}{12} - \frac{1.20(1.30)^3}{12} = 0.174\ in^4$$

$$M_{MAX} = \frac{\sigma_d I}{c} = \frac{7750\ lb}{in^2}\cdot\frac{.174\ in^4}{0.75\ in} = 1798\ lb\cdot in$$

$$W_{MAX} = \frac{M}{L} = \frac{1798\ lb\cdot in}{24\ in} = \underline{75\ lb}$$

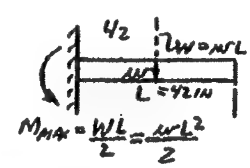


7-87 FROM P6-19: $I = 1.2506\ in^4$; $c = 1.570\ in$

$\sigma_d = S_u/6 = 45\ ksi/6 = 7.5\ ksi = 7500\ psi$

$$M_{MAX} = \frac{\sigma_d I}{c} = \frac{7500\ lb}{in^2}\cdot\frac{1.2506\ in^4}{1.570\ in} = 5974\ lb\cdot in$$

$$w_{MAX} = \frac{2M_{MAX}}{L^2} = \frac{2(5974\ lb\cdot in)}{(42\ in)^2} = \underline{6.77\ lb/in}$$



7-88 FROM P6-20: $I = 0.8263\ in^4$; $c = 1.28\ in$; $\sigma_d = \frac{S_u}{6} = \frac{41\ ksi}{6} = 6.83\ ksi$

$$M_{MAX} = \frac{\sigma_d I}{c} = \frac{6830\ lb}{in^2}\cdot\frac{0.8263\ in^4}{1.28\ in} = 4409\ lb\cdot in$$

$$w = \frac{2M_{MAX}}{L^2} = \frac{2(4409\ lb\cdot in)}{(42\ in)^2} = \underline{5.00\ lb/in}$$

BEAMS WITH STRESS CONCENTRATIONS AND VARYING CROSS SECTIONS

7-89

M_x = ALLOWABLE MOMENT AT JOINT

$\sigma = K_t M_x / S$ ($S = 2.394\,in^3$ FOR $3\frac{1}{2}$ in PIPE)

$M_x = \frac{\sigma_d S}{K_t} = \frac{(20000\,LB/IN^2)(2.394\,IN^3)}{1.78} = 26900$

$\left.\begin{array}{l} r/d = \frac{0.25}{4.00} = 0.063 \\ D/d = 4.50/4.00 = 1.125 \end{array}\right\} K_t = 1.78$

$M_x = 26900\,LB{\cdot}IN\,(1FT/12IN) = 2242\,LB{\cdot}FT$

LET $M_x = -4900 + 700X = -2242$

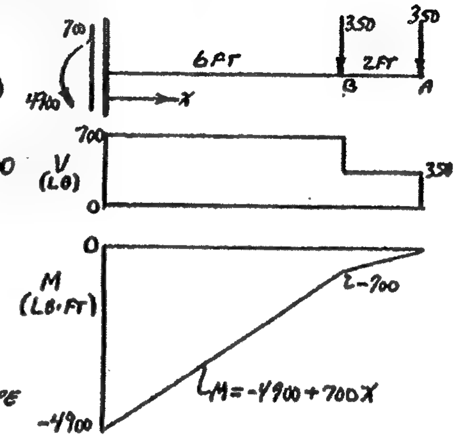
$X = 2658/700 = \underline{3.80\,FT}$ AT JOINT

AT D AT WALL: $S = 3.215\,IN^3$ FOR 4 IN PIPE

$\sigma = \frac{K_t M}{S} = \frac{(1.0)(4900\,LB{\cdot}FT)}{3.215\,IN^3} \times \frac{12\,IN}{FT}$

$\underline{\sigma = 18\,300\,PSI}$ IF NO SIGNIFICANT K_t EXISTS AT WALL.

<u>OK</u>



7-90

$M \approx 0$ AT A, E

POINT B IS CRITICAL

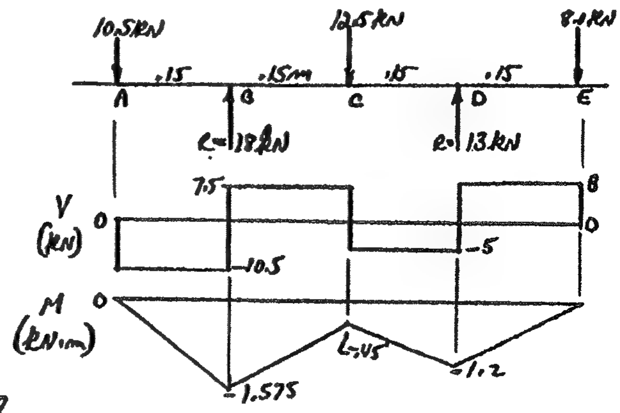
$\sigma = \frac{M K_t}{S}$

$S = \frac{\pi (45)^3}{32} = 8946\,mm^3$

FOR FILLET

$\left.\begin{array}{l} \frac{r}{d} = \frac{2}{45} = 0.044 \\ \frac{D}{d} = \frac{55}{45} = 1.22 \end{array}\right\} K_t = 2.17$

$\sigma = \frac{M K_t}{S} = \frac{1.575 \times 10^3\,N{\cdot}m\,(2.17)}{8946\,mm^3} \times \frac{10^3\,mm}{m} = \underline{382\,MPa}$



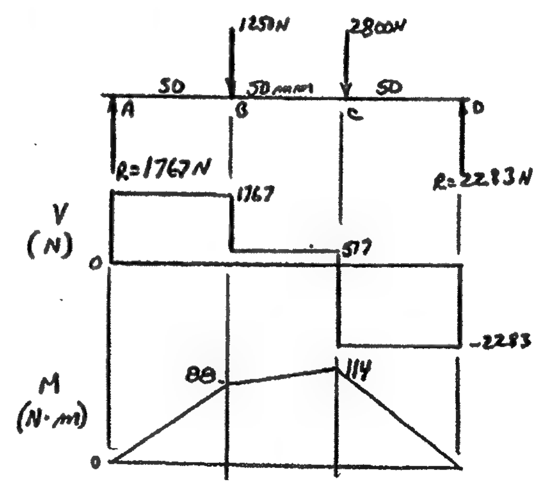
7-91

AT C

$S = \frac{\pi (18)^3}{32} = 572.6\,mm^3$

$\sigma = \frac{M K_t}{S} = \frac{(114\,N{\cdot}m)(2.0)}{572.6\,mm^3} \times \frac{10^3\,mm}{m}$

$\underline{\sigma = 398\,MPa}$



7-92

FOR D_1: $S = \pi D_1^3/32 = \pi(0.68)^3/32 = 0.03087\ IN^3$

FOR D_2: $S = \pi(1.00)^3/32 = 0.0982\ IN^3$

FOR D_3: $S = \pi(0.94)^3/32 = 0.08154\ IN^3$

AT C: $K_t = 2.0$ (KEYSEAT)

$\sigma = \frac{K_t \cdot M_C}{S} = \frac{2.0(1535)}{.0982} = 31270\ PSI$

AT D: $r/d = .01/.94 = .0106$: $D/d = 1.00/.94 = 1.06$

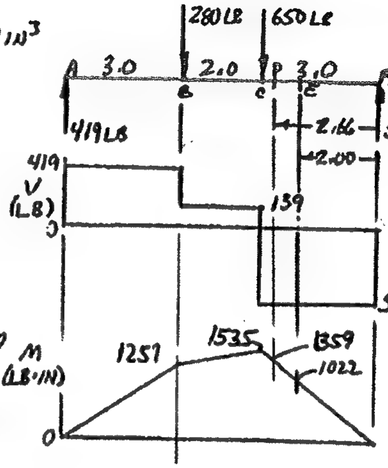
$K_t = 2.55$; $M_D = 1359\ LB{\cdot}IN$ (GROOVE)

$\sigma = (2.55)(1359)/.08154 = 42500\ PSI$

AT E: $r/d = .06/.68 = 0.088$: $D/d = 1.00/0.68 = 1.47$

$K_t = 1.78$: $M_E = 1022\ LB{\cdot}IN$ (STEP)

$\sigma = \frac{1.78(1022)}{.03087} = 58930\ PSI$ HIGHEST



7-93

AT FULCRUM C:

$S = bh^2/6 = .75(2.00)^2/6 = 0.50\ IN^3$

$\sigma = \frac{M}{S} = \frac{4000\ LB{\cdot}IN}{0.50\ IN^3} = 8000\ PSI$

AT HOLE B_1: $D_H = 0.75 = d$

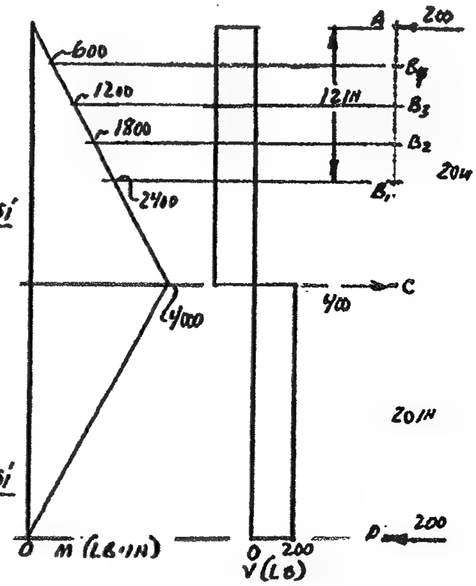
$d/w = .75/2.0 = 0.375 \rightarrow K_t = 1.0$

$\sigma = \frac{K_t\, 6Mw}{(w^3-d^3)(t)} = \frac{(1.0)(6)(2400)(2.0)}{(2.0^3-.75^3)(0.75)} = 5067\ PSI$

AT B_2: $M = 1800$: $\sigma = 3800\ PSI$

AT B_3: $M = 1200$: $\sigma = 2534\ PSI$

AT B_4: $M = 600$: $\sigma = 1267\ PSI$



7-94

AT C: $\sigma = 8000\ PSI$ (FROM 7-93)

AT B_1: $d/w = 1.38/2.0 = 0.69 \rightarrow K_t = 1.40$

$\sigma = \frac{K_t\, 6Mw}{(w^3-d^3)t} = \frac{1.40(6)(2400)(2.00)}{(2.0^3-1.38^3)(.75)} = 10000\ PSI$

AT B_2: $\sigma = 7506\ PSI$: AT B_3: $\sigma = 5004\ PSI$

AT B_4: $\sigma = 2500\ PSI$

7-95

$d/w = 1.25/2.00 = 0.625 \rightarrow K_t = 1.27$ AT HOLE

$\sigma_{B_1} = \frac{K_t\, 6M_{B_1}w}{(w^3-d^3)t} = \frac{(1.27)(6)(2.00)}{(2.0^3-1.25^3)(.75)} \cdot M_{B_1} = 3.36\, M_{B_1}$

AT FULCRUM, C

$S_C = 0.50\ IN^3$ (SEE 7-93)

$\sigma_C = M_C/S_C = 2.0\, M_C$

PIVOT	CD	M_C	σ_C	R_A	AB_1	M_{B_1}	σ_{B_1}
a) END HOLE	20 IN.	4000 LB·IN.	8000 PSI	200 LB	12 IN.	2400 LB·IN.	8064 PSI
b) HOLE B_4	17 IN.	3400	6800	170	9	1530	5141
c) HOLE B_3	14 IN.	2800	5600	140	6	840	2822
d) HOLE B_2	11 IN.	2200	4400	110	3	330	1109
e) HOLE B_1	8 IN.	1600	3200	80	0	0	0

7-96

$M = F(52 + 25/2) = (2500\ N)(64.5\ mm) = 161250\ N{\cdot}mm$

AT A-A; $S = bh^2/6 = 16(25)^2/6 = 1667\ mm^3$

$\sigma = M/S = (161250\ N{\cdot}mm)/1667\ mm^3 = 96.8\ MPa$

7-97

AT B-B: $d/w = 12/25 = 0.48 \rightarrow K_t = 1.0$

$\sigma = \frac{K_t\, 6Mw}{(w^3-d^3)(t)} = \frac{(1.0)(6)(161250)(25)}{(25^3-12^3)(16)} = 108.8\ MPa$

7-98 AT B-B: $d/w = 15/25 = 0.6 \rightarrow K_t = 1.21$

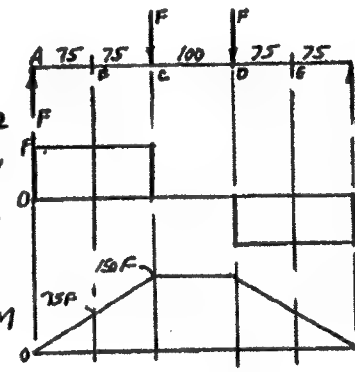
$$\sigma = \frac{(1.21)(6)(161250)(25)}{(25^3 - 15^3)(16)} = \underline{149.3\ MPa}$$

7-99 LET $\sigma = \sigma_d = S_u/8$: REQ'D. $S_u = 8\sigma = 8(149.3) = \underline{1195\ MPa}$

POSSIBLE STEEL: $\underline{\text{AISI 4140 OQT 900, } S_u = 1289\ MPa\text{; 15\% ELONG.}}$

7-100

AT C: $S = bh^2/6 = (12)(60)^2/6 = 7200\ mm^3$



$\underline{\text{AT B}}$: $S = 12(40)^2/6 = 3200\ mm^3$

$r/h = 10/40 = 0.25$: $H/h = 60/40 = 1.50 \rightarrow K_t = 1.42$

$\sigma_d = S_u/8 = 669/8 = 83.6\ MPa = \frac{K_t M}{S}$

$$M_{MAX} = \frac{\sigma_d S}{K_t} = \frac{83.6 N}{mm^2} \cdot \frac{3200 mm^3}{1.42} = 188450\ N\cdot mm$$

$M_B = 75F$: $F = \frac{M}{75} = \frac{188450\ N\cdot mm}{75\ mm} = \underline{\underset{MAX}{2513 N}}$

$\underline{\text{AT C}}$: $M_{MAX} = \frac{(83.6)(7200)}{1.0} = 602100\ N\cdot mm$

$M_C = 150F$: $F = \frac{M}{150} = \frac{602100}{150} = 4014 N$

7-101 $\underline{\text{AT B}}$: $r/h = 2.0/40 = 0.05$: $H/h = 1.50$: $K_t = 2.2$

$$M_{MAX} = \frac{\sigma_d S}{K_t} = \frac{83.6 N\ (3200 mm^3)}{mm^2\ (2.20)} = 121636 = 75F$$

$F = M_{MAX}/75 = \underline{1622\ N}$

7-102 LET DISTANCE FROM A TO B BE $\underline{a}$. THEN $M_B = Fa$: LET $\sigma_B = \sigma_C$

$$\sigma_B = \frac{K_t M_B}{S_B} = \frac{1.42(F \cdot a)}{3200} = \frac{F \cdot a}{2254} \quad \text{(DATA FROM PROB. 7-100)}$$

$$\sigma_C = \frac{K_t M_C}{S_C} = \frac{(1.0)(150F)}{7200} = \frac{F}{48}$$

$$\frac{F \cdot a}{2254} = \frac{F}{48} \ : \ a = \frac{2254}{48} = \underline{47\ mm}$$

7-103

$$\frac{K_t M_B}{S_B} = \frac{\cancel{K_t}^{1.0} M_C}{S_C} \ : \ K_{tB} = \frac{M_C}{M_B} \cdot \frac{S_B}{S_C} \cdot K_{tC} = \frac{150F}{75F} \cdot \frac{3200}{7200} \cdot 1.0 = 0.889 \ \underline{\text{IMPOSSIBLE}}$$

7-104 AT B: $S = 3200\ mm^3$: $r/h = 0.25$: $H/h = 75/40 = 1.88$: $K_t = 1.42$ $\underline{\text{SAME AS PROB. 7-100}}$

NO CHANGE IN LIMITING VALUE OF $\underline{F = 2513 N}$

7-105 BECAUSE MAXIMUM STRESS OCCURS AT B, A HOLE CAN BE DRILLED AT C.

$$\sigma = \sigma_d = 83.6\ MPa = \frac{K_t 6 M w}{(w^3 - d^3)(t)}$$

$F = 2513$ N MAXIMUM FROM PROB 7-100. THEN $M_C = 150F = 3.77 \times 10^5\ N\cdot mm$

$w = 60$, $t = 12$: BUT IF $d > 0.5w$, $K_t > 1.0$ (APP. A21-4)

SOLVE FOR d:

$$d = \left[w^3 - \frac{K_{tC}\, 6 M_C w}{\sigma_d t}\right]^{1/3} = \left[60^3 - \frac{(6)(60)(3.77\times10^5) K_t}{83.6\ (12)}\right]^{1/3}$$

$$d = \left[2.16\times10^5 - (1.35\times10^5) K_t\right]^{1/3}$$

BY ITERATION: IF $K_t = 1.0$, $d = 43.3$; BUT $d/w = 0.72$ AND $K_t = 1.42$

$K_t = 1.30$, $d = 34.3$: $d/w = 0.57$, $K_t = 1.14$

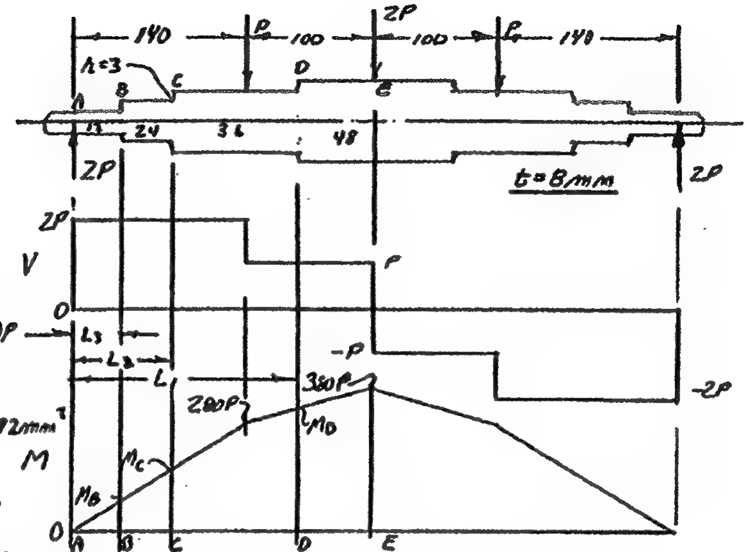
$K_t = 1.20$, $d = 37.8$: $d/w = 0.63$, $K_t = 1.24$

$K_t = 1.22$, $\underline{d = 37.2\ mm}$; $d/w = 0.62$, $K_t = 1.22$ $\underline{\text{OK}}$

MAXIMUM HOLE SIZE

7-106

$\sigma = \dfrac{K_t M}{S}$



$M_B = 2PL_3$

$M_C = 2PL_2$

$M_D = 280P + (L_1 - 140)P$

$M_D = 140P + L_1 P$

$S = th^2/6$

$S_B = 8(12)^2/6 = 192\ mm^3$

$S_C = \dfrac{8(24)^2}{6} = 768$

$S_D = \dfrac{8(36)^2}{6} = 1728$

$S_E = \dfrac{8(48)^2}{6} = 3072$

$L_1 = 180\ mm$; $L_2 = 80\ mm$; $L_3 = 40\ mm$; $P = 200\ N$

POINT	M(N·mm)	S(mm³)	r/h	H/h	Kt	σ (MPa)
B	16000	192	.25	2.0	1.42	118.3 MAXIMUM
C	32000	768	.125	1.50	1.68	70.0
D	64000	1728	.083	1.33	1.82	67.4
E	76000	3072	—	—	1.0	24.7

7-107

LET $\sigma = \sigma_d = S_u/8$

ONE POSSIBLE CHOICE

REQD $S_u = 8\sigma = 8(118.3\ MPa) = 946\ MPa$ ∴ AISI 1141 OQT 900, $S_u = 1007\ MPa$

7-108

DATA FROM 7-106 EXCEPT:

AT B: $r/h = 1.5/12 = 0.125 \rightarrow K_t = 1.75$: $\sigma_B = \underline{145.8\ MPa}$ MAXIMUM

AT C: $r/h = 1.5/24 = 0.063 \rightarrow K_t = 2.00$: $\sigma_C = 83.3\ MPa$

AT D: $r/h = 1.5/36 = 0.042 \rightarrow K_t = 2.27$: $\sigma_D = 84.1\ MPa$

7-109

$P = 400\ N$, K_t AND S FROM PROB 7-106

$\sigma_d = S_u/8 = 1170\ MPa/8 = 146\ MPa = \dfrac{K_t M}{S}$: $M_{MAX} = \dfrac{\sigma_d S}{K_t}$

AT B: $M_{B_{MAX}} = \dfrac{146 N}{mm^2} \dfrac{192\ mm^3}{1.42} = 19740\ N\cdot mm = 2PL_3$

$\underline{L_3} = \dfrac{19740\ N\cdot mm}{2(400)\ N} = \underline{24.7\ mm}$

AT C: $M_{C_{MAX}} = \dfrac{(146)(768)}{1.68} = 66742\ N\cdot mm = 2PL_2$

$\underline{L_2} = \dfrac{66742}{2(400)} = \underline{83.4\ mm}$

AT D: $M_{D_{MAX}} = \dfrac{(146)(1728)}{1.82} = 138620\ N\cdot mm = P(140 + L_1)$

$\underline{L_1} = \dfrac{M_D}{P} - 140 = \dfrac{138620}{400} - 140 = \underline{206.5\ mm}$

7-110

$\sigma_a = S_u/8 = 1014\,MPa/8 = 126.8\,MPa = \frac{K_t M}{S}$; K_t AND S FROM 8-106

AT B: $M_{MAX} = \frac{\sigma_a S_B}{K_{tB}} = \frac{126.8\,N}{mm^2}\cdot\frac{192\,mm^3}{1.42} = 17138\,N\cdot mm = 2PL_3$

$P_{MAX} = \frac{M}{2L_3} = \frac{17138\,N\cdot mm}{2(40\,mm)} = 214\,N$ GOVERNING VALUE

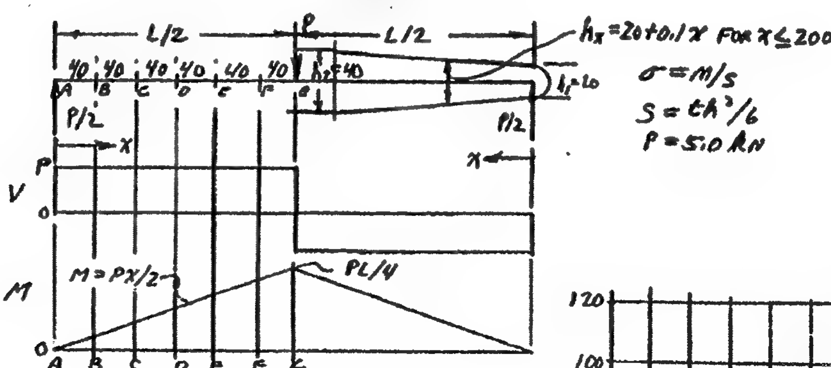
AT C: $M_{MAX} = \frac{(126.8)(768)}{1.68} = 57943\,N\cdot mm = 2PL_2$

$P_{MAX} = \frac{M}{2L_2} = \frac{57943}{2(80)} = 362\,N$

AT D: $M_{MAX} = \frac{(126.8)(1728)}{1.82} = 120342\,N\cdot mm = P(L_1+140)$

$P_{MAX} = \frac{M}{L_1+140} = \frac{120342}{180+140} = 376\,N$

7-111

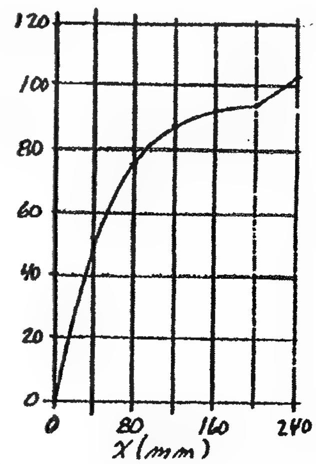


$\sigma = M/S$

$S = th^2/6$

$P = 5.0\,kN$

x (m)	M (kN·m)	h (mm)	S mm³	σ MPa
0	0	20	1333	0
0.040	.020P = 0.10	24	1920	52.1
0.080	.040P = 0.20	28	2613	76.5
0.120	.060P = 0.30	32	3413	87.9
0.160	.080P = 0.40	36	4320	92.6
0.200	.100P = 0.50	40	5333	93.8
0.240	.120P = 0.60	40	5333	112.5



7-112

$\sigma_a = \sigma'_f/4 = 76\,MPa/4 = 19.0\,MPa = M/S$

$M = Px/2$; $h_x = 20+0.2x$ FOR $x \le 200$; $S = th^2/6$

x	h	S	M	σ = M/S	P FOR σ_a = 19.0 MPa
0	20	1333	0	0	–
40	28	2613	20P	P/130.7	2483 N
80	36	4320	40P	P/108	2052 N
120	44	6453	60P	P/107.6	2044 N — GOVERNING VALUE WITHIN TAPER.
160	52	9013	80P	P/112.7	2141 N
200	60	12000	100P	P/120	2280 N
240	60	12000	120P	P/100	1900 N MAXIMUM PERMISSIBLE

7-113

$\sigma_d = S_u/8 = 793\,MPa/8 = 99.1\,MPa$; $P = 1.20\,kN = 1200\,N$

BASED ON RESULTS OF PROBS. 111 AND 112, h_1 IS CRITICAL AT MIDDLE

$M = PL/4 = (1200N)(480mm)/4 = 144\,000\,N\cdot mm$

REQD $S = \dfrac{M}{\sigma} = \dfrac{144\,000\,N\cdot mm}{99.1\,N/mm^2} = 1453\,mm^3 = th^2/6$

$h_1 = \sqrt{\dfrac{6S}{t}} = \sqrt{\dfrac{6(1453\,mm^3)}{20\,mm}} = 20.9\,mm$

AT $x = 120\,mm$: $M = 60P = 60(1200N) = 72000\,N\cdot mm$

REQD $S = \dfrac{M}{\sigma} = \dfrac{72000}{99.1} = 727\,mm^3$

$h_{120} = \sqrt{\dfrac{6(727)}{20}} = 14.8\,mm$

$\Delta h = 6.1\,mm$

FOR LINEAR SIDES; $h_2 = h_{120} - 6.1 = 8.7\,mm$

LET $h_2 = 8\,mm$; $h_1 = 22\,mm$ FOR CONVENIENT DIMENSIONS

A CHECK OF STRESS AS IN 7-111 SHOWS $\sigma < \sigma_d$ FOR ALL SECTIONS.

7-114

$\sigma_d = \dfrac{S_y}{4} = \dfrac{60\,000}{4} = 15\,000\,psi$

at B, $M = 10\,000\,lb\cdot ft$

REQ'D $S = \dfrac{M}{\sigma_d} = \dfrac{10000\,lb\cdot ft\,(12)\,in}{15000\,lb/in^2\,(ft)} = 8.00\,in^3$

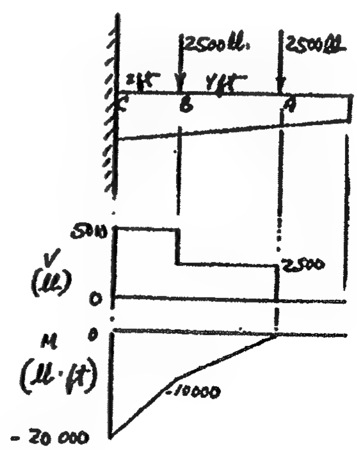
$S = \dfrac{bh^2}{6} = \dfrac{(1.5)\,h^2}{6} = 0.25h^2$

$h = \sqrt{S/0.25} = \sqrt{8.00/0.25} = 5.66\,in$

at C, $M = 20\,000\,lb\cdot ft$

$S = \dfrac{M}{\sigma_d} = \dfrac{20000\,(12)}{15000} = 16.00\,in^3$

$h = \sqrt{S/0.25} = 8.00\,in$

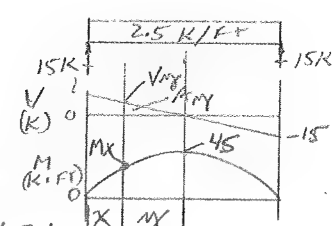


7-115 $\sigma_d = 0.66\,S_y = 0.66(50\,ksi) = 33\,ksi = M/S$

REQ'D. $S = M/\sigma_d = (45\,K\cdot FT)(12\,IN/FT)/33\,K/IN^2$

$S_{MIN} = 16.36\,IN^3$: USE W12×16; $S = 17.1\,IN^3$

WEIGHT $= (16\,LB/FT)(12\,FT) = 192\,LB$



7-116 USE W10×12 WITH COVER PLATES ON MIDDLE PART

$I_x = 53.8\,IN^4$, DEPTH $= 9.87\,IN$, $S = 10.9\,IN^3$

WITH COVER PL.: $I_T = I_x + 2Ad^2 = 53.8 + (.875)(5.31)^2(2) = 103.1\,IN^4$

$S = I/c = 103.1/5.185 = 19.88\,IN^3$ — OK FOR M_{MAX}

FIND ALLOWABLE M FOR W 10×12 ONLY

$M_x = \sigma_d S = (33\,ksi)(10.9\,in^3) = 359\,K\cdot IN = 29.925\,K\cdot FT$

ΔM FROM M_{MAX} TO M_x: $45.0 - 29.925 = 15.025\,K\cdot FT = A_y$

$V_y = 2.5\,y$; $\Delta M = V_y \cdot y/2 = 2.5\,y^2/2 = 15.025$

$y = 3.467\,FT$. SPECIFY $x = 2.5\,FT$, $y = 3.5\,ft$

PLATES COVER MIDDLE 7.0 FT. OF BEAM

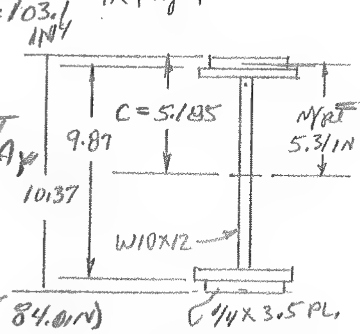
WT. OF PLATES: $= 2\rho AL = 2(0.283\,lb/in^3)(0.875\,in^2)(84.0\,in)$

$= 41.6\,LB$; WT OF W10×12 $= (12\,LB/FT)(12\,FT)$

$= 144\,LB$

TOTAL WT $= 144 + 41.6 = 185.6\,LB$

WT SAVINGS $= 192 - 185.6 = 6.4\,LB$ - SMALL



$A_{PL} = (0.25)(3.50) =$

$A_{PL} = 0.875\,IN^2$

7-117

COMPOSITE: FIG P6-26: $I = 469.4\ \text{IN}^4$; $C = 7.90\ \text{IN}$

$M_{MAX} = \dfrac{\sigma_d I}{C} = \dfrac{(23760)(469.4)}{7.90\ \text{IN}} = 1.41 \times 10^6\ \text{LB}\cdot\text{IN}(1\text{FT}/12\text{IN}) = 117.6\ \text{K}\cdot\text{FT} = wL^2/8$

$w = \dfrac{8M}{L^2} = \dfrac{8(117.6)\text{K}\cdot\text{FT}}{(15\text{FT})^2} = \underline{4.18\ \text{K/FT}}$

FOR S12X50: $S = 50.8\ \text{IN}^3$: $M_{MAX} = \sigma_d S = (23760)(50.8) = 1.207 \times 10^6\ \text{LB}\cdot\text{IN} = 100.6\ \text{K}\cdot\text{FT}$

$w = \dfrac{8M}{L^2} = \dfrac{8(100.6)\text{K}\cdot\text{FT}}{(15\text{FT})^2} = \underline{3.58\ \text{K/FT}}$

FLEXURAL CENTER

7-118

$e = \dfrac{b^2 h^2 t}{4 I_x}$; $I_x = \dfrac{38(76)^3}{12} - \dfrac{34(68)^3}{12} = 4.992 \times 10^5\ \text{mm}^4$

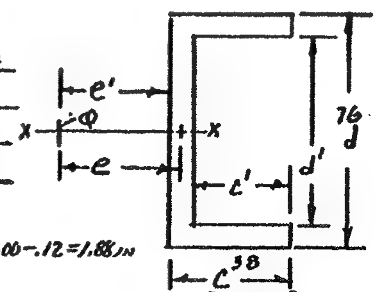
$b = 38 - 2 = 36\ \text{mm}$; $h = 76 - 4 = 72\ \text{mm}$; $t = 4\ \text{mm}$

$e = \dfrac{(36^2)(72)^2(4)}{4(4.992 \times 10^5)} = 13.5\ \text{mm}$ FROM MIDDLE OF WEB

Q IS AT $e - 2 = \underline{11.5\ \text{mm}}$ FROM LEFT FACE OF WEB.

7-119

t	c'	d'	I_x	b	h	e (mm)	$e' = e - t/2$
.50	37.5	75	71731	37.75	75.5	14.16	13.91
1.60	36.4	72.8	219745	37.20	74.4	13.94	13.14
3.00	35.0	70.0	389674	36.50	73.0	13.66	12.16



$I_x = \dfrac{cd^3}{12} - \dfrac{c'd'^3}{12}$

7-120

$h = 2.00 - 0.12 = 1.88\ \text{IN}$; $C = .50 + \frac{.12}{2} = 0.56\ \text{IN}$; $b = 2.00 - .12 = 1.88\ \text{IN}$

$b/h = 1.00$; $c/h = 0.298$; $e/h = 0.46$

$e = 0.46\,h = 0.46(1.88) = 0.865\ \text{IN}$

$e' = e - t/2 = .865 - .06 = \underline{0.805\ \text{IN}}$ FROM LEFT FACE

7-121

t	h	c	b	c/h	b/h	e/h	e	e'
.020	1.980	.5100	1.980	.258	1.00	.48	.950	.940
.063	1.937	.5315	1.937	.274	1.00	.46	.891	.860
.125	1.875	.5625	1.875	.300	1.00	.45	.844	.781

ALL DIMENSIONS IN INCHES

7-122

$h = 80 - 3 = 77\ \text{mm}$: $c = 20 - 1.5 = 18.5\ \text{mm}$: $b = 50 - 3 = 47\ \text{mm}$

$b/h = 47/77 = 0.61$; $c/h = 18.5/77 = 0.24$: THEN $e/h = 0.35$

$e = 0.35\,h = 0.35(77) = 27.0\ \text{mm}$

$e' = e - t/2 = 27.0 - 1.50 = \underline{25.5\ \text{mm}}$ FROM LEFT FACE

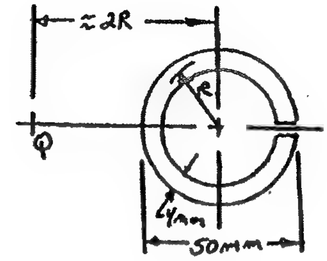
7-123

t	h	c	b	c/h	b/h	e/h	e	e'
0.50	79.5	19.75	49.5	0.248	0.623	.38	30.2	30.0 mm
1.60	78.4	19.20	48.4	0.245	0.617	.37	29.0	28.2 mm
3.00	77.0	18.5	47.0	0.24	0.61	0.35	27.0	25.5 mm

7-124

$R = \frac{D}{2} - \frac{t}{2} = 25 - 2 = 23\ mm$

Q IS AT $2R$ = 46 mm FROM CENTER



7-125

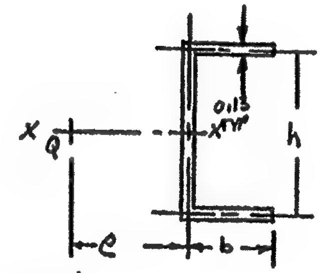
$I_x = 0.288\ IN^4$ (FROM APPENDIX)

OVERALL DIMENSIONS 2.00 IN. DEPTH = D

1.00 IN. WIDTH = B

$h = D - t = 2.00 - 0.13 = 1.87\ IN$

$b = B - \frac{t}{2} = 1.00 - \frac{0.13}{2} = 0.935\ IN.$



$$e = \frac{b^2 h^2 t}{4 I_x} = \frac{(0.935)^2 (1.87)^2 (0.13)}{4(0.288)}\ in. = \underline{0.345\ in.}$$

7-126

$h = 25 + 5 = 30\ mm$: $c = 20 - 2.5 = 17.5\ mm$: $b = 45 - 5 = 40\ mm$

$c/h = 0.58$: $b/h = 1.33$: THEN $e/h \approx 0.43$ BY EXTRAPOLATION

$e = 0.43h = 0.43(30) = \underline{12.9\ mm}$

BEAMS MADE FROM ANISOTROPIC MATERIALS

7-127

FROM P6-15: $I = 0.3572\ IN^4$; $C_b = 1.068\ IN$; $C_t = 1.332\ IN$; $L = 6.5\ FT = 78\ IN$

FOR TENSION ON BOTTOM: $M_{MAX} = \frac{\sigma_d I}{C_b} = \frac{19000\ LB}{IN^2} \cdot \frac{0.3572\ IN^4}{1.068\ IN} = 6355\ LB \cdot IN$

FOR COMP. ON TOP: $M_{MAX} = \frac{\sigma_d I}{C_t} = \frac{14000\ LB}{IN^2} \cdot \frac{0.3572\ IN^4}{1.332\ IN} = \underline{3754\ LB \cdot IN}$ LIMITING VALUE

FROM 8-47: $M_{MAX} = wL^2/8$

$$w_{MAX} = \frac{8 M_{MAX}}{L^2} = \frac{8(3754\ LB \cdot IN)}{(78\ IN)^2} = \underline{4.94\ LB/IN}$$

7-128

TENSION AT BOTTOM: $M_{MAX} = \frac{\sigma_d I}{c} = \frac{(19000)(0.3572)}{1.332} = \underline{5095\ LB \cdot IN}$ LIMITING VALUE

COMP. ON TOP: $M_{MAX} = \frac{(21000)(0.3572)}{1.068} = 7024\ LB \cdot IN$

$$w_{MAX} = \frac{8M}{L^2} = \frac{8(5095)}{(78)^2} = \underline{6.70\ LB/IN}$$

7-129

FROM P6-6: $I = 6.167 \times 10^4\ mm^4$; $C_b = 12.5\ mm$; $C_t = 17.5\ mm$

TENSION AT BOTTOM: $M_{MAX} = \frac{\sigma_d I}{C_b} = \frac{(100\ N)(6.167 \times 10^4\ mm^4)}{mm^2\ (12.5\ mm)} = 4.93 \times 10^5\ N \cdot mm$

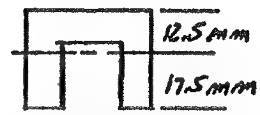
COMPRESSION AT TOP: $M_{MAX} = \frac{\sigma_d I}{C_t} = \frac{(70)(6.167 \times 10^4)}{17.5} = 2.47 \times 10^5\ N \cdot mm$ LIMITING VALUE

$M_{MAX} = WL/4$ (SEE 8-81)

$$W_{MAX} = \frac{4 M_{MAX}}{L} = \frac{4(2.47 \times 10^5\ N \cdot mm)}{1200\ mm} = \underline{822\ N}$$

7-130

FROM P6-6 : $I = 6.167 \times 10^4\ mm^4$



$C_t = 12.5\,mm$, $C_b = 17.5\,mm$

COMPRESSION AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t}$

$$M_{MAX} = \frac{70\,N\,(6.167 \times 10^4\,mm^4)}{mm^2\,(12.5\,mm)} = 3.45 \times 10^5\ N\cdot mm \text{ — LIMITING VALUE}$$

TENSION AT BOTTOM: $M_{MAX} = \dfrac{\sigma_d I}{C_b} = \dfrac{(100)(6.167 \times 10^4)}{17.5} = 3.52 \times 10^5\ N\cdot mm$

$$W_{MAX} = \frac{4\,M_{MAX}}{L} = \frac{4(3.45 \times 10^5\,N\cdot mm)}{1200\,mm} = \underline{1150\,N}$$

7-131

FROM P6-8 : $I = 5.36 \times 10^4\ mm^4$; $C_b = C_t = 20\,mm$

COMPR. AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{(70\,N)(5.36 \times 10^4\,mm^4)}{mm^2 (20.0\,mm)} = 1.876 \times 10^5\ N\cdot mm$ LIMITING VALUE

TENSION AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(100)(5.36 \times 10^4)}{20.0} = 2.68 \times 10^5\ N\cdot mm$

$$W_{MAX} = \frac{4\,M_{MAX}}{L} = \frac{4(1.876 \times 10^5\,N\cdot mm)}{1200\,mm} = \underline{625\,N}$$

7-132

FROM P6-9 : $I = 1.35 \times 10^5\ mm^4$; $C_b = C_t = 20\,mm$

COMPRESSION AT TOP IS LIMITING

$$M_{MAX} = \frac{\sigma_{dc} I}{C_t} = \frac{70\,N\,(1.35 \times 10^5\,mm^4)}{mm^2\,(20\,mm)} = 4.73 \times 10^5\ N\cdot mm$$

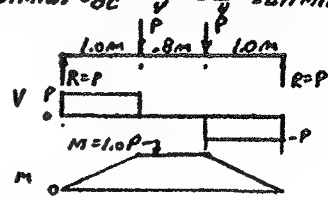
$$W_{MAX} = \frac{4\,M_{MAX}}{L} = \frac{4(4.73 \times 10^5\,N\cdot mm)}{1200\,mm} = \underline{1575\,N}$$

7-133

FROM P6-4 : $I = 4.64 \times 10^7\,mm^4$; $C_b = \bar{Y} = 152.5\,mm$; $C_t = 72.5\,mm$

CI-ASTM A48, GR40 : $\sigma_{dt} = \dfrac{S_{ut}}{4} = \dfrac{276\,MPa}{4} = 69\,MPa$; $\sigma_{dc} = \dfrac{S_{uc}}{4} = \dfrac{965}{4} = 241\,MPa$

$\sigma = \dfrac{Mc}{I}$: $M_{MAX} = \dfrac{\sigma_d I}{c}$



COMP. AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{241\,N\,(4.64 \times 10^7\,mm^4)}{mm^2\,(72.5\,mm)}$

$M_{MAX} = 1.54 \times 10^8\ N\cdot mm$

TENS. AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(69)(4.64 \times 10^7)}{152.5}$

$M_{MAX} = 2.10 \times 10^7\ N\cdot mm$ — LIMITING VALUE $= (1.0\,m)(P)$

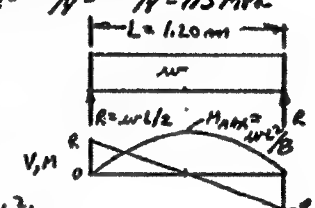
$P_{MAX} = M_{MAX}/1.0\,m = 2.10 \times 10^7\,N\cdot mm / 1000\,mm = 21.0 \times 10^3\,N = \underline{21.0\,kN}$

7-134

FROM P6-5 : $I = 2.66 \times 10^5\,mm^4$; $C_b = \bar{Y} = 35.0\,mm$; $C_t = 25.0\,mm$

ASTM A220, 80002 : $\sigma_{dt} = S_{ut}/4 = 655/4 = 164\,MPa$; $\sigma_{dc} = S_{uc}/4 = 1650/4 = 413\,MPa$

$\sigma = \dfrac{Mc}{I}$: $M_{MAX} = \dfrac{\sigma_d I}{c}$



COMP. AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{413\,N\,(2.66 \times 10^5\,mm^4)}{mm^2\,(25.0\,mm)}$

$M_{MAX} = 4.39 \times 10^6\ N\cdot mm$

TENS. AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(164)(2.66 \times 10^5)}{35.0}$

$M_{MAX} = 1.25 \times 10^6\ N\cdot mm$ — LIMITING VALUE $= wL^2/8$

$$w_{MAX} = \frac{8\,M_{MAX}}{L^2} = \frac{8(1.25 \times 10^6\,N\cdot mm)}{(1200\,mm)^2} = 6.94\,\frac{N}{mm} \cdot \frac{10^3\,mm}{m} = \underline{6.94\,kN/m}$$

TOTAL LOAD $= w \cdot L = (6.94\,kN/m)(1.20\,m) = \underline{8.33\,kN}$

7-135

$I = 2.66 \times 10^5\ mm^4$; $C_b = 25\ mm$; $C_t = 35\ mm$ (FROM PG-5 UPSIDE DOWN)

COMPR. AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{(413)(2.66\times10^5)}{35} = 3.14 \times 10^6\ N \cdot mm$

TENS. AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{(164)(2.66\times10^5)}{25} = 1.74 \times 10^6\ N \cdot mm$

$w = \dfrac{8M}{L^2} = \dfrac{8(1.74\times10^6)\ N\cdot mm}{(1200\ mm)^2} = 9.69\ N/mm = 9.69\ kN/m$

TOTAL LOAD $= wL = (9.69\ kN/m)(1.20\ m) = \underline{\underline{11.63\ kN}}$

7-136

$\sigma_{dt} = S_u/10 = 827\ MPa/10 = 82.7\ MPa$ & $\sigma_{dc} = S_{uc}/10 = 1240\ MPa/10 = 124\ MPa$

PART	A	y	Ay	I	d	Ad^2	$I + Ad^2$	
1	18750	62.5	1.17×10^6	24.4×10^6	75	105.5×10^6	129.9×10^6	3 RIBS
2	56250	162.5	9.14×10^6	26.4×10^6	25	35.16×10^6	61.56×10^6	TOP

$\Sigma A = 75000$ $\quad \Sigma Ay = 10.31\times10^6$ $\quad I = 191.4\times10^6\ mm^4$

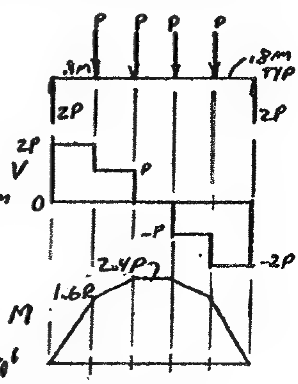
$\bar{Y} = 137.5\ mm = C_b$; $C_t = (200 - \bar{Y}) = 62.5\ mm$

TENSION AT BOTTOM: $M_{MAX} = \dfrac{\sigma_{dt} I}{C_b} = \dfrac{82.7\ N}{mm^2} \cdot \dfrac{191.4\times10^6\ mm^4}{137.5\ mm} = 115.1\times10^6\ N\cdot mm$

COMPR. AT TOP: $M_{MAX} = \dfrac{\sigma_{dc} I}{C_t} = \dfrac{(124)(191.4\times10^6)}{62.5} = 379.8\times10^6\ N\cdot mm$

$M_{MAX} = 2.4P$

$P_{MAX} = \dfrac{M_{MAX}}{2.4} = \dfrac{115.1\times10^6\ N\cdot mm}{2.4\ m(10^3\ mm/m)} = \underline{48.0\ kN}$



7-137

INCREASE DEPTH OF RIBS TO 250 mm

THEN $\bar{Y} = 222.5\ mm = C_b$; $C_t = (325 - \bar{Y}) = 102.5\ mm$

$I = 815.8 \times 10^6\ mm^4$

TENSION AT BOTTOM: $M_{MAX} = \dfrac{(82.7)(815.8\times10^6)}{222.5}$

$M_{MAX} = 303.2 \times 10^6\ N\cdot mm$

COMPR. AT TOP: $M_{MAX} = \dfrac{(124)(815.8\times10^6)}{102.5} = 986\times10^6$

$P_{MAX} = \dfrac{M_{MAX}}{2.4\ m} = \dfrac{303\times10^6\ N\cdot mm}{2400\ mm} = \underline{126\ kN}$

7-138 DESIGN PROBLEM - MULTIPLE SOLUTIONS POSSIBLE

7-139 ASTM A48 CAST IRON. $S_u = 276\ MPa$ – BRITTLE

$\sigma_d = S_u/6 = 276\ MPa/6 = 46.0\ MPa$

$$\text{ACTUAL } \sigma_{MAX} = \frac{M}{S} = \frac{(2.4\times10^3\ N)(350\ mm)}{\pi(50\ mm)^3/32} = 68.4\ MPa > \sigma_d$$

UNSAFE

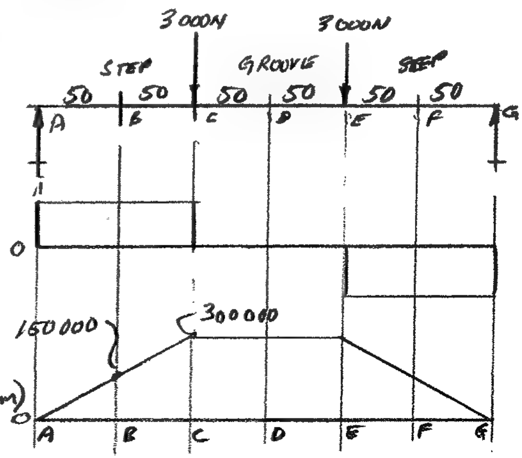
7-140 FROM FIG. 7-15: $M_{MAX} = 45900$ LB·IN. $\sigma_d = S_u/8$ REPEATED LOAD

AL. 6061-T6 $S_u = 45000$ PSI. $\sigma_d = 45000\ PSI/8 = 5625\ PSI$

$\sigma = M/S$. REQ'D $S = M/\sigma_d = 45900\ LB\text{-}IN / 5625\ LB/IN^2 = 8.16\ IN^3$

SPECIFY 6I × 4.692 ALUMINUM I-BEAM SHAPE. $S = 8.50\ IN^3$

7-141 FIND MAX. STRESS. POINTS B OR D POSSIBLE SECTIONS



AT B: $d = 40\ mm$, $M_B = 150\ 000\ N\cdot mm$

STEP: $d/D = 80/40 = 2.00$; $r/d = 3.0/40 = 0.075$ } $K_t = 1.97$ APP A-21-9

$$S_B = \frac{\pi d^3}{32} = \frac{\pi(40\ mm)^3}{32} = 6283\ mm^3$$

$$\sigma_B = \frac{M_B \cdot K_t}{S_B} = \frac{(150\ 000\ N\cdot mm)(1.97)}{6283\ mm^3} = 47.0\ MPa$$

AT D: $d_g = 60\ mm$, $M_D = 300000\ N\cdot mm$

GROOVE: $D/d_g = 80/60 = 1.33$; $r/d_g = 6.0/60 = 0.10$ } $K_t = 2.03$ APP A21-8

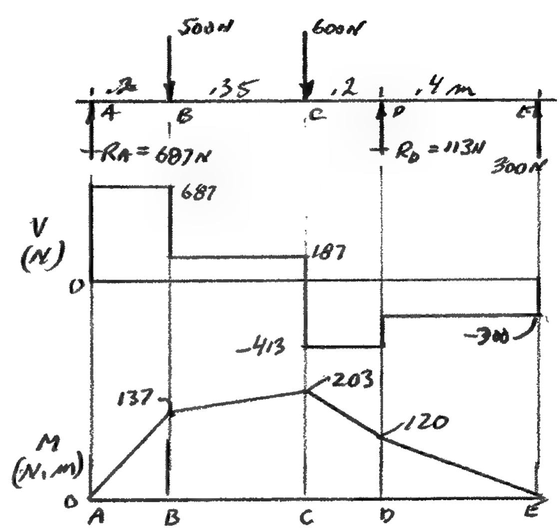
$$S_D = \pi(d_g)^3/32 = \pi(60\ mm)^3/32 = 21206\ mm^3$$

$$\sigma_D = \frac{M_D K_t}{S_D} = \frac{(300\ 000\ N\cdot mm)(2.03)}{21\ 206\ mm^3} = 28.7\ MPa$$

$\sigma_{MAX} = \sigma_B = 47.0\ MPa$

AT STEP AT B.

7-142 SHAFT $D = 30.0\ mm$. $M_{MAX} = 203\ N\cdot m$



$S = \pi D^3/32 = \pi(30\ mm)^3/32 = 2651\ mm^3$

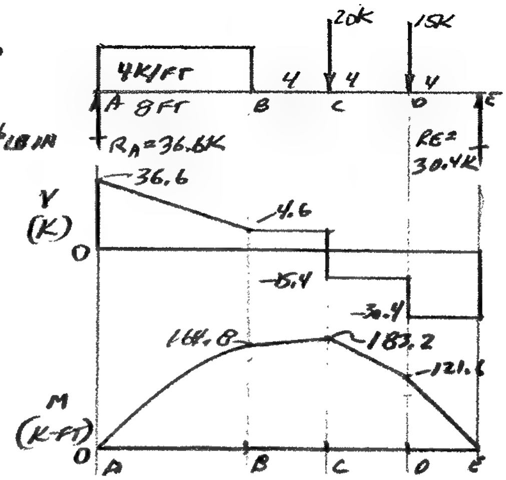
$$\sigma_{MAX} = \frac{M}{S} = \frac{203\ N\cdot m}{2651\ mm^3}\cdot\frac{10^3\ mm}{m} = 76.6\ MPa$$

$\sigma_d = S_u/8 = 600\ MPa/8 = 75.0\ MPa$

AISI 1040 WQT 1300. $S_u = 600\ MPa$

BECAUSE $\sigma_{MAX} > \sigma_d$ – UNSAFE

7-143 ASTM A992, $S_y = 50000$ PSI, STATIC LOAD



AISC: $\sigma_d = 0.66\,S_y = 0.66\,(50000) = 33000$ PSI

$M_{max} = 183.2\text{K-FT} \times \frac{1000\,LB}{K} \times \frac{12\,IN}{FT} = 2.20 \times 10^6\ LB\cdot IN$

$$\text{REQ'D } S = \frac{M_{max}}{\sigma_d} = \frac{2.20 \times 10^6\ LB\cdot IN}{33000\ LB/IN^2}$$

$S_{MIN} = 66.7\ IN^3$

SPECIFY W18×40; $S = 68.4\ IN^3$

CHECK SHEAR STRESS IN WEB.
CHECK LATERAL BRACING AND DEFLECTION

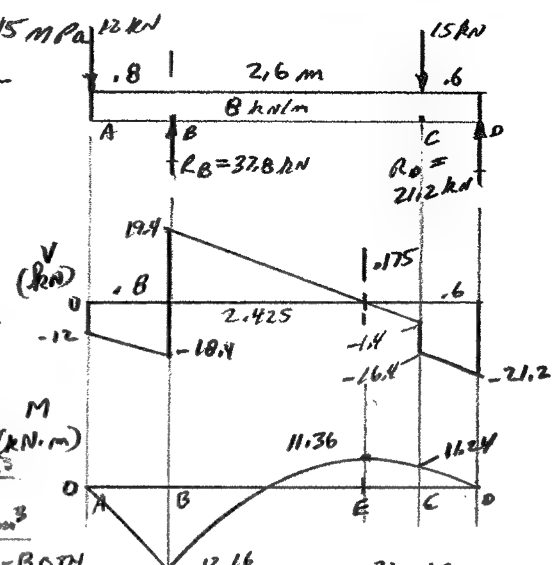
7-144 SAME AS 7-143 BUT ASTM A572, GR 65

$S_y = 65000$ PSI; $\sigma_d = 0.66\,S_y = 0.66\,(65000\,PSI) = 42900$ PSI

$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{2.20 \times 10^6\ LB\cdot IN}{42900\ LB/IN^2} = 51.3\ IN^3$$

SPECIFY W18×40; $S = 68.4\ IN^3$. NO BENEFIT, BUT CHOICES ARE LIMITED IN THIS BOOK. SEE AISC MANUAL FOR LARGER SELECTION. COST/LB MAY BE HIGHER. ALSO CHECK WEB SHEAR, DEFLECTION, AND LATERAL BRACING REQUIREMENTS FROM AISC SPECIFICATIONS.

7-145 ASTM A500 GRADE C: $S_y = 50$ KSI $= 345$ MPa



AISC: $\sigma_d = 0.66\,S_y = 0.66\,(345\,MPa) = 228$ MPa

$M_{MAX} = -12.16$ kN·m AT B.

$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{12.16\ kN\cdot m}{228\ N/mm^2} \times \frac{10^3\,N}{kN} \times \frac{10^3\,mm}{m}$$

$S_{MIN} = 5.33 \times 10^4\ mm^3$

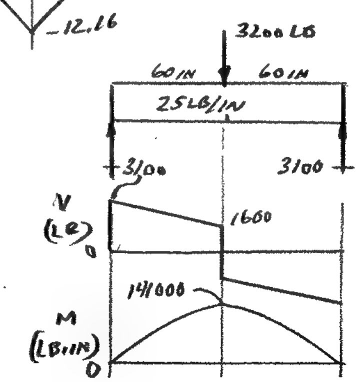
CONVERT TO IN^3:

$$S_{MIN} = 5.33 \times 10^4\ mm^3 \times \frac{(1\,IN)^3}{(25.4\,mm)^3} = 3.25\ IN^3$$

SPECIFY EITHER:

	SI SIZES
4×4×1/4; $S = 4.11\ IN^3$	102×102×6.4 $S = 6.39 \times 10^4\ mm^3$
6×2×1/4; $S = 4.60\ IN^3$	152×51×6.4 $S = 7.16 \times 10^4\ mm^3$
BOTH WEIGH 12.2 LB/FT	18.2 kg/m - BOTH

7-146 ASTM A500-C: $S_y = 46$ KSI



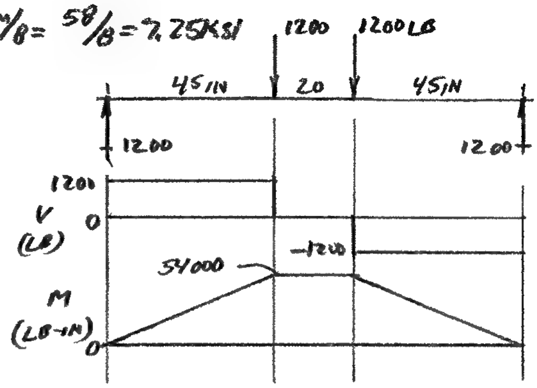
$\sigma_d = S_y/4 = 46\ KSI/4 = 11500$ PSI

$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{141000\ LB\cdot IN}{11500\ LB/IN^2} = 12.26\ IN^3$$

SPECIFY 8-IN SCHEDULE 40 STEEL PIPE

$S = 16.81\ IN^3$

7-147 ASTM A501: $S_u = 58\,KSI$; $\sigma_d = S_u/8 = 58/8 = 7.25\,KSI$



$$\text{REQ'D } S = \frac{M_{MAX}}{\sigma_d} = \frac{54000\ LB \cdot IN}{7250\ LB/IN^2}$$

$S_{MIN} = 7.45\ IN^3$

SPECIFY $8 \times 2 \times 1/4$ STEEL TUBE (HSS)

$S = 7.52\ IN^3$

$w = 15.6\ LB/FT$

7-148 $M_{MAX} = 36F$ AT SUPPORT

ASTM A48 GRADE 60: $S_{ut} = 55\,KSI$

$S_{uc} = 170\,KSI$

$$\sigma_{dt} = S_{ut}/4 = 55000\,PSI/4 = 13750\,PSI$$

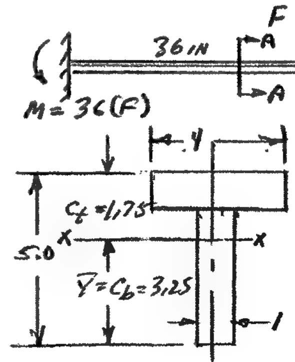
$$\sigma_{dc} = S_{uc}/4 = 170000\,PSI/4 = 42500\,PSI$$

TOP IS IN TENSION: $\sigma_T = \dfrac{M\,C_t}{I}$

$$M_{MAX} = \frac{\sigma_{dt}\,I}{C_t} = \frac{(13750\,LB/IN^2)(18.16\,IN^4)}{1.75\,IN}$$

$M_{MAX} = 142685\ LB \cdot IN = (36\,IN)(F)$

$$F_{ALL} = \frac{M_{MAX}}{36\,IN} = \frac{142685\,LB \cdot IN}{36\,IN} = 3963\ LB.$$ ANSWER



SECTION A-A

$I_x = 18.16\ IN^4$

BOTTOM IS IN COMPRESSION: $\sigma_b = \dfrac{M\,C_b}{I}$

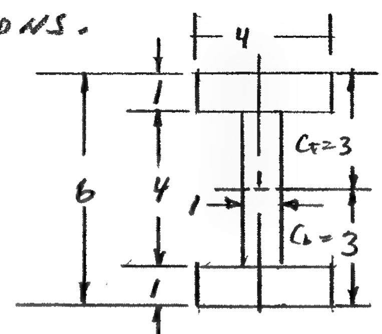
$$M_{MAX} = \frac{\sigma_{dc}\,I}{C_b} = \frac{(42500)(18.16)}{3.25} = 237477\ LB \cdot IN > M \text{ AT TOP}$$

7-149 DESIGN PROBLEM. MANY POSSIBLE SOLUTIONS.

ONE DESIGN: MAKE SHAPE A FULL I-BEAM

THEN $I = 4(6)^3/12 - 3(4)^3/12 = 56.0\ IN^4$

$$S = \frac{I}{c} = \frac{56.0\ IN^4}{3\,IN} = 18.67\ IN^3$$ FOR BOTH TENSION AT TOP AND COMPRESSION AT BOTTOM



AT TOP & BOTTOM: $\sigma = \dfrac{M}{S} = \dfrac{(6000\,LB)(36\,IN)}{18.67\,IN^3} = 11571\,PSI < \sigma_{dt}$ OK

SHAPE MAY BE OPTIMIZED BY MAKING TOP FLANGE LARGER THAN BOTTOM FLANGE TO TAKE ADVANTAGE OF $\sigma_{dc} >> \sigma_{dt}$. SEE EXAMPLE PROBLEM 7-10. TRIAL & ERROR SOLUTION REQ'D.

7-150 ASTM A992; $S_y = 50000$ PSI

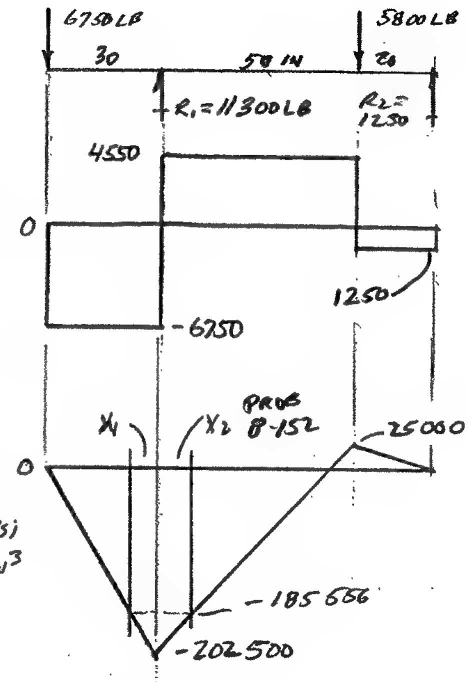
$\sigma_d = 0.66(50000) = 33000$ PSI

REQ'D $S = \dfrac{M_{MAX}}{\sigma_d} = \dfrac{202\,500 \text{ LB-IN}}{33000 \text{ LB/IN}^2} = 6.14 \text{ IN}^3$

SPECIFY <u>W8×10</u> $S = 7.81 \text{ IN}^3$

CHECK BEAM FOR COMPACTNESS, WEB SHEAR, AND LATERAL SUPPORT REQUIRED.

<u>TOTAL WT</u> $= 10\dfrac{\text{LB}}{\text{FT}} \times \dfrac{1\text{FT}}{12\text{IN}} \times 100\text{IN} = \underline{83 \text{ LB}}$



7-151 SHAPE IS IDENTICAL TO THAT IN PROBLEM 6-44.

$I_x = 34.95 \text{ IN}^4$

$C_t = C_b = 3.50 \text{ IN}$

$\sigma_d = 0.66(36000 \text{ PSI}) = 23760 \text{ PSI}$

$S = M/\sigma_d = \dfrac{202500}{23760} = 8.52 \text{ IN}^3$

$S = I/c = 34.95/3.50 = 9.99 \text{ IN}^3 > 8.52 \text{ IN}^3$

<u>SAFE</u>

6×2×1/4 TUBE WEIGHS $12.2 \text{ LB/FT} \times 100\text{IN} \times \dfrac{1\text{FT}}{12\text{IN}} = 101.7$ LB.

PLATES: VOL. IN 1.0 FT

$2[(2(0.5)\text{ IN}^2)12\text{IN}] = 24 \text{ IN}^3$

$WT = 24\text{IN}^3 \times 0.283 \text{ LB/IN}^3 = 6.79 \text{ LB}$; $6.79\dfrac{\text{LB}}{\text{FT}} \times 100\text{IN} \times \dfrac{1\text{FT}}{12\text{IN}} = 56.6 \text{ LB}$

<u>TOTAL WT</u> $= 101.7 + 56.6 = \underline{158.3 \text{ LB}}$ MUCH HEAVIER THAN W 8×10.

7-152 DESIGN PROBLEM – MANY POSSIBLE SOLUTIONS.

CONSIDER USING A SMALLER, LIGHTER SECTION FOR RIGHT PORTION OF THE BEAM WHERE M IS MUCH SMALLER. THEN ENHANCE THE SECTION NEAR R1 TO ENABLE IT TO WITHSTAND THE LOCALLY HIGH M. PLATES COULD BE ADDED TO THE TOP AND BOTTOM AS IN PROBLEM 7-151 BUT USING A SMALLER TUBE OR OTHER SHAPE AS A BASE.

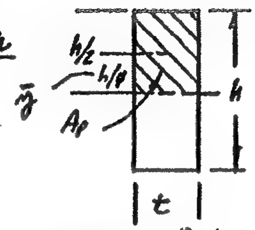
GENERAL SHEAR FORMULA

8-1

$$\tau = \frac{VQ}{It} = \frac{(1500\,N)(2.5\times10^5\,mm^3)}{(33.3\times10^6\,mm^4)(50\,mm)} = 1.125\,N/mm^2 = \underline{1.125\,MPa}$$

$$I = \frac{th^3}{12} = \frac{50(200)^3}{12} = 33.3\times10^6\,mm^4$$

$$Q = A_p\bar{y} = (100)(50)(50) = 2.5\times10^5\,mm^3$$



PROBLEMS 8-1 THRU 8-4

8-2

$$I = (38)(180)^3/12 = 18.47\times10^6\,mm^4$$

$$Q = (90)(38)(45) = 1.539\times10^5\,mm^3$$

$$\tau = \frac{VQ}{It} = \frac{(5000)(1.539\times10^5)}{(18.47\times10^6)(38)} = \underline{1.10\,MPa}$$

8-3

$$I = (1.5)(7.25)^3/12 = 47.63\,in^4 : Q = (3.625)(1.5)(1.813) = 9.86\,in^3$$

$$\tau = \frac{VQ}{It} = \frac{(12\,500\,lb)(9.86\,in^3)}{(47.63\,in^4)(1.5\,in)} = \underline{1724\,psi}$$

8-4

$$I = (3.5)(11.25)^3/12 = 415\,in^4 : Q = (5.625)(3.5)(2.813) = 55.37\,in^3$$

$$\tau = \frac{VQ}{It} = \frac{(20\,000)(55.37)}{(415)(3.5)} = \underline{762\,psi}$$

8-5

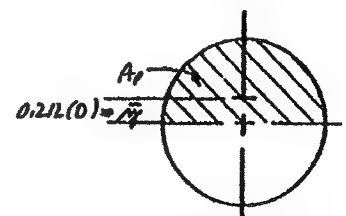
$$I = \pi D^4/64 = \pi(50^4)/64 = 3.07\times10^5\,mm^4$$

$$A_p = \pi D^2/8 = \pi(50)^2/8 = 982\,mm^2$$

$$\bar{y} = 0.212\,D = 0.212(50) = 10.6\,mm$$

$$Q = A_p\bar{y} = (982)(10.6) = 10\,407\,mm^3$$

$$\tau = \frac{VQ}{It} = \frac{(4500)(10\,407)}{(3.07\times10^5)(50)} = \underline{3.05\,MPa}$$



PROBLEMS 9-5 THRU 9-8

8-6

$$I = \pi(38)^4/64 = 1.024\times10^5\,mm^4 : Q = \frac{\pi(38)^2}{8}\times 0.212(38) = 4568\,mm^3$$

$$\tau = \frac{(2500)(4568)}{(1.024\times10^5)(38)} = \underline{2.94\,MPa}$$

8-7

$$I = \pi(2.0)^4/64 = 0.785\,in^4 : Q = [\pi(2.0)^2/8][0.212(2.0)] = 0.666\,in^3$$

$$\tau = \frac{(7500)(0.666)}{(0.785)(2.0)} = \underline{3180\,psi}$$

8-8

$$I = \pi(0.63)^4/64 = 0.00773\,in^4 : Q = [\pi(0.63)^2/8][0.212(0.63)] = 0.0208\,in^3$$

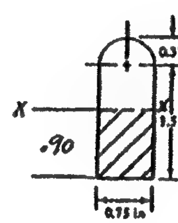
$$\tau = \frac{VQ}{It} = \frac{(850)(0.0208)}{(0.00773)(0.63)} = \underline{3632\,psi}$$

8-9

$$I_x = 0.366\,in^4 : \bar{Y} = 0.90\,in \text{ FROM P7-16.}$$

$$Q = (0.75)(0.90)(0.45) = 0.304\,in^3$$

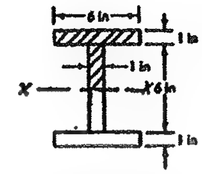
$$\tau = \frac{VQ}{It} = \frac{(1500)(0.304)}{(0.366)(0.75)} = \underline{1661\,psi}$$



8-10 $I_x = 166\ IN^4$ FROM P6-2.

$Q = A_1 y_1 + A_2 y_2 = 3(1.5) + 6(3.5) = 25.5\ IN^3$

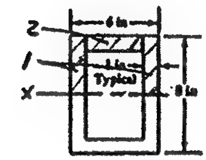
$\tau = \frac{VQ}{It} = \frac{(850)(25.5)}{(166)(1.0)} = \underline{131\ PSI}$



8-11 $I_x = 184\ IN^4$ FROM P6-3 $\therefore t = 2(1.0\ IN) = 2.0\ IN$

$Q = A_1 y_1 + A_2 y_2 = (2)(4)(2) + (4)(3.5) = 30.0\ IN^3$

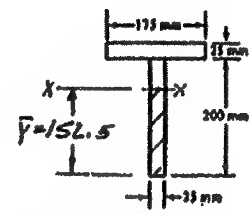
$\tau = \frac{(850)(30.0)}{(184)(2.0)} = \underline{69.3\ psi}$



8-12 $I_x = 4.64 \times 10^7\ mm^4$ FROM P6-4

$Q = A_p \bar{y} = (25)(152.5)(76.25) = 2.907 \times 10^5\ mm^3$

$\tau = \frac{(112 \times 10^3\ N)(2.907 \times 10^5\ mm^3)}{(4.64 \times 10^7\ mm^4)(25\ mm)} = \underline{28.1\ MPa}$

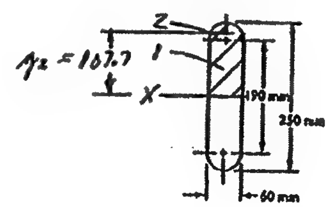


8-13 $I_x = 6.73 \times 10^7\ mm^4$ FROM P6-17

$Q = A_1 y_1 + A_2 y_2 = (60)(95)(47.5) + \frac{\pi (60)^2}{8}(107.7)$

$Q = 4.23 \times 10^5\ mm^3$

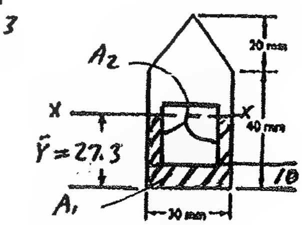
$\tau = \frac{(71.2 \times 10^3)(4.23 \times 10^5)}{(6.73 \times 10^7)(60)} = \underline{7.46\ MPa}$



8-14 $I_x = 3.08 \times 10^5\ mm^4$ FROM P6-18. $t = 10\ mm$

$Q = A_p \bar{y} = (300)(22.3) + (173)(8.65) = 8186\ mm^3$

$\tau = \frac{(1780)(8186)}{(3.08 \times 10^5)(10)} = \underline{4.73\ MPa}$

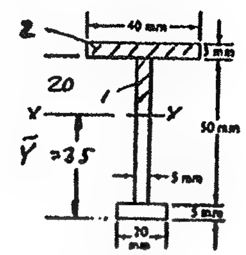


8-15 $I_x = 2.66 \times 10^5\ mm^4$ FROM P6-5

$Q = A_1 y_1 + A_2 y_2 = (20)(5)(10) + (40)(5)(22.5)$

$Q = 5500\ mm^3$

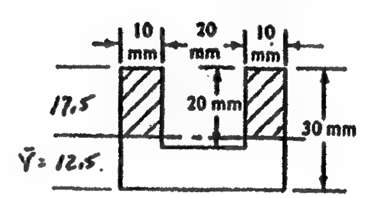
$\tau = \frac{(675)(5500)}{(2.66 \times 10^5)(5)} = \underline{2.79\ MPa}$



8-16 $I_x = 6.167 \times 10^4\ mm^4$ FROM P6-6

$Q = A_1 y_1 = (17.5)(20)(17.5/2) = 3063\ mm^3$

$\tau = \frac{(2500\ N)(3063\ mm^3)}{(6.167 \times 10^4\ mm^4)(20)\ mm} = \underline{6.21\ MPa}$



8-17 $I_X = 5.36 \times 10^4\ mm^4$ FROM P6-8.

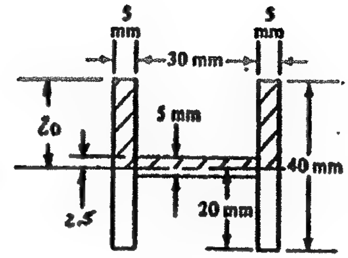
$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(5)(20)(20/2) + (30)(2.5)(1.25)$

$Q = 2094\ mm^3$

$\tau = \frac{VQ}{It} = \frac{(10500\ N)(2094\ mm^3)}{(5.36 \times 10^4\ mm^4)(40\ mm)} = \underline{10.3\ MPa}$

NOTE: $\tau_{MAX} = 30.0$ MPa JUST ABOVE HOR. WEB WHERE $Q = 1531\ mm^3$ AND $t = 10\ mm$.



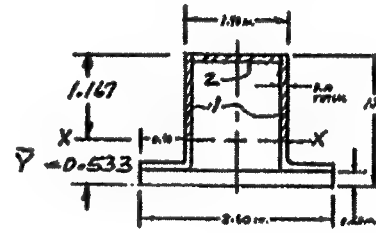
8-18 $I_X = 0.3672\ IN^4$ FROM P6-14

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(.10)(1.167)(1.167/2) + (0.1)(1.2)(1.117)$

$Q = 0.270\ IN^3$

$\tau = \frac{(1200)(0.270)}{(0.3672)(0.20)} = \underline{4416\ psi}$

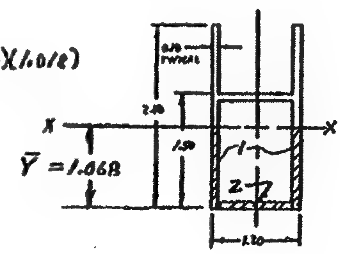


8-19 $I_X = 0.3572\ IN^4$ FROM P6-15

$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (2)(.10)(1.068)(1.068/2) + (0.1)(1.0)(1.018)$

$Q = 0.2159\ IN^3$

$\tau = \frac{(775)(0.2159)}{(0.3572)(0.20)} = \underline{2342\ psi}$



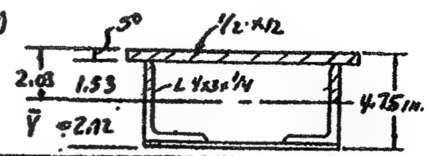
8-20 $I_X = 46.76\ IN^4$ FROM P6-33.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(0.25)(1.53)(1.53/2) + (0.5)(12)(1.78)$

$Q = 11.27\ IN^3$

$\tau = \frac{(2500)(11.27)}{(46.76)(0.50)} = \underline{1205\ psi}$



PROBLEMS 8-21 THRU 8-30: GIVEN $\tau_{MAX} = 70\ psi = \frac{VQ}{It}$

$V_{MAX} = \frac{\tau I t}{Q}$

8-21 $t = 1.50\ in$; $h = 3.50\ in$; $I = th^3/12 = 5.36\ in^4$; $Q = t(h/2)(h/4) = 2.297\ in^3$

$V = \frac{(70)(5.36)(1.50)}{2.297} = \underline{245\ LB}$

8-22 $t = 3.5\ in$; $h = 1.50\ in$; $I = (3.5)(1.5)^3/12 = 0.984\ in^4$; $Q = (3.5)(1.50/2)(1.50/4) = 0.984\ in^3$

$V = (70)(0.984)(3.5)/0.984 = \underline{245\ LB}$

8-23 $t = 1.50\ in$; $h = 11.25\ in$; $I = 178\ in^4$; $Q = (1.50)(11.25/2)(11.25/4) = 23.73\ in^3$

$V = (70)(178)(1.50)/23.73 = \underline{788\ LB}$

8-24 $t = 11.25\ in$; $h = 1.50\ in$; $I = 11.25(1.50)^3/12 = 3.16\ in^4$; $Q = (11.25)(1.50/2)(1.50/4) = 3.16\ in^3$

$V = (70)(3.16)(11.25)/3.16 = \underline{788\ LB}$

8-25 $t = 9.5\ IN$; $h = 11.5\ IN$; $I = 1204\ IN^4$; $Q = (9.5)(11.5/2)(11.5/4) = 157.\ IN^3$

$V = (70)(1204)(9.5)/157 = \underline{5098\ LB}$

8-26 $t = 11.5\ IN$; $h = 9.5\ IN$; $I = 11.5(9.5)^3/12 = 822\ IN^4$;

$Q = (11.5)(9.5/2)(9.5/4) = 129.7\ IN^3$

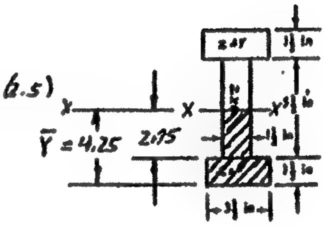
$V = (70)(822)(11.5)/129.7 = \underline{5098\ LB}$

8-27 $I_X = 151.4\ IN^4$ FROM P6-21.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (2.75)(1.5)(2.75/2) + (3.5)(1.5)(6.5)$

$Q = 24.05\ IN^3$

$V = (70)(151.4)(1.50)/24.05 = \underline{661\ LB}$



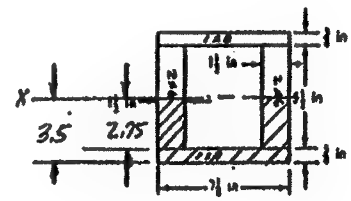
8-28 $I_X = 148.3\ IN^4$ FROM P6-22.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(1.5)(2.75)(2.75/2) + (.75)(7.25)(3.125)$

$Q = 28.33\ IN^3$

$V = (70)(148.3)(3.0)/28.33 = \underline{1099\ LB}$



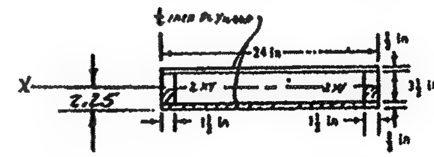
8-29 $I_X = 107.2\ IN^4$ FROM P6-23.

$Q = A_1\bar{y}_1 + A_2\bar{y}_2$

$Q = (2)(1.5)(1.75)(1.75/2) + (.5)(24)(2.00)$

$Q = 28.59\ IN^3$

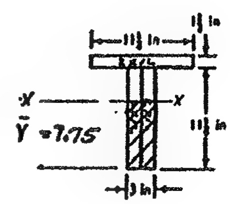
$V = (70)(107.2)(3.0)/28.59 = \underline{787\ LB}$



8-30 $I_X = 816.3\ IN^4$ FROM P6-24.

$Q = A_p\bar{y} = (7.75)(3.0)(7.75/2) = 90.09\ IN^3$

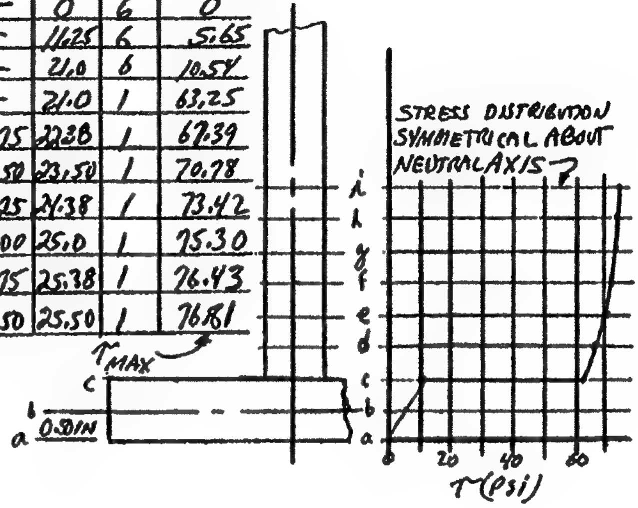
$V = (70)(816.3)(3.0)/90.09 = \underline{1903\ LB}$



8-31 $I_X = 166\ IN^4$ FROM P6-2 : $V = 500\ LB$; $\tau = \dfrac{VQ}{It}$

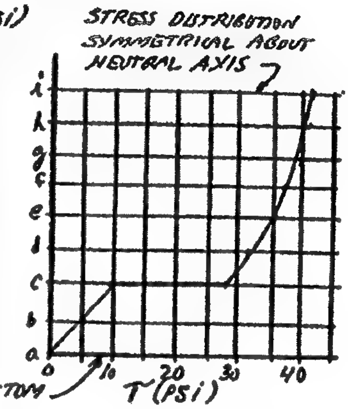
SECTION	A_1	$\bar{y}_1$	A_2	$\bar{y}_2$	Q	t	τ (PSI)
a-a	0	—	—	—	0	6	0
b-b	3.0	3.75	—	—	11.25	6	5.65
c-c	6.0	3.50	—	—	21.0	6	10.54
c-c'	6.0	3.5	—	—	21.0	1	63.25
d-d	6.0	3.5	0.50	2.75	22.38	1	67.39
e-e	6.0	3.5	1.0	2.50	23.50	1	70.78
f-f	6.0	3.5	1.5	2.25	24.38	1	73.42
g-g	6.0	3.5	2.0	2.00	25.0	1	75.30
h-h	6.0	3.5	2.5	1.75	25.38	1	76.43
i-i	6.0	3.5	3.0	1.50	25.50	1	76.81

τ_{MAX}



8-32 $I_x = 184\ IN^4$ FROM P6-3 : $V = 500\ LB$; $\tau = VQ/It$

SECTION	A_1	y_1	A_2	y_2	Q	t	τ (PSI)
a-a	0	—	—	—	0	6	0
b-b	3.0	3.75	—	—	11.25	6	5.10
c-c	6.0	3.50	—	—	21.0	6	9.51
c-c'	6.0	3.50	—	—	21.0	2	28.5
d-d	6.0	3.50	1.00	2.75	23.75	2	32.3
e-e	6.0	3.5	2.00	2.50	26.0	2	35.3
f-f	6.0	3.5	3.00	2.25	27.75	2	37.7
g-g	6.0	3.5	4.00	2.00	29.0	2	39.4
h-h	6.0	3.5	5.00	1.75	29.75	2	40.4
i-i	6.0	3.5	6.00	1.50	30.0	2	40.8

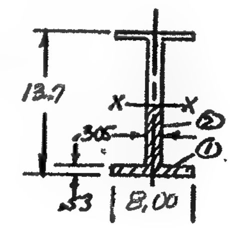


8-33 W14X43 : $I_x = 428\ IN^4$

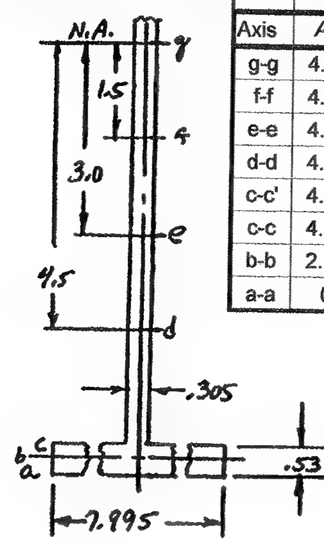
$Q = A_1 y_1 + A_2 y_2 = (8.00)(.53)(6.59) + (6.32)(0.305)(3.16)$

$Q = 34.03\ IN^3$

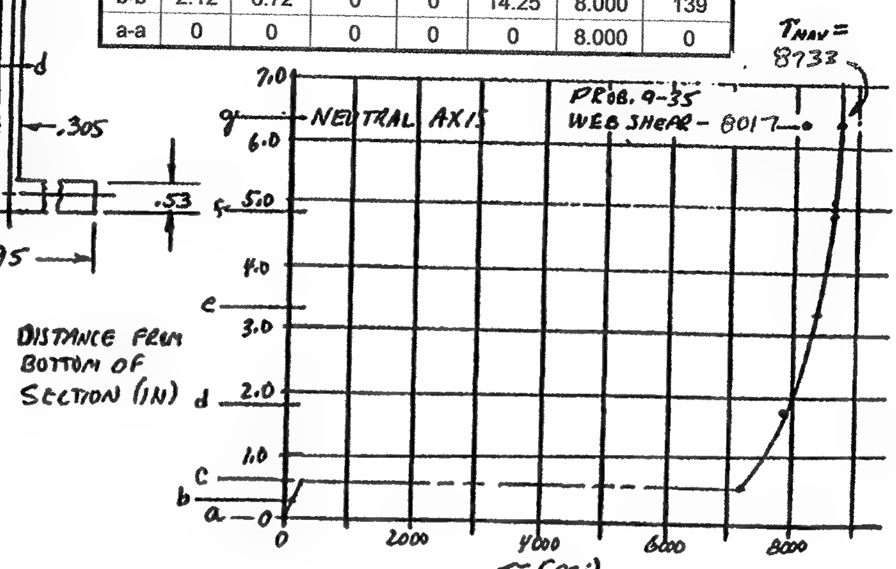
$\tau = \dfrac{VQ}{It} = \dfrac{(33500)(34.03)}{(428)(0.305)} = \underline{8733\ PSI}$ AT X-X



8-34 W14X43 : $I_x = 428\ IN^4$



Axis	A_1	y_1	A_2	y_2	Q	t	τ
g-g	4.24	6.59	1.928	3.16	34.03	0.305	8733
f-f	4.24	6.59	1.470	3.91	33.69	0.305	8646
e-e	4.24	6.59	1.013	4.66	32.66	0.305	8381
d-d	4.24	6.59	0.555	5.41	30.94	0.305	7940
c-c'	4.24	6.59	0	0	27.94	0.305	7170
c-c	4.24	6.59	0	0	27.94	8.000	273
b-b	2.12	6.72	0	0	14.25	8.000	139
a-a	0	0	0	0	0	8.000	0



8-35 W14 X 43 : FOR WEB SHEAR FORMULA : $t = 0.305$ IN; $h = 13.7$ IN

$$\tau_{ws} \approx \frac{V}{th} = \frac{33500 \text{ LB}}{(.305)(13.7)\text{IN}^2} = \underline{8017 \text{ PSI}}$$ GRAPH ON PREVIOUS PAGE

FROM PROB 8-34 : $\tau_{MAX} = 8733$ PSI

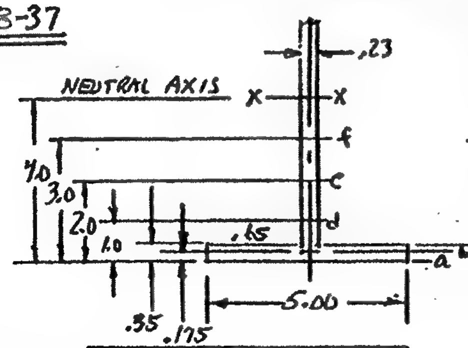
$$\tau_{ws}/\tau_{MAX} = 8017/8733 = \underline{0.92}$$

8-36 I 8 × 6.181 ALUMINUM I-BEAM : $I_x = 59.69 \text{ IN}^4$

$$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = (5.00)(0.35)(3.825) + (3.65)(0.23)(1.825) = 8.226 \text{ IN}^3$$

$$\tau = \frac{VQ}{It} = \frac{(13500 \text{ LB})(8.226 \text{ IN}^3)}{(59.69 \text{ IN}^4)(0.23 \text{ IN.})} = \underline{8089 \text{ PSI}}$$

8-37



Axis	A_1	$\bar{y}_1$	A_2	$\bar{y}_2$	Q	t	τ
X-X	1.75	3.825	.840	1.825	8.226	.23	8089
f	1.75	3.825	.610	2.325	8.111	.23	7976
e	1.75	3.825	.380	2.825	7.767	.23	7638
d	1.75	3.825	.150	3.325	7.193	.23	7073
c'	1.75	3.825	–	–	6.694	.23	6582
c	1.75	3.825	–	–	6.694	5.00	303
b	.875	3.913	–	–	3.423	5.00	155
a	0	–	–	–	0	5.00	0

8-38 ALUMINUM I 8 × 6.181

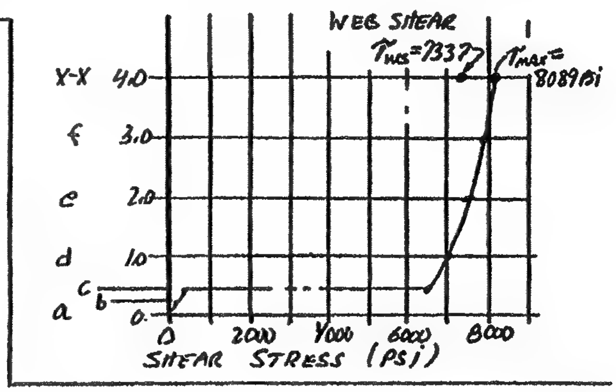
$t = 0.23$ IN ; $h = 8.00$ IN

WEB SHEAR FORMULA

$$\tau_{ws} \approx \frac{V}{th} = \frac{13500 \text{ LB}}{(.23)(8.0)\text{IN}^2}$$

$\tau_{ws} \approx \underline{7337 \text{ PSI}}$

$$\tau_{ws}/\tau_{MAX} = \frac{7337}{8089} = \underline{0.907}$$



8-39 SEE PROB. 5-4. $V_{MAX} = 10K = 10\,000$ PSI; $M_{MAX} = 30K\text{-FT} = 3.6 \times 10^5$ LB-IN

W12 X 16 : $S = 17.1 \text{ IN}^3$; DEPTH = 12.00 IN, $t_w = 0.220$ IN.

$$\tau_{ws} = \frac{V}{th} = \frac{10000 \text{ LB}}{(0.22)(12.00)\text{IN}^2} = \underline{3788 \text{ PSI}}; \quad \tau_d = 0.40(50000) = \underline{20000 \text{ PSI}} \text{ OK}$$

$$\sigma = \frac{M}{S} = \frac{3.60 \times 10^5 \text{ LB}\cdot\text{IN}}{17.1 \text{ IN}^3} = \underline{21053 \text{ PSI}}; \quad \sigma_d = 0.66(50000) = \underline{33000 \text{ PSI}} \text{ OK}$$

8-40 SEE PROB 5-4 AND 8-39; $\sigma_d = 0.66 S_y = 0.66(50000) = 33000$ PSI

$$M_{MAX} = 3.60 \times 10^5 \text{ LB}\cdot\text{IN}; \quad \underline{\text{REQ'D } S} = M/\sigma_d = \frac{3.60 \times 10^5 \text{ LB}\cdot\text{IN}}{33000 \text{ LB/IN}^2} = \underline{10.9 \text{ IN}^3}$$

SPECIFY $\underline{\text{W10 X 12}}$; $S = 10.9 \text{ IN}^3$; DEPTH = 9.87 IN, $t_w = 0.190$ IN

$$\tau_{ws} = \frac{V}{th} = \frac{10000 \text{ LB}}{(9.87)(0.19)} = \underline{5332 \text{ PSI}}$$

$$\tau_d = 0.4 S_y = (0.4)(50000 \text{ PSI}) = \underline{20000 \text{ PSI}} \text{ - OK}$$

8-41 FROM PROB. 5-52: $V_{MAX} = 44.5K = 44500LB$ ASTM A992 STEEL

$M_{MAX} = 148K\text{-}FT = 1.776 \times 10^6 LB\cdot IN$; $\sigma_d = (0.66)(50000) = 33000 PSI$

$REQD.\ S = \frac{M}{\sigma_d} = \frac{1.776 \times 10^6 LB\cdot IN}{33000 LB/IN^2} = 53.8 IN^3 \rightarrow$ SPECIFY W18×40

$S = 68.4 IN^3$; DEPTH = 17.9 IN; $t_w = 0.315 IN$.

$\tau_{WS} = \frac{V}{A_W} = \frac{44500LB}{(0.315)(17.9)IN^2} = 7892\ PSI$; $\tau_d = 0.4S_Y = 0.4(50KSI) = 20KSI$ OK

8-42 FROM PROB 5-54: $V_{MAX} = 162.9 kN = 162.9 \times 10^3 N$

$M_{MAX} = 228 kN\cdot m \times 10^3 N/kN \times 10^3 mm/m = 228 \times 10^6 N\cdot mm$ ASTM A992

$REQ'D.\ S = M/\sigma_d$; $\sigma_d = 0.66(S_Y) = 0.66(345 MPa = 227.7 MPa$;

$S_{MIN} = \frac{228 \times 10^6 N\cdot mm}{227.7 N/mm^2} = 1.00 \times 10^6 mm^3 \rightarrow$ W460×60

$S = 1.12 \times 10^6 mm^3$; DEPTH = 455 mm; $t_w = 8.00 mm$

$\tau = \frac{V}{th} = \frac{162.9 \times 10^3 N}{(8.00)(455)mm^2} = 44.75 MPa$; $\tau_d = 0.4(345) = 138 MPa$ OK

8-43 FROM PROB. 5-51: $V_{MAX} = 804 LB$; $M_{MAX} = 2528 LB\cdot IN$

ASTM A53, GR.B: $S_Y = 35KSI$; $\sigma_d = S_Y/3 = 35/3 = 11.67KSI = 11667PSI$

$REQ'D\ S = \frac{M}{\sigma_d} = \frac{2528 LB\cdot IN}{11667 LB/IN^2} = 0.217 IN^3 \rightarrow$ 1¼ SCH 40 STEEL PIPE

$S = 0.2346 IN^3$, $A = 0.669 IN^2$ PIPE 1¼ STD

$\tau = \frac{2V}{A} = \frac{2(804LB)}{0.669 IN^2} = 2404\ PSI$; LET $\tau_d = \frac{0.5S_Y}{N}$; $N = \frac{0.5S_Y}{\tau}$

$N = \frac{0.5(35000PSI)}{2304\ PSI} = 7.28$ OK

8-44

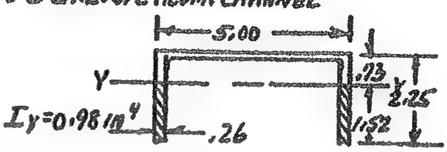
FROM PROB 5-9 : $V_{MAX} = 1557LB$: $M_{MAX} = 6228 LB\cdot IN$

$\sigma_d = S_Y/4 = 40000PSI/4 = 10000\ PSI$: $\tau_d = 0.5S_Y/4 = 5000\ PSI$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{6228 LB\cdot IN}{10000 LB/IN^2} = 0.623 IN^3$: C 5×2.212 ALUM. CHANNEL

$\tau = \frac{VQ}{It} = \frac{(1557)[(2)(.26)(1.52)](1.52/2)}{(0.98)(.52)}$

$\tau = 1835\ PSI$ SAFE



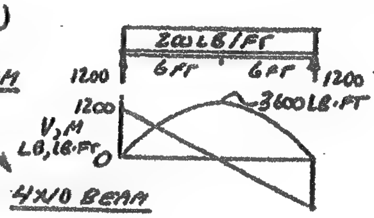
8-45

SHEAR: $\tau = \frac{3V}{2A} = \tau_d = 75PSI$ (RECT. SECTION)

$REQD\ A = \frac{3V}{2\tau_d} = \frac{3(1200LB)}{2(75LB/IN^2)} = 24 IN^2$ ~~4×8 BEAM~~

BENDING: $\sigma = \frac{M}{S} = \sigma_d = 1150PSI$

$REQD\ S = \frac{M}{\sigma_d} = \frac{(3600LB\cdot FT)(12IN/FT)}{1150LB/IN^2} = 37.6 IN^3$ 4×10 BEAM



8-46

FROM PROB 5-53 : $V_{MAX} = 2950N$: $M_{MAX} = 3350 N\cdot m$

SHEAR: $A_{MIN} = \frac{3V}{2\tau_d} = \frac{(3)(2950N)}{(2)(0.66 N/mm^2)} = 6705 mm^2$ ~~2×8 BEAM~~ OR ~~4×4 BEAM~~

BENDING: $S = \frac{M}{\sigma_d} = \frac{3350N\cdot m}{5.5 N/mm^2} \times \frac{10^3 mm}{m} = 609 \times 10^3 mm^3$: USE 4×10 BEAM

8-47 FROM PROB 8-28: $I_x = 148.3\ IN^4$; $C = 3.50\ IN$; $Q = 28.33\ IN^3$. SEE ALSO P6-22.

$V_{MAX} = P$: $M_{MAX} = P(3\ FT) = 36(P)\ LB\cdot IN$: $\tau_d = 80\ PSI$; $\sigma_{MAX} = 1400\ PSI$

$\tau = \frac{VQ}{It}$: $V_{MAX} = P_{MAX} = \frac{\tau I t}{Q} = \frac{(80)(148.3)(3.0)}{28.33} = \underline{1256\ LB = P_{MAX}}$

$\sigma = \frac{Mc}{I} = \frac{36Pc}{I}$: $P = \frac{\sigma_d I}{36c} = \frac{(1400)(148.3)}{(36)(3.50)} = \underline{1648\ LB}$

8-48 FROM PROB P5-8 : $V_{MAX} = 21.36\ kN = 21360\ N$: $\underline{I\ 229 \times 12.44}$

$I\ 9 \times 8.361$: $t = 0.27\ IN(25.4\ mm/IN) = 6.86\ mm$: $h = 9.00\ IN(25.4) = 229\ mm$

$\tau_{WS} \approx \frac{V}{th} = \frac{21360\ N}{(6.86)(229)\ mm^2} = \underline{13.6\ MPa}$

8-49 FROM PROB P5-8 : $M_{MAX} = 43.2\ kN\cdot m$: $\underline{I\ 229 \times 12.44}$

$I\ 9 \times 8.361$: $S = 22.67\ IN^3 \times \frac{(25.4\ mm)^3}{IN^3} = 371\ 495\ mm^3$

$\sigma = \frac{M}{S} = \frac{43.2 \times 10^3\ N\cdot m}{371\ 495\ mm^3} \times \frac{10^3\ mm}{m} = \underline{116.3\ MPa}$

8-50 TOTAL LOAD $= W = wL = (80\ LB/FT)(12\ FT) = 960\ LB$: $V_{MAX} = W/2 = 480\ LB$

$\tau = \frac{3V}{2A} = \frac{(3)(480\ LB)}{2(10.87\ IN^2)} = \underline{66.2\ PSI}$: $\underline{\tau_d = 70\ PSI}$ $\underline{OK}$

CHECK BENDING: $M_{MAX} = wL^2/8 = \frac{(80)(12)^2}{8} = 1440\ LB\cdot FT(12\ IN/FT) = 17\ 280\ LB\cdot IN$

$\sigma = \frac{M}{S} = \frac{17280\ LB\cdot IN}{13.14\ IN^3} = \underline{1315\ PSI}$: $\sigma_d = \underline{1000\ PSI}$ $\underline{UNSAFE}$

8-51 FROM PROB P5-10 : $V_{MAX} = 1500\ LB$: $M_{MAX} = 9000\ LB\cdot IN$

(a) SHEAR: $\tau = \frac{3V}{2A} = \frac{3(1500\ LB)}{(2)(.50)(4.0)\ IN^2} = \underline{1125\ PSI}$

(b) BENDING: $\sigma = \frac{M}{S} = \frac{9000\ LB\cdot IN}{(.5)(4.0)^2/6\ IN^3} = \underline{6750\ PSI}$

(c) $\tau_d = 0.5\ S_y/3$: REQD $S_y = 3\tau_d/0.5 = 3(1125)/0.5 = 6750\ PSI$

$\sigma_d = S_y/3$: REQD $S_y = 3\sigma_d = 3(6750) = \underline{20250\ PSI}$ $\underline{ANY\ STEEL}$

8-52 FROM PROB P5-6 : $V_{MAX} = 1451\ N$: $M_{MAX} = 318\ N\cdot m$

(a) SHEAR: $\tau = \frac{3V}{2A} = \frac{3(1451\ N)}{(2)(16)(60)\ mm^2} = \underline{2.267\ MPa}$

(b) BENDING: $\sigma = \frac{M}{S} = \frac{318\ N\cdot m\ (10^3\ mm/m)}{(16)(60)^2/6\ mm^3} = \underline{33.1\ MPa}$

(c) $\tau_d = 0.5\ S_y/3$: REQD $S_y = \frac{3(\tau)}{0.5} = \frac{3(2.267)}{0.5} = \underline{13.6\ MPa}$

$\sigma_d = S_y/3$: REQD $S_y = 3\sigma = 3(33.1) = \underline{99.3\ MPa}$ 6061-T4: $S_y = 145\ MPa$ OR SEVERAL OTHERS

8-53 FROM PROB. P5-47 : $V_{MAX} = 450\ N$: $M_{MAX} = 172.5\ N\cdot m$

$\sigma_d = S_y/N = 276\ MPa/4 = 69\ MPa = M/S$:

REQD $S = \frac{M}{\sigma_d} = \frac{172\ 500\ N\cdot mm}{69\ N/mm^2} = 2500\ mm^3 = \frac{bh^2}{6}$

REQD $h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(2500)\ mm^3}{12\ mm}} = \underline{35.4\ mm}$

$\tau = \frac{3V}{2A} = \frac{3(450\ N)}{(2)(12)(35.4)\ mm^2} = \underline{1.59\ MPa} = \frac{0.5S_y}{N}$

$N = \frac{0.5S_y}{\tau} = \frac{0.5(276\ MPa)}{1.59\ MPa} = \underline{86.7}$ SAFE. VERY HIGH N.

8-54 FROM PROB P5-48 : $V_{MAX} = 1290\ N$: $M_{MAX} = 370.8\ N\cdot m$

(a) $\tau = \dfrac{4V}{3A} = \dfrac{4(1290N)}{3(\pi)(40)^2/4\ mm^2} = \underline{1.37\ MPa} = \tau_d = \dfrac{0.5S_y}{4} = \dfrac{S_y}{8}$

(b) $\sigma = \dfrac{M}{S} = \dfrac{370.8 \times 10^3\ N\cdot mm}{\pi(40)^3/32} = \underline{59.0\ MPa} = \sigma_d = \dfrac{S_y}{4}$;

(C) FOR SHEAR : REQD $S_y = 8\tau = 8(1.37) = \underline{11.0\ MPa}$

FOR BENDING : REQD $S_y = 4\sigma = 4(59.0) = \underline{236\ MPa}$ $\underline{\text{AISI 1020 HR } S_y = 331\ MPa}$

8-55 FROM PROB P5-47 : $M_{MAX} = 172.5\ N\cdot m$: $V_{MAX} = 450\ N$

REQD. $S = \dfrac{M}{\sigma_d} = \dfrac{(172.5\ N\cdot m)(10^3\ mm/m)}{120\ N/mm^2} = 1438\ mm^3 = \pi(D)^3/32$

$D = \left[\dfrac{32S}{\pi}\right]^{1/3} = \left[\dfrac{32(1438)}{\pi}\right]^{1/3} = \underline{24.5\ mm}$: $A = \pi D^2/4 = 470\ mm^2$

$\tau = \dfrac{4V}{3A} = \dfrac{4(450N)}{3(470\ mm^2)} = \underline{1.28\ MPa}$

8-56 $\tau = 4V/3A$: $V_{MAX} = \dfrac{3A\tau_d}{4} = \dfrac{3(\pi)(1.50)^2 IN^2 (70\ LB/IN^2)}{(4)(4)} = \underline{92.8\ LB}$

8-57 $\tau_d = \dfrac{0.5S_y}{6} = \dfrac{0.5(48000)}{6} = 4000\ PSI \approx \dfrac{2V}{A}$

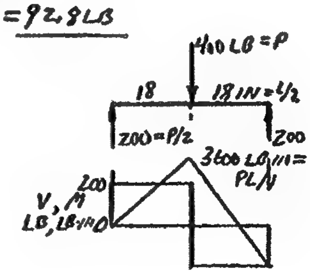
$\sigma_d = S_y/6 = 48000/6 = 8000\ PSI = M/S$

REQD $S = \dfrac{M}{\sigma_d} = \dfrac{3600\ LB\cdot IN}{8000\ LB/IN^2} = 0.45\ IN^3$

$\underline{2\text{-IN SCH 40 PIPE}}$

$A = 1.075\ IN^2$

$\tau = 2V/A = 2(200)/1.075 = \underline{372\ PSI}$ OK



8-58 $\sigma_d = S_y/4 = 48000/4 = 12000\ PSI$; $\tau_d = 0.5S_y/4 = 6000\ PSI$; $P = 2800\ LB$

$V_{MAX} = P/2 = 1400\ LB$: $M_{MAX} = PL/4$ (PROB 9-57) : REQD $S = M/\sigma_d$: $\tau \approx 2V/A$

	L(IN)	M (LB·IN)	REQD S(IN³)	PIPE	A	$\tau = \frac{2V}{A}$
(a)	1.50	1050	0.0875	1	.494	5668 OK
(b)	3.00	2100	0.175	1¼	.669	4185 OK
(c)	4.50	3150	0.263	1½	.799	3504 OK
(d)	6.00	4200	0.350	2	1.075	2605 OK

8-59 SHEAR FLOW AT JOINT $= q = VQ/I$

FROM FIG P6-14 : $I = 0.3672\ IN^4$; $\bar{Y} = 0.533\ IN$

$Q = A_p\bar{y} = (2.60)(0.20)(0.533 - 0.10) = 0.225\ IN^3$

$q = (1200\ LB)(0.225\ IN^3)/0.3672\ IN^4 = \underline{736\ LB/IN}$

ON 1.0 IN OF LENGTH, AREA OF GLUE $= (1.0\ IN)(2)(0.7\ IN) = 1.40\ IN^2 = A_s$

$\tau = \dfrac{736\ LB}{IN} \times \dfrac{1\ IN}{1.40\ IN^2} = \underline{526\ PSI}$

8-60 FROM PROB. 6-26 : $I = 467.0\ IN^4$; $\bar{Y} = 7.913\ IN$; $A_{CH} = 7.35\ IN^2$

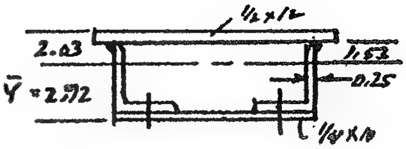
$\bar{y}_{CH} = 11.713 - 7.913 = 3.80\ IN$ $Q = A\bar{y} = (7.35)(3.80) = 27.93\ IN^3$

$q = \dfrac{VQ}{I} = \dfrac{(2500)(27.93)}{467.0} = \underline{149.5\ LB/IN}$; $\tau = q/A_s$

FLANGE WIDTH FOR S12×50 $= 5.48\ IN$; $A_s = (1.0\ IN)(5.48\ IN) = 5.48\ IN^2/IN$

$\tau = \dfrac{q}{A_s} = \dfrac{149.5\ LB/IN}{5.48\ IN^2/IN} = \underline{27.3\ PSI}$ $\underline{\text{REQ'D FOR GLUE}}$

8-61 FROM PROB P6-33 : $I = 46.8\ IN^4$



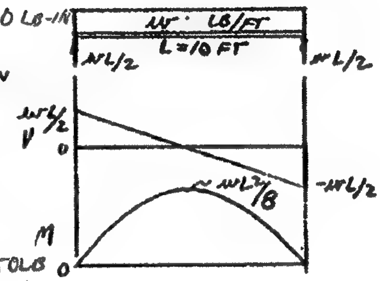
$\sigma_d = S_y/4 = 21000/4 = 5250\ PSI$

$\tau_d = 0.5 S_y/4 = 2625\ PSI$

BENDING: $\sigma = Mc/I$

$M_{ALLOW} = \dfrac{\sigma_d I}{c} = \dfrac{(5250)(46.8)}{2.72} = 90\,330\ LB\text{-}IN$

$w = \dfrac{8M}{L^2} = \dfrac{8(90\,330\ LB\text{-}IN)}{(120\ IN)^2} = 50.2\ LB/IN$



SHEAR AT NEUTRAL AXIS:

$Q = (0.5)(12)(2.03 - 0.25) + 2(1.53)(0.25)(1.53/2)$

$Q = 11.27\ IN^3$: $t = 2(0.25) = 0.50\ IN$

$\tau = \dfrac{VQ}{It}$; $V = \dfrac{\tau_d I t}{Q} = \dfrac{(2625)(46.8)(0.50)}{11.27} = 5450\ LB$

$w = \dfrac{2(V)}{L} = \dfrac{2(5450\ LB)}{120\ IN} = \underline{90.8\ LB/IN}$

SHEAR FLOW AT WELDS: $q = \dfrac{VQ}{I}$; $V = \dfrac{qI}{Q} = \dfrac{(1800)(46.8)}{(6.0)(2.03 - 0.25)} = 7888\ LB$

$w = 2V/L = 2(7888)/120 = \underline{131\ LB/IN}$

RIVETS: S = SPACING = 4.0 IN : $F_{sd} = 2(600) = 1200\ LB = S q_d$

$q_d = \dfrac{F_{sd}}{S} = \dfrac{1200\ LB}{4.0\ IN} = 300\ LB/IN$

$q = \dfrac{VQ}{I}$: $V = \dfrac{qI}{Q} = \dfrac{(300\ LB/IN)(46.8\ IN^4)}{(2.50)(2.72 - 0.125)\ IN^3} = 2164\ LB$

$w = 2V/L = 2(2164\ LB)/120\ IN = \underline{36.1\ LB/IN}\ (12\ IN/FT) = \underline{433\ LB/FT}$ LIMITING VALUE

8-62 FROM PROB P6-24 : $I = 816.3\ IN^4$: $\bar{Y} = 7.75\ IN$

#3 SOUTHERN PINE

$\sigma_d = 650\ PSI$

$\tau_d = 70\ PSI$

BENDING: $\sigma = Mc/I$

$M = \dfrac{\sigma_d I}{c} = \dfrac{(650)(816.3)}{7.75} = 68\,464\ LB\text{-}IN = wL^2/8$

$w = \dfrac{8M}{L^2} = \dfrac{8(68\,464)}{(120)^2} = \underline{38.0\ LB/IN}$

SHEAR AT NEUTRAL AXIS: $Q = A\bar{y} = (3.0)(7.75)(7.75/2) = 90.09\ IN^3$

$\tau = \dfrac{VQ}{It}$: $V = \dfrac{\tau_d I t}{Q} = \dfrac{(70)(816.3)(3.0)}{90.09} = 1903\ LB = wL/2$

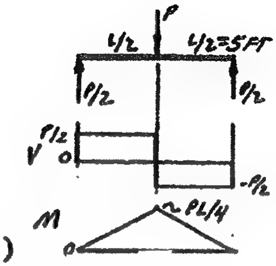
$w = \dfrac{2V}{L} = \dfrac{2(1903)}{120} = \underline{31.7\ LB/IN}$

NAILS: $S = 6.0\ IN$: $F_{sd} = 2(160) = 320\ LB = S q_d$: $q_d = \dfrac{F_{sd}}{S} = \dfrac{320\ LB}{6.0\ IN} = 53.3\ LB/IN$

$q_d = \dfrac{VQ}{I}$: $V = \dfrac{q_d I}{Q} = \dfrac{(53.3)(816.3)}{(1.5)(11.25)(12.75 - 7.75 - 0.75)} = 607\ LB$

$w = \dfrac{2V}{L} = \dfrac{2(607)}{120} = \underline{10.1\ LB/IN}\ (12\ IN/FT) = \underline{121\ LB/FT}$ LIMITING

8-63 FROM PROB P6-21 : $I = 151.4\ IN^4$; $\bar{Y} = 4.25\ IN$



BENDING : $\sigma = Mc/I$; $\sigma_d = 1450\ psi$

$$M = \frac{\sigma_d I}{c} = \frac{(1450)(151.4)}{4.25} = 51\,654\ LB\cdot IN = PL/4$$

$$P = \frac{4M}{L} = \frac{4(51\,654\ LB\cdot IN)}{120\ IN} = \underline{1722\ LB}\quad \underline{\text{LIMITING}}$$

SHEAR : $\tau = VQ/It$: $V = \tau_d It/Q$; $\tau_d = 95\ psi$

AT NEUTRAL AXIS : $Q = (3.5)(1.5)(3.5) + (2.75)(1.5)(2.75/2)$

$Q = 24.05\ IN^3$

$$V = \frac{\tau_d I t}{Q} = \frac{(95)(151.4)(1.50)}{24.05} = 897\ LB = \frac{P}{2} : P = 2V = \underline{1794\ LB}$$

GLUE AT TOP OF WEB :

$$q = \frac{VQ}{I} = \frac{PQ}{2I} = \frac{P(3.5)(1.5)(3.5)}{2(151.4)} = 0.06068\ P\ LB/IN$$

$$\tau_G = \frac{q}{A_s} : q_d = \tau_G A_s = \frac{800\ LB}{IN^2} \times \frac{(1.5)(1.0)\ IN^2}{IN} = 1200\ LB/IN$$

$$P = \frac{q_d}{0.0608} = \frac{1200}{0.0608} = \underline{19\,775\ LB}$$

8-64 $I = 151.4\ IN^4$: $Q = (3.5)(1.5)(3.5) = 18.375\ IN^3$

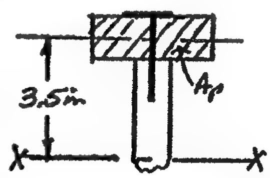


FIGURE P6-21

$$q = \frac{VQ}{I} = \frac{(300)(18.375)}{151.4} = 36.4\ LB/IN$$

$$S = \frac{F_{sd}}{q} = \frac{180\ LB}{36.4\ LB/IN} = \underline{4.94\ IN}$$

8-65 $I = 148.3\ IN^4$: $Q = (7.25)(0.75)(3.125) = 17.0\ IN^3$

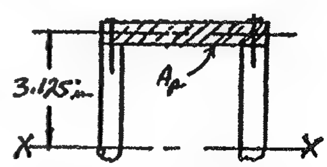


FIG. 6-22

$q = VQ/I = (600)(17.0)/148.3 = 68.8\ LB/IN$

$S = F_{sd}/q = 2(150)/68.8 = \underline{4.36\ IN}$

8-66 $I = 107.2\ IN^4$; $Q = (24)(0.5)(2.0) = 24.0\ IN^3$

$q = VQ/I = (500)(24.0)/107.2 = 112\ LB/IN$

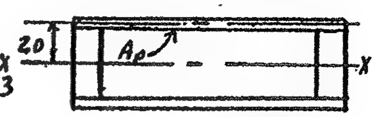


FIGURE P6-23

8-67 $V = 175\ kN = 175\,000\ N$; $V = \dfrac{175\,000\ N}{4.448\ N/LB} = 39\,344\ LB$

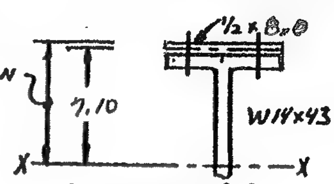


FIGURE P6-25

FROM PROB P6-25 : $I = 831\ IN^4$; $\bar{Y} = 7.35\ IN$

$Q = (0.5)(8.00)(7.10) = 28.4\ IN^3$

$q = VQ/I = (39\,344)(28.4)/831 = 1345\ LB/IN$

$$S = \frac{F_{sd}}{q} = \frac{(2)(2650)\ LB}{1345\ LB/IN} = 3.94\ IN$$

8-68 $V = 50\,000\ N(1\ LB/4.448\ N) = 11241\ LB$

FROM PROB P6-26 : $I = 467.0\ IN^4$

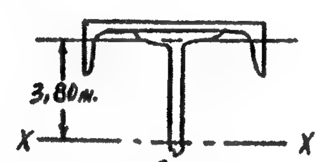


FIGURE P6-26

$Q = A\bar{y} = (7.35)(3.80) = 27.93\ IN^3$

$$q = \frac{VQ}{I} = \frac{(11241)(27.93)}{467.0} = 672\ LB/IN$$

$$S = \frac{F_{sd}}{q} = \frac{2(1750)}{672} = \underline{5.21\ IN}$$

8-69 W18×55: $t = 0.390$ IN, $h = 18.1$ IN. ASTM A992, $S_y = 50\,000$ psi

$$\tau \cong \frac{V}{th} = \frac{36\,600 \text{ lb}}{(0.39)(18.10)\,\text{IN}^2} = \underline{5185 \text{ psi}} \quad \underline{\text{SAFE}}$$

$$\tau_d = 0.40\,S_y = 0.40(50\,000\text{ psi}) = 20000 \text{ psi} \quad \text{OK}$$

8-70 W18×40: $t = 0.315$ IN, $h = 17.90$ IN. ASTM A992, $S_y = 50\,000$ psi

$$\tau \cong \frac{V}{th} = \frac{36\,600 \text{ lb}}{(0.315)(17.90)\,\text{IN}^2} = \underline{6491 \text{ psi}} \quad \underline{\text{SAFE}}$$

$$\tau_d = 0.40\,S_y = 0.40(50\,000 \text{ psi}) = 20000 \text{ psi} \quad \text{OK}$$

8-71 W14×26: $t = 0.255$ IN, $h = 13.90$ IN. ASTM A992, $S_y = 50\,000$ psi

$$\tau = \frac{V}{th} = \frac{10\,000 \text{ lb}}{(0.255)(13.90)\,\text{IN}^2} = \underline{2821 \text{ psi}} \quad \underline{\text{SAFE}}$$

$\tau_d = 20000$ psi (PROB 9-70)

8-72 6I×4.692 ALUMINUM: $t = 0.21$ IN $h = 6.00$ IN

$$\tau = \frac{V}{th} = \frac{10\,000 \text{ lb}}{(0.21)(6.00)} = \underline{7937 \text{ psi}} \quad \underline{\text{SAFE}}$$

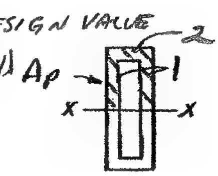
AL 6061-T6, $S_y = 40\,000$ psi; $\tau_d = S_y/2N = S_y/2(2) = 0.25\,S_y$

$$\tau_d = (0.25)(40\,000 \text{ psi}) = 10\,000 \text{ psi} \quad \text{OK.}$$

8-73 W10×12: $t = 0.190$ IN $h = 9.87$ IN, ASTM A992, $\tau_d = 20000$ psi (PROB 8-69)

$$\tau = \frac{V}{th} = \frac{6250 \text{ LB}}{(0.190)(9.87)\,\text{IN}^2} = \underline{3599 \text{ psi}} \quad \underline{\text{SAFE}}$$

8-74 HSS 6×2×1/4. $I = 13.1$ IN^4, $t_w \cong 0.233$ IN – DESIGN VALUE



$$Q = A_1\bar{y}_1 + A_2\bar{y}_2 = 2(2.767)(0.233)(1.384) + (2.0)(0.233)(2.884)$$

$$Q = 1.798 + 1.344 = 3.142 \text{ IN}^3$$

$$\tau = \frac{VQ}{It} = \frac{(6750 \text{ lb})(3.142 \text{ IN}^3)}{(13.1 \text{ IN}^4)(0.466 \text{ IN})} = \underline{3474 \text{ psi}} \quad \underline{\text{SAFE}}$$

$$\tau_d = \frac{S_y}{2N} = \frac{46\,000 \text{ psi}}{2(2)} = 11\,500 \text{ psi} \quad \text{OK}$$

ASTM A500, GR B
$S_y = 46\,000$ psi

8-75 2×8 WOOD BEAM: $A = 10.87$ IN^2. No. 2 SOUTH. PINE, $\tau_d = 70.0$ psi

$$\tau = \frac{3V}{2A} = \frac{3(480 \text{ lb})}{2(10.87 \text{ IN}^2)} = \underline{66.2 \text{ psi}} \quad \underline{\text{SAFE}}$$

8-76 FIG. P8-29: $Q = 28.59$ IN^3 FROM PROB 8-29., $I_x = 107.2$ IN^4

$$\tau = \frac{VQ}{It} = \frac{(750 \text{ lb})(28.59 \text{ IN}^3)}{(107.2 \text{ IN}^4)(3.0 \text{ IN})} = \underline{66.7 \text{ psi}} \quad \underline{\text{SAFE}}$$

No. 2 DOUGLAS FIR, $\tau_d = 95$ psi OK

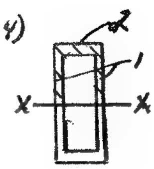
8-77 HSS 8×2×1/4 : $I = 28.5\,IN^4$, $t_w = 0.233\,IN$ - DESIGN VALUE

$Q = A_1 y_1 + A_2 y_2 = 2(0.233)(3.767)(1.884) + (2.0)(0.233)(3.884)$

$Q = 3.307 + 1.810 = 5.117\,IN^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(1200\,lb)(5.117)}{(28.5\,IN^4)(0.466)\,IN} = \underline{4623\,PSI}$ $\underline{SAFE}$

$\tau_d = S_y/2N = 46000\,PSI/2(2) = 11500\,PSI$ OK



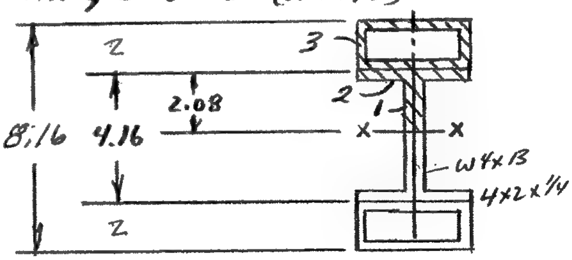
ASTM A500, GR. B
$S_y = 46000\,PSI$

8-78 FROM PROBLEM 6-42, $I = 60.55\,IN^4$, $t = 0.280$ (W4×13)

$Q = A_1 y_1 + A_2 y_2 + A_3 y_3$

	A	y	Ay
1	.486	.868	.422
2	1.401	1.908	2.673
3	2.440	3.080	7.515

$Q = 10.610\,IN^3$



$\tau_{MAX} = \dfrac{VQ}{It} = \dfrac{(1800\,lb)(10.61\,IN^3)}{(60.55\,IN^4)(0.280)\,IN} = \underline{1126\,PSI}$ $\underline{SAFE}$ AT AXIS X-X

$\tau_d = S_y/2N = 50\,000\,PSI/2(2) = 12\,500\,PSI$ FOR ASTM A992 - W4×13

8-79 C 10 × 6.136 ALUMINUM CHANNEL

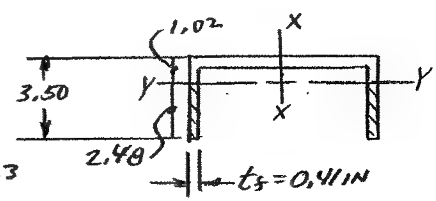
$I_y = 6.33\,IN^4$, $t_f = 0.41\,IN$

Q FOR LOWER PART OF FLANGES

$Q = Ay = 2(0.41)(2.48)(2.48/2) = 2.522\,IN^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(430\,lb)(2.522\,IN^3)}{(6.33\,IN^4)(2 \times 0.41\,IN)} = \underline{209\,PSI}$ $\underline{SAFE}$

$\tau_d = S_y/2N = 40000/2(2) = 10000\,PSI$ OK



6061-T6 ALUM.
$S_y = 40{,}000\,PSI$

8-80 DATA OF PROB. 8-78. SHEAR FLOW $q = \dfrac{VQ}{I}$

$I = 60.55\,IN^4$, $V = 1800\,lb$

$Q = Ay$ FOR ONE 4×2×1/4 TUBE

$Q = (2.44\,IN^2)(3.08)\,IN = 7.515\,IN^3$

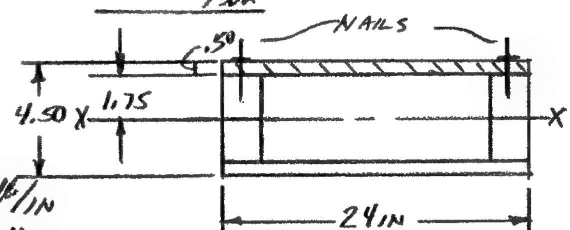
$q = \dfrac{VQ}{I} = \dfrac{(1800\,lb)(7.515\,IN^3)}{60.55\,IN^4} = \underline{223\,lb/in}$

8-81 FIG. P8-29: $I = 107.2\,IN^4$

$Q = AY$ FOR TOP PANEL

$Q = (0.50)(24)(2.00) = 24.0\,IN^3$

$q = \dfrac{VQ}{I} = \dfrac{(500\,lb)(24.0\,IN^3)}{107.2\,IN^4} = 112\,lb/IN$



$F_{sd} = 135\,lb/NAIL \times 2\,NAILS = 270\,lb$

MAX. SPACING $= S_{MAX} = F_{sd}/q = 270\,lb / 112\,lb/IN = \underline{2.41\,IN\ MAXIMUM}$

SPECIFY $\underline{S = 2.25\,IN}$

9-1

A23-(a): $y = -\dfrac{P\,l^3}{48EI} = \dfrac{(3000)(700)^3}{(48)(207\times10^3)(5.147\times10^4)} = 2.01\ mm$

$E = 207\times10^9\ \dfrac{N}{m^2} \times \dfrac{1m^2}{(10^3mm)^2} = 207\times10^3\ N/mm^2$

$I = \dfrac{\pi D^4}{64} = \dfrac{\pi(32\ mm)^4}{64} = 5.147\times10^4\ mm^4$

9-2

$E = 69\ GPa = 69\times10^3\ N/mm^2$; y INVERSELY PROPORTIONAL TO E.

$y = -2.01\ mm \times \dfrac{207}{69} = -6.03\ mm$

9-3

A25-(e):

$y = \dfrac{-Pl^3}{192EI} = \dfrac{(-3000)(700)^3}{192(207\times10^3)(5.147\times10^4)} = -0.503\ mm$

9-4

$y = \dfrac{-Pl^3}{48EI} = \dfrac{-3000(350)^3}{48(207\times10^3)(5.147\times10^4)} = -0.252\ mm$

9-5

$I = \pi D^4/64 = \pi(25)^4/64 = 19\,175\ mm^4$

$y = -\dfrac{3000(700)^3}{48(207\times10^3)(19175)} = -5.40\ mm$

9-6

A-23-(b):

AT LOAD: $y = \dfrac{-Pa^2b^2}{3EIl} = \dfrac{-3000(175)^2(525)^2}{3(207\times10^3)(5.147\times10^4)(700)} = -1.13\ mm$

AT CENTER: $y = \dfrac{-Pbx}{6EIl}(l^2 - x^2 - b^2)$, USE $x = 350\ mm$

$y = \dfrac{-3000(175)(350)}{6(207\times10^3)(5.147\times10^4)(700)}\left[(700)^2 - (350)^2 - (175)^2\right]$

$y = -1.38\ mm$

9-7

A23-(c): W12×16 : $I = 103\ in^4$; LOADING IN P5-4.

$P = 10000\ lb$; $a = 36\ in$; $l = 168\ in$; $E = 30\times10^6\ psi$

AT LOADS: $y = \dfrac{-Pa^2}{6EI}(3l - 4a) = \dfrac{-(10000)(36)^2}{6(30\times10^6)(103)}\left[(3)(168) - 4(36)\right]$

$y = -0.251\ in$

AT CENTER: $y = \dfrac{-Pa}{24EI}(3l^2 - 4a^2) = \dfrac{-10000(36)}{24(30\times10^6)(103)}\left[3(168)^2 - 4(36)^2\right]$

$y = -0.385\ in$

9-8

A23-(a): $I = 0.310\ in^4$

$$y = \frac{-Pl^3}{48EI} = \frac{-650(28)^3}{48(30\times10^6)(0.310)} = \underline{-0.032\ in}$$

9-9

A23-(d) : $I = 59.69\ in^4$; $E = 10\times10^6\ psi$

$w = (1125\ lb/ft)(10\ ft) = 11250\ lb$

$l = 10\ ft \times 12\ in/ft = 120\ in$

$$y = \frac{5wl^3}{384EI} = \frac{-5(11250)(120)^3}{384(10\times10^6)(59.69)} = \underline{-0.424\ in}$$

9-10

A23-(d): $x = 3.5\ FT\ (12\ IN/FT) = 42\ IN.$

$w = (1125\ LB/FT)(1\ FT/12\ IN) = 93.75\ LB/IN$

$$y = \frac{-wx}{24EI}(L^3 - 2Lx^2 + x^3) = \frac{-(93.75)(42)}{24(10\times10^6)(59.69)}(120^3 - 2(120)(42)^2 + 42^3)$$

$\underline{y = -0.379\ IN}$

9-11

LOADING IN FIGURE P5-12.

$I = 238\ in^4$: A23-(g) ; $a = 48\ in$; $l = 120\ in$; $P = 15000\ lb$

$$y = \frac{-Pa^2}{3EI}(a+l) = \frac{-15000(48)^2}{3(30\times10^6)(238)}(48+120) = \underline{-0.271\ in}$$

9-12

A23-(g): $a = 24\ IN$; $L = 144\ IN$

$$y = \frac{-Pa^2}{3EI}(a+L) = \frac{-15000(24)^2}{3(30\times10^6)(238)}(24+144) = \underline{-0.0678\ IN}$$

9-13

$+y_{MAX}$ at $x = 0.577\ (l) = 0.577\ (120) = 69.2\ in$ FROM A

$$y = \frac{.06415\ PaL^2}{EI} = \frac{(.06415)(15000)(48)(120)^2}{(30\times10^6)(238)} = \underline{0.093\ in}\ \text{(UPWARD)}.$$

9-14

$$\text{A24-(a)} : y = \frac{-Pl^3}{3EI} = \frac{-120(8)^3}{3(30\times10^6)(0.08734)} = -0.0078\ in.$$

9-15 A23-(a)

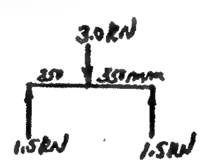
$$y = \frac{-Pl^3}{48EI} \therefore \text{REQ'D } I = \frac{-Pl^3}{48Ey} = \frac{-3000(700)^3}{48(207\times10^3)(-2.12)} = 8.63\times10^5\ mm^4$$

$$I = \frac{\pi D^4}{64} \therefore D = [64I/\pi]^{1/4} = [64(8.63\times10^5)/\pi]^{1/4} = \underline{64.8\ mm}$$

9-16

$$\sigma = \frac{Mc}{I} = \frac{\overbrace{(1500N)(350mm)}^{M}\overbrace{(32.4mm)}^{c}}{8.63\times10^5\ mm^4} = \underline{19.7\ MPa}$$

REQ'D $S_u = 8(19.7\ MPa) = \underline{158\ MPa\ (\text{ANY STEEL})}$

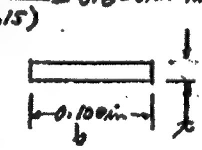


9-17

A24(a)

$$y_{MAX} = \frac{-Pl^3}{3EI} \therefore \text{REQ'D } I = \frac{-Pl^3}{3Ey} = \frac{-0.52(1.20)^3}{3(30\times10^6)(-0.15)} = 6.656\times10^{-8}\ in^4$$

$$I = \frac{bt^3}{12} \therefore t = \left[\frac{12I}{b}\right]^{1/3} = \left[\frac{12(6.656\times10^{-8})}{0.100}\right]^{1/3} = \underline{0.020\ in}$$



9-18

$$I = \frac{bh^3}{12} = \frac{(1.50)(9.25)^3}{12} = 98.93\ in^4$$

$$S = \frac{bh^2}{6} = \frac{(1.50)(9.25)^2}{6} = 21.39\ in^3$$

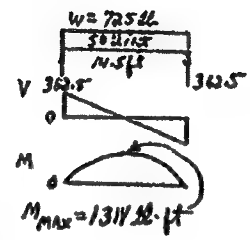
A23 (d)

$$y_{MAX} = \frac{-5wL^3}{384EI} = \frac{-5(725)(174)^3}{384(1.76\times10^6)(98.93)} = \underline{-0.286\ in}$$

$$\sigma = \frac{M}{S} = \frac{1314\ lb\text{-}ft\,(12\ in/ft)}{21.39} = \underline{737\ psi}$$

$$\tau = \frac{3V}{2A} = \frac{3(362.5)}{2(1.50)(9.25)} = \underline{39.2\ psi}$$

OK PER TABLE A19



Statically Indeterminate Beams

9-19

CASE A25 (a); P = 35 kN; L = 4.0 m

$R_A = (11/16)P = (11/16)\,35000\,N = \underline{24{,}063\,N = V_A}$

$R_C = (5/16)P = (5/16)\,35000\,N = \underline{10938\,N = V_C}$

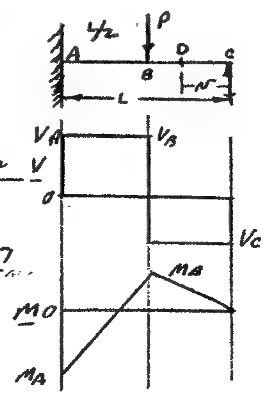
$M_A = -3/16 \cdot PL = -3/16\,(35000\,N)(4.0\,m) = \underline{-26250\,N\cdot m}$

$M_B = 5/32 \cdot PL = 5/32\,(35000)(4.0) = \underline{21875\,N\cdot m}$

$$y_B \text{ AT LOAD} = \frac{-7PL^3}{768EI} = \frac{-7(35000)(4)^3}{768EI} = \frac{-20417}{EI}$$

AT D: $v = 0.447L = 0.447(4.0\,m) = 1.788\,m$

$$y_{D\,MAX} = \frac{-PL^3}{107EI} = \frac{(-35000)(4.0)^3}{107EI} = \frac{20934}{EI}$$



LET $y_{MAX} = L/360 = 4.0\,m/360 = 0.0111\,m$

SPECIFY STEEL BEAM $E = 207 \times 10^9\,Pa$

$$\text{REQ'D. } I = \frac{20934\,N\cdot m^3}{E\,y_{MAX}} = \frac{20934\,N\cdot m^3}{207\times10^9\,N/m^2\,(0.0111\,m)} = 9.102\times10^{-6}\,m^4 = 9.102\times10^6\,mm^4$$

$$I = 9.102\times10^{-6}\,m^4 \times \frac{(10^3\,mm)^4}{1\,m^4} \times \frac{1\,IN^4}{(25.4\,mm)^4} = 21.87\,IN^4$$

SPECIFY W200 x 15 $I = 1.28\times10^7\,mm^4$ $S = 1.28\times10^5\,mm^3$ LIGHTEST

OR W8 x 10: $I = 30.8\,IN^4$; $S = 7.81\,IN^3$

CHECK STRESS:

$$\sigma = \frac{M}{S} = \frac{26250\,N\cdot m}{1.28\times10^5\,mm^3} \times \frac{10^3\,mm}{1\,m} = 205\,N/mm^2 = 205\,MPa$$

LET $\sigma = 0.66\,S_Y$; REQ'D $S_Y = \sigma/0.66 = \frac{205\,MPa}{0.66} = 311\,MPa$

SPECIFY ASTM A992, $S_Y = 345\,MPa$

SUMMARY

REACTIONS: $R_A = 24.06\,kN$, $R_C = 10.94\,kN$

$V_{MAX} = V_A = 24.06\,kN$ AT A. $M_{MAX} = 26.25\,kN\cdot m$ AT A.

REQ'D. $I = 9.102\times10^{-6}\,m^4$ ($21.87\,IN^4$) TO LIMIT DEFL. TO $L/360$.

SPECIFY LIGHTEST STEEL BEAM: W8 x 10, $I = 30.8\,IN^4$; $S = 7.81\,IN^3$

9-20 CASE A25(b); $P = 35\text{ kN}$, $L = 4.0\text{ m}$, $a = 1.50\text{ m}$

$b = L - a = 4.0 - 1.5 = 2.5\text{ m}$

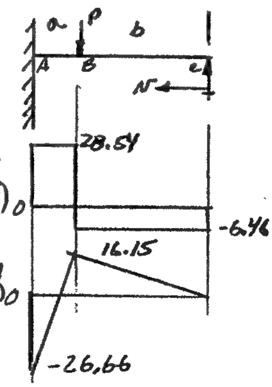
$$R_A = \frac{Pb}{2L^3}(3L^2 - b^2) = \frac{35\text{ kN}(2.5)}{2(4.0)^3}(3(4.0)^2 - 2.5^2) = 28.54\text{ kN}$$

$$R_C = \frac{Pa^2}{2L^3}(b + 2L) = \frac{35\text{ kN}(1.5)^2}{2(4)^3}(2.5 + 2(4)) = 6.46\text{ kN}$$

$$M_A = \frac{-Pab}{2L^2}(b + L) = \frac{-35\text{ kN}(1.5)(2.5)}{2(4.0)^2}(2.5 + 4.0) = -26.66\text{ kN·m}$$

$$M_B = \frac{Pa^2 b}{2L^3}(b + 2L) = \frac{35\text{ kN}(1.5)^2(2.5)}{2(4.0)^3}(2.5 + 2(4.0)) = 16.15\text{ kN·m}$$

DEFLECTION



MAX. DEFLECTION OCCURS IN BC.

$$y_{BC} = \frac{-Pa^2 x}{12EIL^3}\left[3L^2 b - x^2(3L - a)\right]$$

x MEASURED FROM C - VARIABLE DISTANCE

REDUCE TO EXPRESSION OF THE FORM:

$$y_{BC} = \frac{a_1 x^3 - a_2 x}{EI}$$

USE $P = 35000\text{ N}$, DISTANCES IN m.

$$y_{BC} = \frac{(-35000)(1.50)^2 x}{12\,EI(4.0)^3}\left[3(4.0)^2(2.5) - x^2(3(4) - 1.5)\right]$$

$$= \frac{-102.54x}{EI}\left[120 - 10.5x^2\right] = \frac{1}{EI}\left[-12305x + 1076.7x^3\right]$$

$$y_{BC} = \frac{1}{EI}\left[1076.7x^3 - 12305x\right]$$

USING GRAPHING CALCULATOR, FUNCTION IS MINIMUM AT $x = 1.952\text{ m}$ FROM C. VALUE OF FUNCTION THERE IS:

$$y_{max} = \frac{-16010}{EI}$$

SPECIFY W200X15 AS IN PROB 9-19 LIGHTEST OR W8X10

$I = 1.28 \times 10^7\text{ mm}^4$

$$y_{max} = \frac{-16010\text{ N·m}^3}{(207 \times 10^9\text{ N/m}^2)(1.28 \times 10^7\text{ mm}^4)} \cdot \frac{(10^3)^5\text{ mm}^5}{\text{m}^5} = -6.03\text{ mm}$$

STRESS:

$$\sigma = \frac{M}{S} = \frac{26.66\text{ kN·m}}{1.28 \times 10^5\text{ mm}^3} \times \frac{10^3\text{ N}}{\text{kN}} \times \frac{10^3\text{ mm}}{\text{m}} = 208\text{ MPa}$$

FOR ASTM A992; $s_y = 345\text{ MPa}$; $\sigma_d = 0.66 s_y = 227\text{ MPa}$ OK

AT LOAD, $y_B = -13939/EI = 5.25\text{ mm}$

9-21

CASE A-25 (b); $P = 35\ kN$, $L = 4.0$, $a = 2.50\ m$

$b = L - a = 4.0 - 2.5 = 1.5\ m$

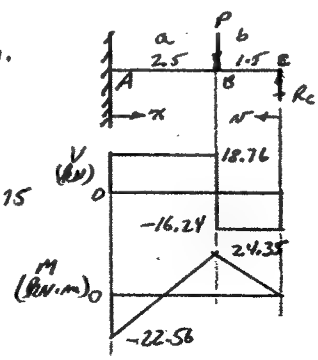
$$\underline{R_A} = \frac{Pb}{2L^3}(3L^2 - b^2) = \frac{35\ kN(1.5)}{2(4)^3}[3(4)^2 - (1.5)^2] = \underline{18.76\ kN} = V_{AB}$$

$$\underline{R_C} = \frac{Pa^2}{2L^3}(b + 2L) = \frac{35\ kN(2.5)^2}{2(4)^3}[1.5 + 2(4)] = \underline{16.24\ kN} = V_{BC}$$

$$\underline{M_A} = \frac{-Pab}{2L^2}(b + L) = \frac{-35(2.5)(1.5)}{2(4)^2}(1.5 + 4) = \underline{-22.56\ kN \cdot m}$$

$$\underline{M_B} = \frac{Pa^2 b}{2L^3}(b + 2L) = \frac{35(2.5)^2(1.5)}{2(4)^3}(1.5 + 2(4)) = \underline{24.35\ kN \cdot m}$$

<u>DEFLECTION</u>



FROM A TO B: $P = 35000\ N$; DISTANCES IN m.

$$y_{AB} = \frac{-Px^2 b}{12EIL^3}(3C_1 - C_2 x)$$

$$C_1 = aL(L + b) = 2.5(4)[4 + 1.5] = 55$$

$$C_2 = (L + a)(L + b) + aL = (6.5)(5.5) + 10 = 45.75$$

$$y_{AB} = \frac{(-35000)(x^2)(1.5)[3(55) - 45.75x]}{12\ EI(4)^3}$$

$$= \frac{-68.36x^2[165 - 45.75x]}{EI}$$

$$y_{AB} = \frac{3127.4x^3 - 11279x^2}{EI}$$

USING A GRAPHING CALCULATOR FUNCTION IS A MINIMUM AT $x = 2.404\ m$ FROM A. THEN y_{MAX} IS:

$$y_{MAX} = \frac{-21734}{EI}$$

At $x = 2.5$, $y_B = \dfrac{-21628}{EI}$

SPECIFY <u>W200×15</u> AS IN PROB 9-19, 9-20 <u>LIGHTEST</u> OR <u>W8×10</u>

$I = 1.28 \times 10^7\ mm^4$

$$\underline{y_{MAX}} = \frac{-21734\ N \cdot m^3}{(207 \times 10^9\ N/m^2)(1.28 \times 10^7\ mm^4)} \times \frac{10^{15}\ mm^5}{m^5} = \underline{-8.19\ mm}$$

$$y_B = \frac{-21628}{EI} = -8.15\ mm$$

<u>STRESS:</u>

$$\sigma = \frac{M}{S} = \frac{24.35\ kN \cdot m}{1.28 \times 10^5\ mm^3} \times \frac{10^3\ N}{kN} \times \frac{10^3\ mm}{m} = \underline{190\ MPa}$$

(OK · SEE 9-20)

ASTM A992 $s_y = 50\ ksi = 345\ MPa$

$\sigma_d = 227\ MPa$

9-22 CASE A-25 (C): $w = 400$ lb/ft, $L = 14.0$ ft

$W = wL = (400 \text{ lb/ft})(14.0 \text{ ft}) = 5600$ lb

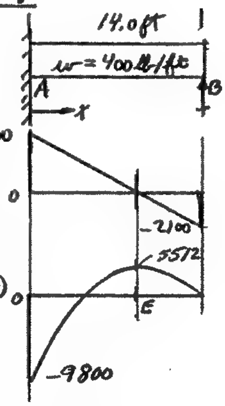
$R_A = 5/8\, W = \underline{3500 \text{ lb}} = V_A$; $R_B = 3/8\, W = \underline{2100 \text{ lb}} = V_B$

$\underline{M_A} = -0.125\,WL = -0.125(5600)(14.0) = \underline{-9800 \text{ lb·ft}}$

$\underline{M_E} = 0.0703\,WL = 0.0703(5600)(14.0) = \underline{5512 \text{ lb·ft}}$

POINT E IS 5/8 L FROM A (FIXED END)

$x_E = 5/8(14.0 \text{ ft}) = 8.75$ ft



DEFLECTION:

AT C AT $x = 0.579L = 0.579(14.0) = 8.11$ ft

$$y_C = y_{MAX} = \frac{-WL^3}{185EI} = \frac{-(5600 \text{ lb})(14 \text{ ft})^3}{185EI}$$

$$y_{MAX} = \frac{-83\,061 \text{ lb·ft}^3}{EI}$$

LET $y_{MAX} \le L/360 = \dfrac{14.0 \text{ ft}\,(12 \text{ in/ft})}{360} = 0.467$ in

$$\text{REQ'D. } I = \frac{-83\,061 \text{ lb·ft}^3}{(30\times10^6 \text{ lb/in}^2)(-0.467 \text{ in})} \times \frac{(12 \text{ in})^3}{1 \text{ ft}^3} = 10.25 \text{ in}^4$$

$\underline{\text{W 8×10 STEEL BEAM IS LIGHTEST}}$, $I = 30.8 \text{ in}^4$, $S = 7.81 \text{ in}^3$

$$\text{ACTUAL } \underline{y_{MAX}} = \frac{-83\,061 \text{ lb·ft}^3}{(30\times10^6 \text{ lb/in}^2)(30.8 \text{ in}^4)} \times \frac{(12 \text{ in})^3}{\text{ft}^3} = \underline{0.155 \text{ in.}}$$

STRESS:

$$\underline{\sigma} = \frac{M}{S} = \frac{9800 \text{ lb·ft}}{7.81 \text{ in}^3} \times \frac{12 \text{ in}}{\text{ft}} = \underline{15\,058 \text{ psi}}$$

CAN USE ASTM A992; $\sigma_d = 0.66(50 \text{ ksi}) = 33.0$ ksi – $\underline{\text{OK}}$

9-23 CASE A-25(C): $w = 50$ lb/in, $L = 16.0$ in

$W = wL = (50 \text{ lb/in})(16.0 \text{ in}) = 800$ lb

$\underline{R_A} = 5/8\, W = \underline{500 \text{ lb}}$; $\underline{R_B} = 3/8\, W = \underline{300 \text{ lb}}$

SKETCH SIMILAR TO 9-22.

$\underline{M_A} = -0.125\,WL = -0.125(800)(16) = \underline{-1600 \text{ lb·in}}$

$\underline{M_E} = 0.0703\,WL = 0.0703(800)(16) = \underline{900 \text{ lb·in}}$

$x_E = 5/8\,L = 5/8(16) = 10.0$ in

DEFLECTION:

AT $C = 0.579L = 0.579(16) = 9.264$ in

$$y_C = y_{MAX} = \frac{-WL^3}{185EI} = \frac{-800(16)^3}{185EI} = \frac{17712}{EI}$$

DESIGN COULD RESULT IN MULTIPLE SOLUTIONS.

9-24 CASE A-25(d); $P = 350\ lb$, $L = 10.8\ in$, $a = 2.50\ in$.

$$R_A = \frac{-3Pa}{2L} = \frac{-3(350)(2.50)}{2(10.8)} = -121.5\ lb \text{ DOWN}$$

$$R_B = P\left(1 + \frac{3a}{2L}\right) = 350\ lb\left[1 + \frac{3(2.50)}{2(10.8)}\right] = 471.5\ lb \text{ UP}$$

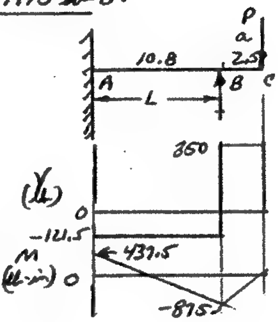
$$M_A = Pa/2 = (350)(2.50)/2 = 437.5\ lb\cdot in$$

$$M_B = -Pa = -350(2.5) = -875\ lb\cdot in$$

$$y_C = \frac{-PL^3}{EI}\left[\frac{a^2}{4L^2} + \frac{a^3}{3L^3}\right]$$

$$= \frac{-350(10.8)^3}{EI}\left[\frac{2.5^2}{4(10.8)^2} + \frac{2.5^3}{3(10.8)^3}\right]$$

$$y_C = \frac{7729\ lb\cdot in^3}{EI}$$



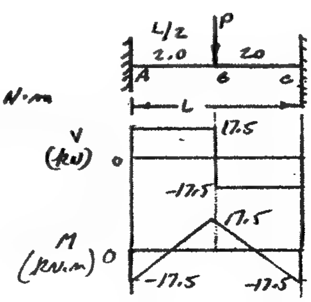
9-25 CASE A-25(e): $P = 35\ kN$; $L = 4.0\ m$

$$R_A = R_C = P/2 = 35/2 = 17.5\ kN$$

$$M_A = M_B = M_C = \frac{PL}{8} = \frac{(35\,000\ N)(4.0\ m)}{8} = 17\,500\ N\cdot m$$

$$y_B = y_{MAX} = \frac{-PL^3}{192EI} = \frac{-35\,000(4.0)^3}{192\ EI} =$$

$$y_B = \frac{11667\ N\cdot m^3}{EI}$$



9-26 CASE A-25(f): $P = 35\ kN$, $L = 4.0\ m$, $a = 1.50\ m$, $b = 4.0 - a = 2.5\ m$

THIS LOADING IS THE MIRROR IMAGE OF THAT IN 9-27

NOTATION OF CASE A-25(f) REQUIRES $a > b$. THEN

CALCULATIONS ARE THE SAME.

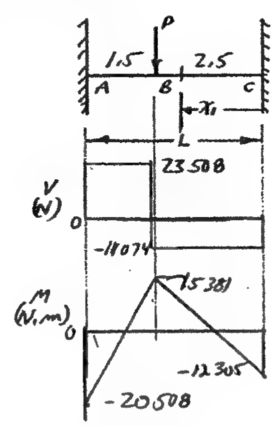
$R_A = 23926\ N$, $R_C = 11074\ N$

$M_A = -20508\ N\cdot m$ $\quad M_B = 15381\ N\cdot m$

$M_C = -12305\ N\cdot m$

$$y_{max} = y_D = \frac{-10127\ N\cdot m^3}{EI}$$

$x_1 = 2.222\ m$ FROM C TO D



9-27

CASE A-25(f): $P = 35\,kN$, $L = 4.0\,m$, $a = 2.5\,m$, $b = L - a = 1.5\,m$

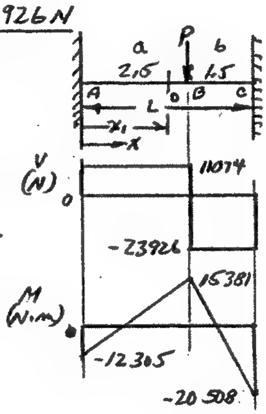
$$\underline{R_A} = \frac{Pb^2}{L^3}(3a+b) = \frac{35000(1.5)^2}{(4.0)^3}(3(2.5)+1.5) = \underline{11074\,N}$$

$$\underline{R_C} = \frac{Pa^2}{L^3}(3b+a) = \frac{35000(2.5)^2}{(4.0)^3}(3(1.5)+2.5) = \underline{23926\,N}$$

$$\underline{M_A} = \frac{-Pab^2}{L^2} = \frac{-35000(2.5)(1.5)^2}{(4.0)^2} = \underline{-12305\,N\cdot m}$$

$$\underline{M_B} = \frac{2Pa^2b^2}{L^3} = \frac{2(35000)(2.5)^2(1.5)^2}{(4.0)^3} = \underline{15381\,N\cdot m}$$

$$\underline{M_C} = \frac{-Pa^2b}{L^2} = \frac{-35000(2.5)^2(1.5)}{(4.0)^2} = \underline{-20508\,N\cdot m}$$



DEFLECTION:

$$y_{MAX} = y_D = \frac{-2Pa^3b^2}{3EI(3a+b)^2}$$

$$y_{MAX} = \frac{2(-35000)(2.5)^3(1.5)^2}{3EI[3(2.5)+1.5]^2} = \frac{-10127\,N\cdot m^3}{EI}$$

$$x_1 = \frac{2aL}{3a+b} = \frac{2(2.5)(4)}{3(2.5)+1.5} = 2.222\,m \text{ FROM A TO D.}$$

9-28

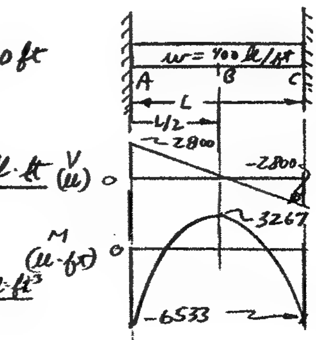
CASE A-25(g): $w = 400\,lb/ft$; $L = 14.0\,ft$

$\underline{W} = wL = (400\,lb/ft)(14.0\,ft) = \underline{5600\,lb}$

$\underline{R_A} = \underline{R_C} = W/2 = \underline{2800\,lb}$

$\underline{M_A} = \underline{M_C} = -WL/12 = -5600(14)/12 = \underline{6533\,lb\cdot ft}$

$\underline{M_B} = WL/24 = \underline{3267\,lb\cdot ft}$



DEFLECTION:

$$y_B = y_{max} = \frac{-WL^3}{384EI} = \frac{-5600(14)^3}{384\,EI} = \frac{-40017\,lb\cdot ft^3}{EI}$$

9-29.

CASE A-25(g): $w = 50\,lb/in$, $L = 16.0\,in$

$W = wL = (50\,lb/in)(16\,in) = 800\,lb$

(SKETCH SIMILAR TO 9-28)

$\underline{R_A} = \underline{R_C} = W/2 = \underline{400\,lb} = V_A = V_C$

$\underline{M_A} = \underline{M_C} = -WL/12 = -800(16)/12 = \underline{-1067\,lb\cdot in}$

$\underline{M_B} = WL/24 = \underline{533\,lb\cdot in}$

DEFLECTION:

$$y_B = y_{max} = \frac{-WL^3}{384EI} = \frac{-800(16)^3}{384\,EI} = \frac{-8533\,lb\cdot in^3}{EI}$$

9-30

CASE A-25(h): $w = 400\ lb/ft$ $L = 7.0\ ft$

$W = wL = (400\ lb/ft)(7\ ft) = 2800\ lb$ ON 1 SPAN.

$\underline{R_A} = \underline{R_C} = 3W/8 = 3(2800)/8 = \underline{1050\ lb} = \underline{V_A} = \underline{V_C}$

$\underline{R_B} = 1.25W = 1.25(2800) = \underline{3500\ lb}$

$\underline{V_B} = 5W/8 = 5(2800)/8 = \underline{1750\ lb}$

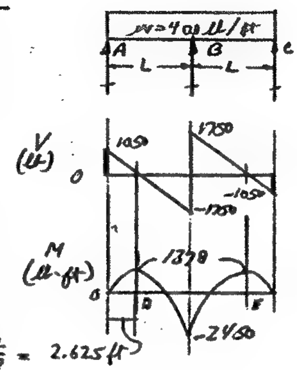
$\underline{M_D} = \underline{M_E} = 0.0703\,WL = 0.0703(2800)(7) = \underline{1378\ lb\cdot ft}$

$\underline{M_B} = -0.125\,WL = -0.125(2800)(7) = \underline{-2450\ lb\cdot ft}$

DEFLECTION: MAX AT $x_1 = 0.4215L$ FROM A OR C.

$x_1 = 0.4215(7.0\ ft) = 2.9505\ ft$

$$y_{MAX} = \frac{-wL^4}{185EI} = \frac{(-400)(7)^4}{185EI} = \frac{-5191\ lb\cdot ft^3}{EI}$$



9-31

CASE A-25(h): $w = 50\ lb/in$, $L = 8.0\ in$

$W = wL = (50\ lb/in)(8\ in) = 400\ lb$ ON EACH SPAN

(SKETCH SIMILAR TO 9-30)

$\underline{R_A} = 3W/8 = 3(400)/8 = \underline{150\ lb} = \underline{R_C} = \underline{V_A} = \underline{V_C}$

$\underline{R_B} = 1.25W = 1.25(400) = \underline{500\ lb}$

$\underline{V_B} = 5W/8 = 5(400)/8 = \underline{250\ lb}$

$\underline{M_D} = \underline{M_E} = 0.0703\,WL = 0.0703(400)(8) = \underline{225\ lb\cdot in}$

$\underline{M_B} = -0.125\,WL = -0.125(400)(8) = \underline{-400\ lb\cdot in}$

DEFLECTION: MAX AT $x_1 = 0.4215L$ FROM A OR C.

$x_1 = 0.4215(8.0\ in) = 3.372\ in$

$$y_{MAX} = \frac{-wL^4}{185EI} = \frac{-50(8)^4}{185EI} = \frac{-1107}{EI}$$

9-32

CASE A-28(i): $w = 400\ lb/ft$, $L = 56\ in$, $W = wL = 1867\ lb$

$\underline{R_A} = \underline{R_D} = 0.4W = 0.4(1867\ lb) = \underline{746.7\ lb} = \underline{V_A} = \underline{V_D}$

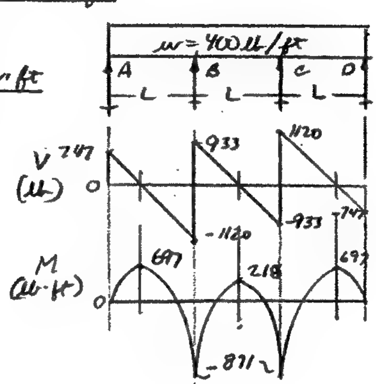
$\underline{R_B} = \underline{R_C} = 1.10W = 1.10(1867) = \underline{2053\ lb}$

$\underline{M_E} = \underline{M_F} = 0.08\,WL = 0.08(1867)(4.67\ ft) = \underline{697\ lb\cdot ft}$

$\underline{M_B} = \underline{M_C} = -0.10\,WL = \underline{-871\ lb\cdot ft}$

$\underline{M_G} = 0.025\,WL = 0.025(1867)(4.67\ ft) = \underline{218\ lb\cdot ft}$

DEFLECTION FORMULAS NOT AVAILABLE



9-33

CASE A-25(i) : $w = 50$ lb/in, $L = 5.333$ in, $W = wL = 266.7$ lb

$R_A = R_D = 0.4W = 106.7$ lb

$R_B = R_C = 1.10W = 293.3$ lb

[SKETCH SIMILAR TO 9-32]

$M_E = M_F = 0.08WL = 0.08(266.7)(5.333) = 113.8$ lb·in

$M_B = M_C = -0.10WL = -0.10(266.7)(5.333) = -142.2$ lb·in

$M_G = 0.025WL = 0.025(266.7)(5.333) = 35.6$ lb·in

9-34

CASE A-25(j) : $w = 400$ lb/ft, $L = 3.5$ ft, $W = wL = 1400$ lb EACH SPAN

$R_A = R_E = 0.393W = 550$ lb

$R_B = R_D = 1.143W = 1600$ lb

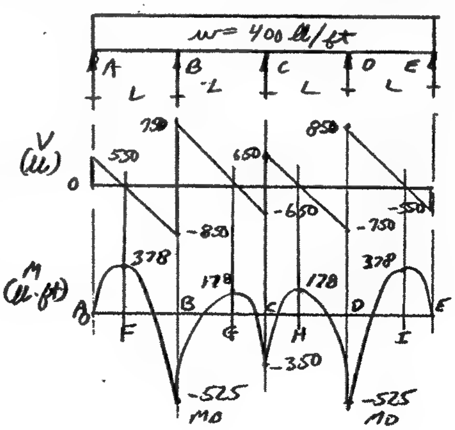
$R_C = 0.928W = 1300$ lb

$M_B = M_D = -0.1071WL = M_{MAX}$
$= -0.1071(1400)(3.5) = 525$ lb·ft

$M_F = M_I = 0.0772WL = 378$ lb·ft

$M_C = -0.0714WL = 350$ lb·ft

$M_G = M_H = 0.0364WL = 178$ lb·ft



9-35

CASE A-25(j) : $w = 50$ lb/in, $L = 4.0$ in $W = wL = 200$ lb

$R_A = R_E = .393W = 78.6$ lb

$R_B = R_D = 1.143W = 228.6$ lb

$R_C = 0.928W = 185.6$ lb

$M_B = M_D = -0.1071WL = -85.68$ lb·in

$M_F = M_I = 0.0772WL = 61.76$ lb·in

$M_C = -0.0714WL = -57.12$ lb·in

$M_G = M_H = 0.0364WL = 29.12$ lb·in

$V_A = R_A = 78.6$ lb

$-V_B = R_A - W = -121.4$ lb

$+V_B = -121.4 + 228.6 = 107.2$ lb

$-V_C = 107.2 - W = -92.8$ lb

$+V_C = -92.8 + 185.6 = 92.8$ lb

$-V_D = 92.8 - W = -107.2$ lb

$+V_D = -107.2 + 228.6 = 121.4$ lb

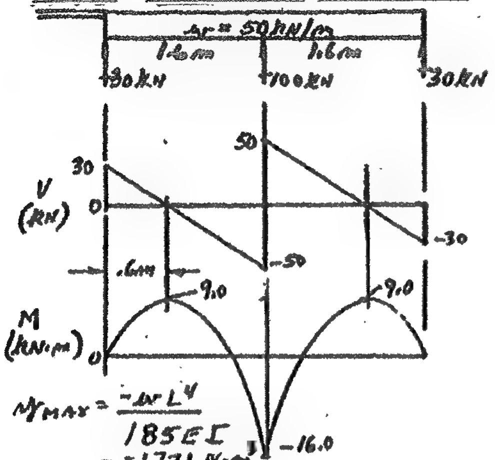
$V_E = 121.4 - W = -78.6$ lb

SKETCH SIMILAR TO 13-16.

$M_{MAX} = 85.7$ lb·in $= M_B = M_D$

$V_{MAX} = 121.4$ lb $= -V_B = +V_D$

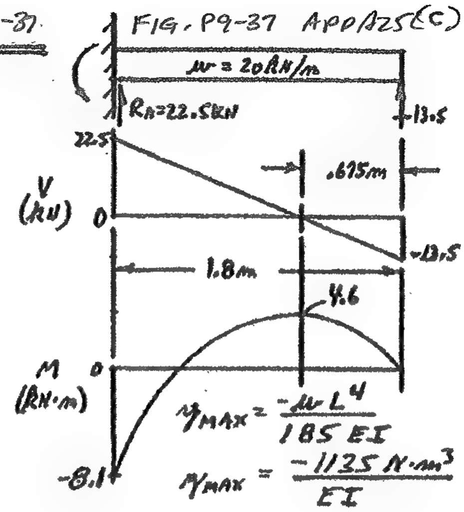
9-36 FIG. P9-36 APP. A25(h)



$y_{MAX} = \frac{-wL^4}{185EI}$

$y_{MAX} = \frac{-1771\ N\cdot m^3}{EI}$

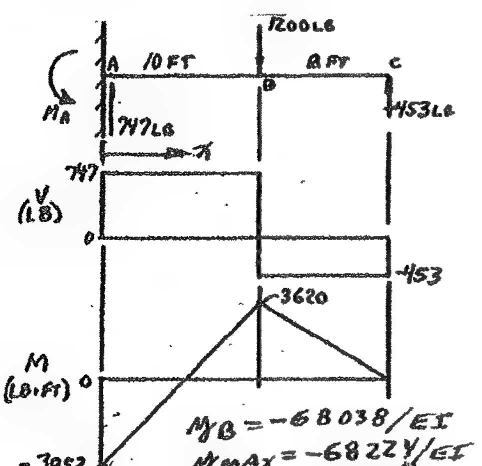
9-37 FIG. P9-37 APP A25(c)



$y_{MAX} = \frac{-wL^4}{185EI}$

$y_{MAX} = \frac{-1125\ N\cdot m^3}{EI}$

9-38 FIG. P9-38 APP A25(b)

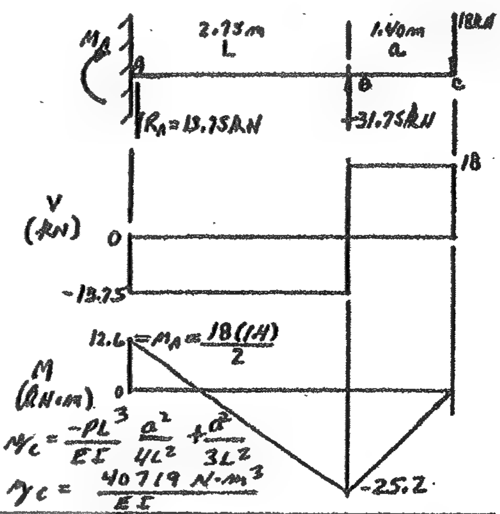


$y_B = -68038/EI$

$y_{MAX} = -68224/EI$

AT $x = 10.32$ ft

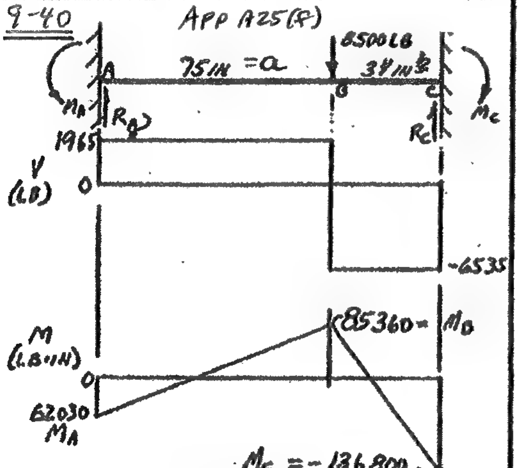
9-39 APP. A25(d)



$y_C = \frac{-PL^3}{EI}\ \frac{a^2}{4L^2} + \frac{a^3}{3L^2}$

$y_C = \frac{40719\ N\cdot m^3}{EI}$

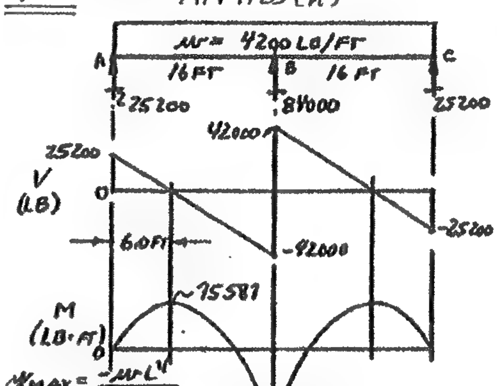
9-40 APP A25(f)



$y_{MAX} = \frac{-2Pa^3b^2}{3EI(3a+b)^2}$

$y_{MAX} = \frac{-41.2\times 10^6\ lb\cdot in^3}{EI}$

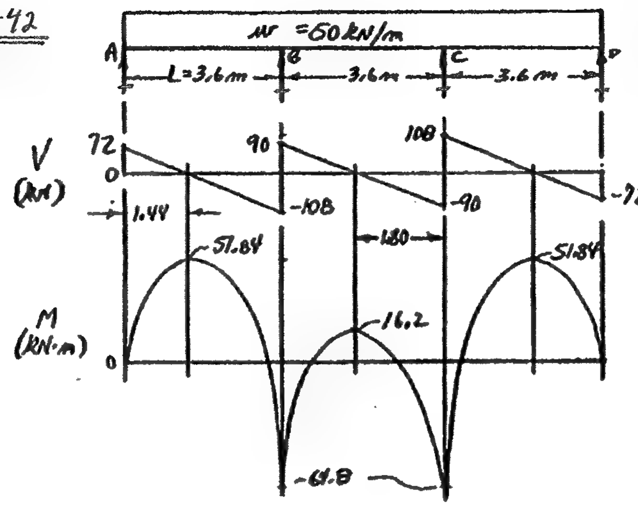
9-41 APP A25(h)



$y_{MAX} = \frac{-wL^4}{185EI}$

$y_{MAX} = \frac{1.488\times 10^6\ lb\cdot ft^3}{EI}$

9-42



$W = wL = (50)(3.6) = 180\ kN$

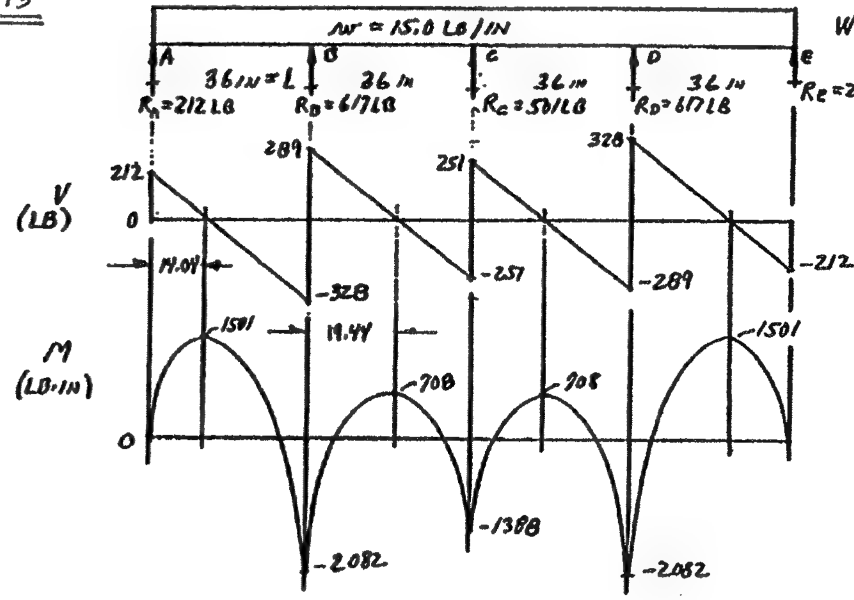
$R_A = R_D = 0.4W = 72\ kN$

$R_B = R_C = 1.10W = 198\ kN$

APP A25(i)

9-43

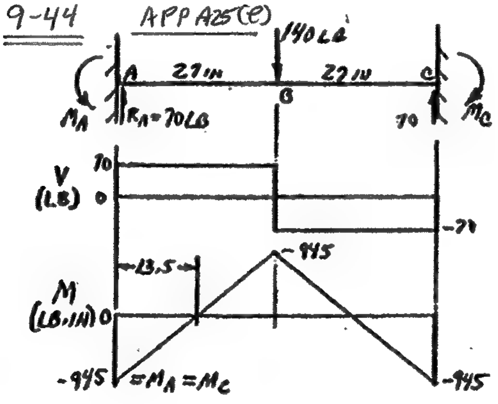
APP A25(j)



$W = wL = 540\ LB.$

9-44

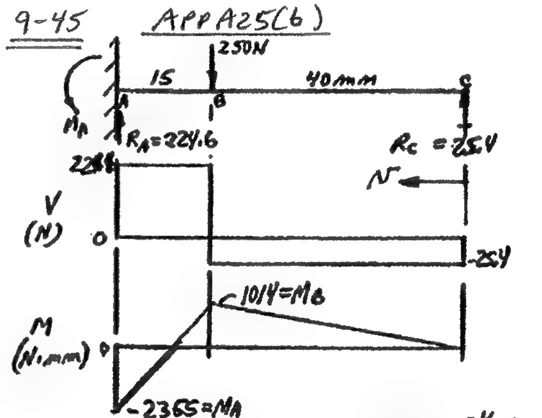
APP A25(e)



$$y_{MAX} = \frac{-PL^3}{192EI} = \frac{1.15\times10^5\ lb\cdot in^3}{EI}$$

9-45

APP A25(b)



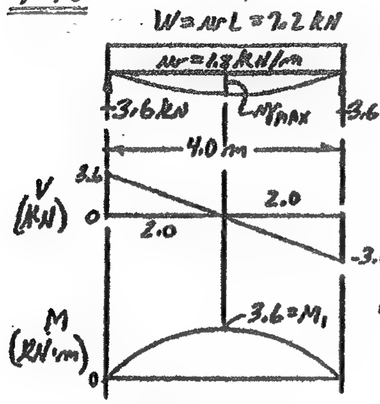
$$y_B = \frac{-Pa^3b^2}{12EIL^3}(3L+b) = \frac{-1.386\times10^{-4}\ N\cdot m^3}{EI}$$

$$y_{MAX} = -1.936\times10^{-4}\ N\cdot m^3/EI$$

AT $x = 0.0284$ m FROM C.

9-46

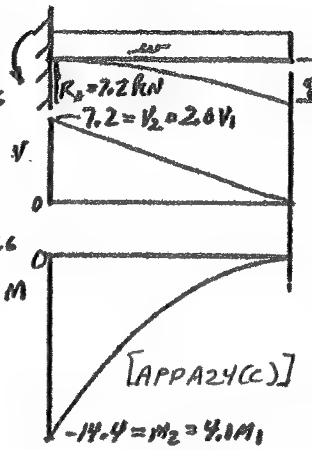
(a)



$y_{1max} = \dfrac{-5WL^3}{384EI} = \dfrac{-6.0}{EI}$

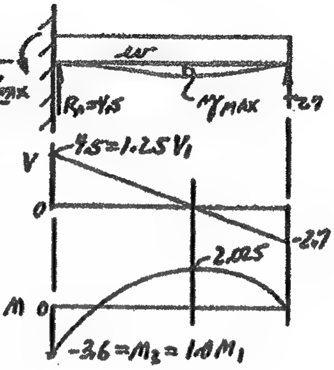
[FROM A23-(d)]

(b)



$y_{2max} = \dfrac{-WL^3}{8EI} = \dfrac{-57.6}{EI} = 9.6 y_1$

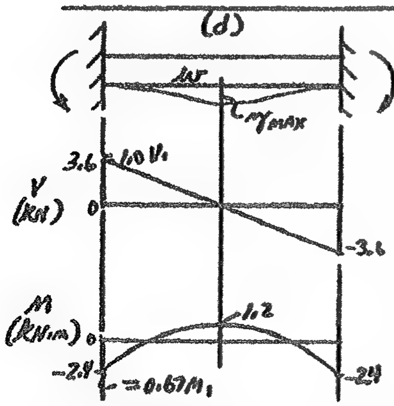
(c)



$y_{3max} = \dfrac{-WL^3}{185EI} = \dfrac{-2.49}{EI} = .415 y_1$

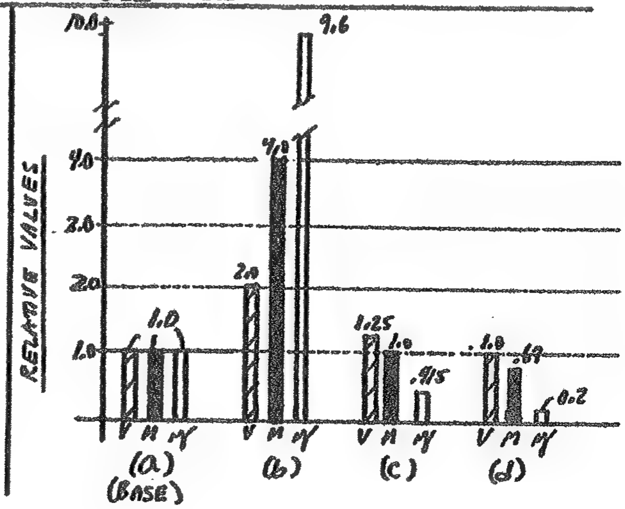
FROM [A25-(c)]

(d)



$y_{4max} = \dfrac{-WL^3}{384EI} = \dfrac{-1.2}{EI} = 0.2 y_1$

FROM [A25(g)]



9-47

The objective is to compare the results of the beam loading and support conditions for five different beams in **problems 9-22, 9-28, 9-30, 9-32, and 9-34**. Details were reported earlier in this solutions manual for each problem. Note that each beam design has a total length of 14.0 ft and carries a uniformly distributed load of 400 lb/ft resulting in a total load of 5600 lb. Changing the manner of support or adding additional supports affects the shearing force, *V*, the bending moment, *M*, and the maximum deflection, *y*, for a given *EI* value for the beam material and shape. Vertical shear stress and bending stress are directly propostional to the values of *V* and *M* respectively. Therefore, a reduction in either value will result in a reduction in stress or will allow the use of a smaller or lighter section for the beam. Comparisons are shown as ratios of *V, M,* and *y/EI* to those values for the first design, a supported cantilever. The other designs are a fixed-end beam and continuous beams on 3, 4, and 5 supports.

Prob.	V_{max}	V/V_1	M_{max}	M/M_1	y_{max}	y/y_1
9-22	3500 lb	1.00	9800 lb in	1.00	-83061/EI	1.00
9-28	2800 lb	0.80	6533 lb in	0.667	-40017/EI	0.482
9-30	1750 lb	0.50	2450 lb in	0.250	-5191/EI	0.0625
9-32	1120 lb	0.32	871 lb in	0.089	N/A	-
9-34	850 lb	0.24	525 lb in	0.054	N/A	-

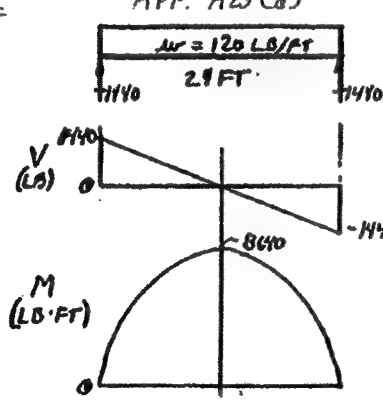
Note that maximum shearing force, bending moment, and deflection all decrease for successive designs. Deflection formulas are not available (N/A) in this book for the last two designs. But it stands to reason that deflection would be reduced by adding additional supports and reducing the span between adjacent supports. The comparison illustrates the advantages of using fixed ends for beams and of using more supports for a given load, thus reducing the effective span between adjacent supports. Fabrication problems and costs must also be considered when selecting a method of supporting a load on a beam.

9-48

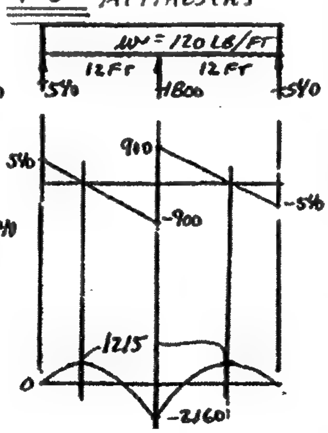
This problem has the same objective as **9-47**. Refer to that problem for a discussion of the objectives and the results. Data listed here are for different beam loadings (w = 50 lb/in; total beam length = 16.0 in) but the support conditions are the same as in **9-47**.

Problem	V_{max}	V/V_1	M_{max}	M/M_1	y_{max}	y/y_1
9-23	500 lb	1.00	1600 lb in	1.00	-17712/EI	1.00
9-29	400 lb	0.80	1067 lb in	0.667	-8533/EI	0.482
9-31	250 lb	0.50	400 lb in	0.250	-1107/EI	0.0625
9-33	160 lb	0.32	142 lb in	0.089	N/A	-
9-35	121 lb	0.24	86 lb in	0.054	N/A	-

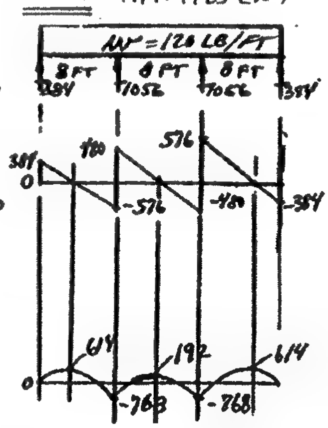
9-49 APP. A23(d)



9-50 APP. A25(h)



9-51 APP. A25(i)



9-52

COMPARISON OF 9-49, 9-50, 9-51

$w = 120\frac{LB}{FT} \times \frac{1FT}{12IN} = 10LB/IN ; L_a = 24FT \times \frac{12IN}{FT} = 288IN$

DEFLECTIONS:

[9-49] $y_{MAX} = \frac{-5wL^4}{384EI} = \frac{-5(10)(288)^4}{384EI} = \frac{-896\times10^6}{EI} = y_a$

[9-50] $y_{MAX} = \frac{-wL^4}{185EI} = \frac{-(10)(288)^4}{185EI} = \frac{-372\times10^6}{EI} = y_b = 0.415 y_a$ IF EI IS EQUAL.

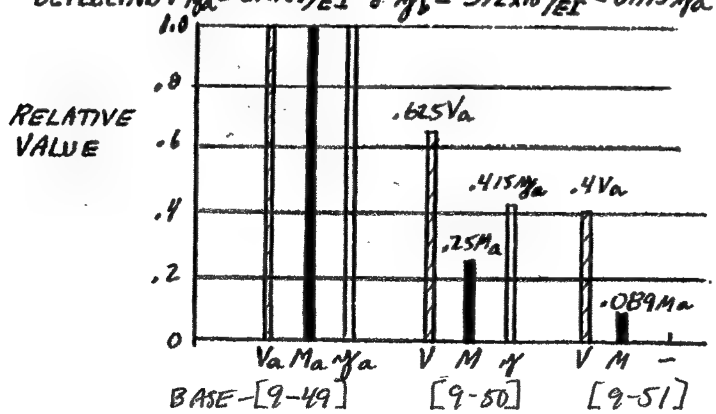
[9-51] DEFLECTION EQUATION NOT AVAILABLE; LESS THAN y_b

SUMMARY:

SHEARING FORCE: $V_{a_{MAX}} = 1440LB$: $V_{b_{MAX}} = 900LB = 0.625V_a$: $V_c = 576LB = 0.4V_a$

MOMENT: $M_{a_{MAX}} = 8640LB\cdot FT$; $M_b = 2160LB\cdot FT = 0.25M_a$: $M_c = 768LB\cdot FT = 0.089M_a$

DEFLECTION: $y_a = 896\times10^6/EI$: $y_b = 372\times10^6/EI = 0.415y_a$ IF EI IS EQUAL.



BEAM SIZE: FOR NO. 2 SOUTHERN PINE: $\tau_d = 70PSI$: $\sigma_d = 1000PSI$

FOR A RECTANGLE: $\tau_{MAX} = 3V/2A$; $S = th^2/6$

[9-49] $V_{MAX} = 1440LB$: $A_{MIN} = \frac{3V}{2\tau_d} = \frac{3(1440)}{2(70)} = 30.86IN^2$

$M_{MAX} = (8640LB\cdot FT)(12IN/FT) = 103680LB\cdot IN$: $\sigma = M/S$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{103680LB\cdot IN}{1000LB/IN^2} = 103.7IN^3$ 6X12 BEAM REQD
$I = 697IN^4, S = 121IN^3; A = 63.3IN^2$

[9-50] $V_{MAX} = 900LB$: $A_{MIN} = \frac{3(900)}{2(70)} = 19.3IN^2$

$M_{MAX} = 2160LB\cdot FT(12IN/FT) = 25920LB\cdot IN$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{25920}{1000} = 25.9IN^3$ 4X8 BEAM REQD
$I = 111.1IN^4, S = 30.7IN^3; A = 25.4IN^2$

[9-51] $V_{MAX} = 576LB$: $A_{MIN} = \frac{3(576)}{2(70)} = 12.34IN^2$

$M_{MAX} = 768LB\cdot FT(12IN/FT) = 9216LB\cdot IN$

$S_{MIN} = \frac{M}{\sigma_d} = \frac{9216}{1000} = 9.22IN^3$ 2X10 BEAM REQD
$I = 98.9IN^4, S = 21.4IN^3; A = 13.87IN^2$

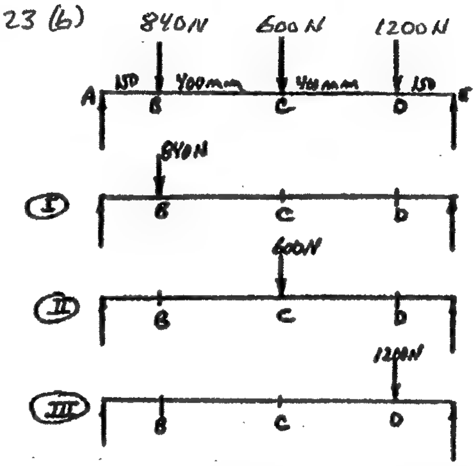
ACTUAL DEFLECTIONS:

[9-49] $y_{MAX} = \frac{-896\times10^6}{EI} = \frac{-896\times10^6}{(1.3\times10^6)(697)} = -0.989IN.$

[9-50] $y_{MAX} = \frac{-372\times10^6}{EI} = \frac{-372\times10^6}{(1.3\times10^6)(111.1)} = -2.58IN.$ (LARGE) HIGHER I MAY BE REQUIRED FOR DEFLECTION.

FOR $y_{MAX} = L/360 = 24FT(12IN/FT)/360 = 0.800IN$

9-53 COMBINE A23(a) AND A23(b)



$v_B = v_{B1} + v_{B2} + v_{B3}$

$v_C = v_{C1} + v_{C2} + v_{C3}$

$v_D = v_{D1} + v_{D2} + v_{D3}$

(I) - A22-2

$v_{B1} = \dfrac{-Pa^2b^2}{3EI\ell}$; $b = 150, a = 950$

[I FROM P6-11]

$EI = (69\times10^3\ N/mm^2)(0.186\times10^6\ mm^4)$

$EI = 1.283\times10^{10}$

$EI\ell = (1.283\times10^{10})(1100) = 1.412\times10^{13}$

$v_{B1} = \dfrac{(-840)(150)^2(950)^2}{3(1.412\times10^{13})} = -0.403\ mm$

$v_{C1} = \dfrac{-Pbx}{6EI\ell}(\ell^2 - x^2 - b^2) = \dfrac{-840(150)(550)}{6(1.412\times10^{13})}(1100^2 - 550^2 - 150^2) = -0.724\ mm$

$v_{D1} = \quad " \downarrow " \quad = \dfrac{-840(150)(150)}{6(1.412\times10^{13})}(1100^2 - 150^2 - 150^2) = -0.260\ mm$

(II) - A23 (a)

$v_{B2} = v_{D2} = \dfrac{-Px}{48EI}(3\ell^2 - 4x^2) = \dfrac{-600(150)}{48(1.283\times10^{10})}(3(1100)^2 - 4(150)^2) = -0.517\ mm$

$v_{C2} = \dfrac{-P\ell^3}{48EI} = \dfrac{-600(1100)^3}{48(1.283\times10^{10})} = -1.297\ mm.$

(III) A23(b)

$v_{B3} = \dfrac{-Pbx}{6EI\ell}(\ell^2 - x^2 - b^2) = \dfrac{-1200(150)(150)}{6(1.412\times10^{13})}(1100^2 - 150^2 - 150^2) = -0.371\ mm$

$v_{C3} = \quad " \downarrow " \quad = \dfrac{-1200(150)(550)}{6(1.412\times10^{13})}(1100^2 - 550^2 - 150^2) = -1.034\ mm$

$v_{D3} = \dfrac{-Pa^2b^2}{3EI\ell} = \dfrac{-1200(150)^2(950)^2}{3(1.412\times10^{13})} = -0.576\ mm$

TOTAL DEFLECTIONS –

$\underline{v_B} = v_{B1} + v_{B2} + v_{B3} = \underline{-1.291\ mm}$

$\underline{v_C} = v_{C1} + v_{C2} + v_{C3} = \underline{-3.055\ mm}$

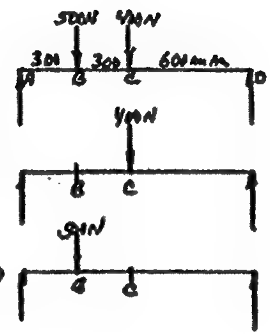
$\underline{v_D} = v_{D1} + v_{D2} + v_{D3} = \underline{-1.353\ mm}$

9-54

$E = 73\,GPa = 73\times10^9\,N/m^2 = 73\times10^3\,N/mm^2$

$EI = (73\times10^3)(16956) = 1.238\times10^9\,N\cdot mm^2$

$EIl = (1.238\times10^9)(1200) = 1.485\times10^{12}\,N\cdot mm^3$



(I) A23-(a) [I FROM P6-12]

$$y_{B1} = \frac{-Px}{48EI}(3l^2-4x^2)$$

$$y_{B1} = \frac{-400(300)}{48(1.238\times10^9)}(3(1200)^2-4(300)^2) = -8.000\,mm$$

$$y_{C1} = \frac{-Pl^3}{48EI} = \frac{-400(1200)^3}{48(1.238\times10^9)} = -11.632\,mm$$

(II) A23-(b)

$$y_{B2} = \frac{-Pa^2b^2}{3EIl} = \frac{-500(300)^2(900)^2}{3(1.485\times10^{12})} = -8.182\,mm$$

$$y_{C2} = \frac{-Pbx}{6EIl}(l^2-x^2-b^2) = \frac{-500(300)(600)}{6(1.485\times10^{12})}(1200^2-600^2-300^2) = -10.000\,mm$$

TOTAL DEFLECTIONS

$\underline{y_B} = y_{B1} + y_{B2} = -8.000\,mm - 8.182\,mm = \underline{-16.182\,mm}$

$\underline{y_C} = y_{C1} + y_{C2} = -11.632\,mm - 10.000\,mm = \underline{-21.632\,mm}$

LARGE DEFLECTIONS

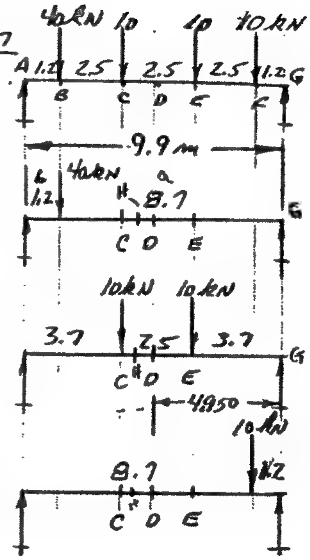
9-55

LOADING IN FIGURE P5-7. FIGURE P5-7 TOTAL LOADING

W18-55 STEEL BEAM: W460×82

USE SUPERPOSITION

SUPERPOSITION



(I) 40 kN LOAD ONLY AT B.
CASE A23 (b): $L = 9900\ mm$
$a = 8700\ mm$, $b = 1200\ mm$

(II) TWO 10 kN LOADS AT C AND E.
CASE A-23 (c): $L = 9900\ mm$
$a = 3700\ mm$

(III) 10 kN LOAD ONLY AT F.
$a = 8700\ mm$, $b = 1200\ mm$
CASE A23 (b): $L = 9900\ mm$

POINT OF MAXIMUM DEFLECTION IS NOT OBVIOUS BECAUSE EACH CASE PRODUCES A MAXIMUM DEFLECTION AT A DIFFERENT POINT. DEFLECTION AT C, D, AND E ARE COMPUTED FOR EACH LOADING, THEN SUMMED. THE MAXIMUM DEFLECTION FOR CASE I OCCURS BETWEEN C AND D AT THE POINT CALLED H, 4226 mm FROM A. DEFLECTION COMPUTED THERE ALSO.

PRODUCT OF EI APPEARS IN ALL EQUATIONS,
$E = 207 \times 10^3\ N/mm^2$
$I = 890\ in^4 \times \dfrac{4.162 \times 10^5\ mm^4}{in^4}$
$I = 3.70 \times 10^8\ mm^4$
$EI = 7.66 \times 10^{13}\ N \cdot mm^2$

SUMMARY OF RESULTS:

I		II		III	
	$y_C = -3.802\ mm$		$y_C = -4.438\ mm$		$y_C = -0.809\ mm$
	$y_D = -3.76\ mm$		$y_D = -4.816\ mm$		$y_D = -0.940\ mm$
	$y_E = -3.235\ mm$		$y_E = -4.438\ mm$		$y_E = -0.951\ mm$
	$y_H = -3.85\ mm$		$y_H = -4.689\ mm$		$y_H = -0.877\ mm$

BY SUPERPOSITION:

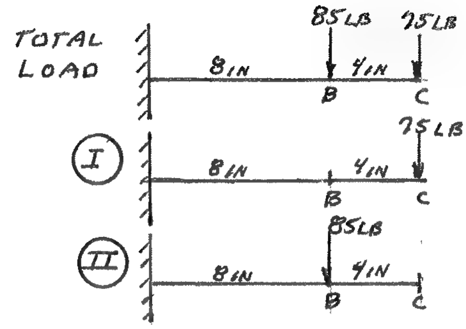
$y_C = -9.049\ mm$

$y_D = -9.516\ mm$ APPARENT MAXIMUM DEFLECTION AT MIDDLE OF BEAM AT D.

$y_E = -8.624\ mm$

$y_H = -9.416\ mm$

9-56 APP. A24 (a) AND A24 (b)



(I) $y_{B_1} = \frac{-Px^2}{6EI}(3l - x)$

$EI = (30\times10^6)(0.08734) = 2.62\times10^6$

$y_{B_1} = \frac{-75(8)^2}{6(2.62\times10^6)}[3(12) - 8] = -0.0085 \text{ in}$

$y_{C_1} = \frac{-Pl^3}{3EI} = \frac{-75(12)^3}{3(2.62\times10^6)} = -0.0165 \text{ in}$

(II) $y_{B_2} = \frac{-Pa^3}{3EI} = \frac{-85(8)^3}{3(2.62\times10^6)} = -0.0055 \text{ in}$

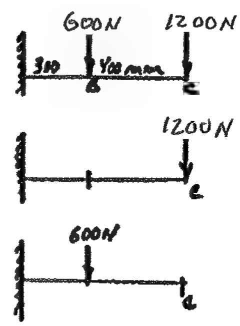
$y_{C_2} = \frac{-Pa^2}{6EI}[3L - a] = \frac{-85(8)^2}{6(2.62\times10^6)}[3(12) - 8] = -0.0097 \text{ in}$

TOTAL DEFLECTION

$y_B = y_{B1} + y_{B2} = -0.0085 - 0.0055 = \underline{-0.0140 \text{ in}}$

$y_C = y_{C1} + y_{C2} = -0.0165 - 0.0097 = \underline{-0.0262 \text{ in}}$

9-57



$I = \frac{bh^3}{12} = \frac{20(80)^3}{12} = 8.533\times10^5 \text{ mm}^4$

$EI = [207\times10^3 \text{ N/mm}^2][8.533\times10^5 \text{ mm}^4] = 1.766\times10^{11}$

(I) A24-(a)

$y_{C1} = \frac{-Pl^3}{3EI} = \frac{-1200(700)^3}{3(1.766\times10^{11})} = -0.777 \text{ mm}$

(II) A24-(b)

$y_{C2} = \frac{-Pa^2}{6EI}[3L - a] = \frac{-600(300)^2}{6(1.766\times10^{11})}[3(700) - 300]$

$y_{C2} = -0.092 \text{ mm}$

TOTAL DEFLECTION

$y_C = y_{C1} + y_{C2} = -0.777 - 0.092 = \underline{-0.869 \text{ mm}}$

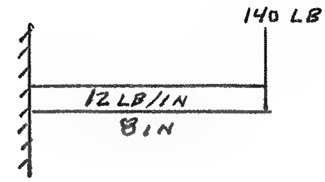
9-58 DEFLECTIONS INVERSELY PROPORTIONAL TO E

$y_C = -0.869 \text{ mm} \times \frac{E_S}{E_{AL}} = -0.869 \times \frac{207 \text{ GPa}}{73 \text{ GPa}} = \underline{-2.464 \text{ mm}}$

9-59

$y_C = -0.869 \text{ mm} \times \frac{E_S}{E_{MAG}} = -0.869 \times \frac{207 \text{ GPa}}{45 \text{ GPa}} = \underline{-3.997 \text{ mm}}$

9-60



$I = \pi D^4/64 = \pi(0.80)^4/64 = 0.0201\ in^4$

$W = \text{TOTAL LOAD} = wL = (12\ LB/IN.)(8\ IN) = 96\ LB$

A24-(c) $y_1 = \frac{-wl^3}{8EI} = \frac{-96(8)^3}{8(30\times10^6)(0.0201)} = -0.0102\ in$

A24-(a) $y_2 = \frac{-Pl^3}{3EI} = \frac{-140(8)^3}{3(30\times10^6)(0.0201)} = -0.0396\ in$

TOTAL $y = y_1 + y_2 = -0.0102 - 0.0396 = \underline{-0.0498\ in}$

9-61

FROM FIG P5-7, $L = 9900\ mm$

$L/360 = 27.5\ mm$; LET $y_{MAX} = -27.5\ mm$

FROM PROBLEM 9-55 ; $y_{MAX} = -9.516\ mm$

FOR A W18-55 BEAM WITH $I = 890\ IN^4$

DEFLECTION INVERSELY PROPORTIONAL TO I.

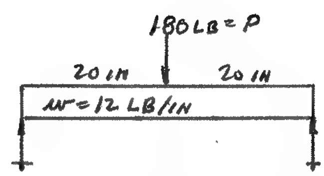
$I_{MIN} = 890\ IN^4 \cdot \frac{9.516}{27.5} = 308\ IN^4$

SPECIFY: W18×40 - LIGHTEST   W460×60 METRIC

OR   W10×60 - LEAST DEPTH   W250×89 METRIC

RESULT COULD HAVE BEEN FOUND BY USING SUPERPOSITION APPROACH OUTLINED IN PROBLEM 9-55 WITH I TREATED AS AN UNKNOWN. THEN SET $y_{MAX} = \frac{C}{I_{MIN}}$ AND SOLVE FOR I_{MIN} FOR $y = 27.5\ mm$.

9-62



$W = wL = 12(40) = 480\ LB$

$$y_{MAX} = -0.080\ in = \underbrace{\frac{-Pl^3}{48EI}}_{A23(a)} - \underbrace{\frac{5W\ l^3}{384EI}}_{A23(d)}$$

$48EIy_{MAX} = -Pl^3 - \frac{5W\ l^3}{8} = l^3[-P - 5W/8]$

REQ'D $I = \frac{l^3}{48Ey_{MAX}}[-P - 5W/8] = \frac{(40)^3[-180 - 5(480)/8]}{48(10\times10^6)(-0.08)} = \underline{0.800\ in^4}$

USE A $\underline{C4\times2.331\ (I_{yy} = 1.02\ in^4)}$

OR $\underline{C5\times2.212\ (I_{yy} = 0.98\ in^4)\ (LIGHTEST)}$

9-63 FIG P9-36

$$y_{C1}=\frac{-5wL^3}{384EI}\ \text{(CASE(d)) TABLE A-23}$$

$$y_{C1}=\frac{-5(160\times10^3)(3200)^3}{384EI}=-6.827\times10^{13}$$

$$y_{C2}=\frac{+PL^3}{48EI}=\frac{+R_B(3200)^3}{48EI}=6.827\times10^{8}(R_B)\ \text{(A23(a))}$$

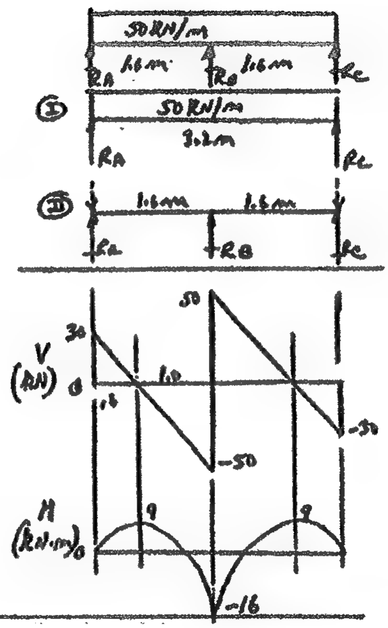
$y_{C1}+y_{C2}=0$

$-6.827\times10^{13}+6.827\times10^{8}(R_B)=0$

$R_B=6.827\times10^{13}/6.827\times10^{8}=1.0\times10^{5}\,N$

$R_B=100\,kN$

THEN $R_A=R_C=30\,kN$



9-64 FIG. P9-37

$$y_{B1}=\frac{-wL^3}{8EI}=\frac{(-36\times10^3)(1800)^3}{8EI}=\frac{-2.624\times10^{13}}{EI}\ \text{[A24-(c)]}$$

$$y_{B2}=\frac{+PL^3}{3EI}=\frac{R_B(1800)^3}{3EI}=\frac{R_B(1.944\times10^9)}{EI}\ \text{[A24-(a)]}$$

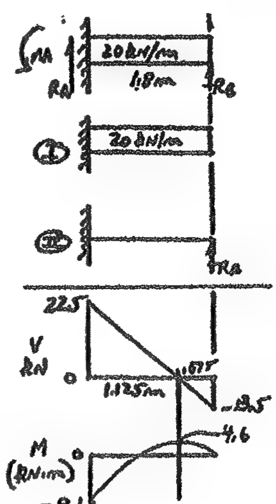
$y_{B1}+y_{B2}=0$

$-2.624\times10^{13}/EI+R_B(1.944\times10^9)/EI=0$

$R_B=2.624\times10^{13}/1.944\times10^9=13.50\,kN=R_B$

THEN $R_A=36-13.5=22.5\,kN=R_A$

$M_A=36(0.9)-13.5(1.8)=8.1\,kN\cdot m$ (NEGATIVE)



9-65 FIG. P9-38

$$y_{C1}=\frac{-Pa^2}{6EI}(3L-a)=\frac{-(1200)(120)^2}{6EI}(3(216)-120)=\frac{-1.521\times10^9}{EI}\ \text{[A24(b)]}$$

$$y_{C2}=\frac{R_CL^3}{3EI}=\frac{R_C(216)^3}{3EI}=\frac{3.359\times10^6(R_C)}{EI}\ \text{[A24(a)]}$$

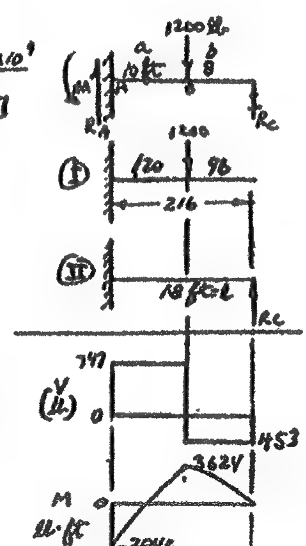
$y_{C1}+y_{C2}=0$

$-1.521\times10^9/EI+3.359\times10^6(R_C)/EI=0$

$R_C=1.521\times10^9/3.359\times10^6=453\,lb=R_C$

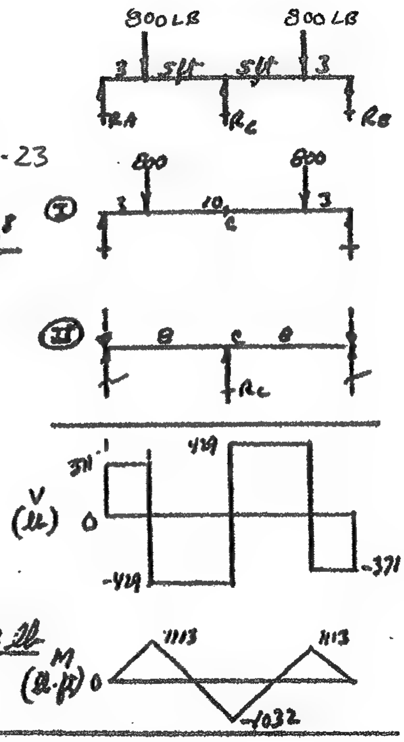
THEN $R_A=1200-453=747\,lb=R_A$

$M_A=1200(10)-R_C(18)=12000-453(18)=3846\,lb\cdot ft$ (NEGATIVE)



9-66

FIG. P9-66



SUPERPOSITION: $y_{c1}+y_{c2}=0$

$$y_{c1}=\frac{-Pa}{24EI}(3l^2-4a^2) \quad \text{(CASE (c)) TABLE A-23}$$

$$y_{c1}=\frac{-800(36)}{24EI}\left[3(192)^2-4(36)^2\right]=\frac{-1.265\times10^8}{EI}$$

$$y_{c2}=+\frac{Pl^3}{48EI} \quad \text{(CASE (a))}$$

$$y_{c2}=\frac{R_c(192)^3}{48EI}=\frac{1.475\times10^5 R_c}{EI}$$

THEN $y_{c1}+y_{c2}=0$

$$\frac{-1.265\times10^8}{EI}+\frac{1.475\times10^5 R_c}{EI}=0$$

$$R_c=1.265\times10^8/1.475\times10^5=858 \text{ lb}$$

BECAUSE OF SYMMETRY; $R_A=R_E=371$ lb

9-67

FIG. P9-67

$$y_{c1}=\frac{-Pa}{24EI}(3l^2-4a^2)=\frac{-1.265\times10^8}{EI}$$

$$y_{c2}=\frac{-5wl^4}{384EI}=\frac{-5(8000)(192)^3}{384EI}=\frac{-7.37\times10^8}{EI}$$

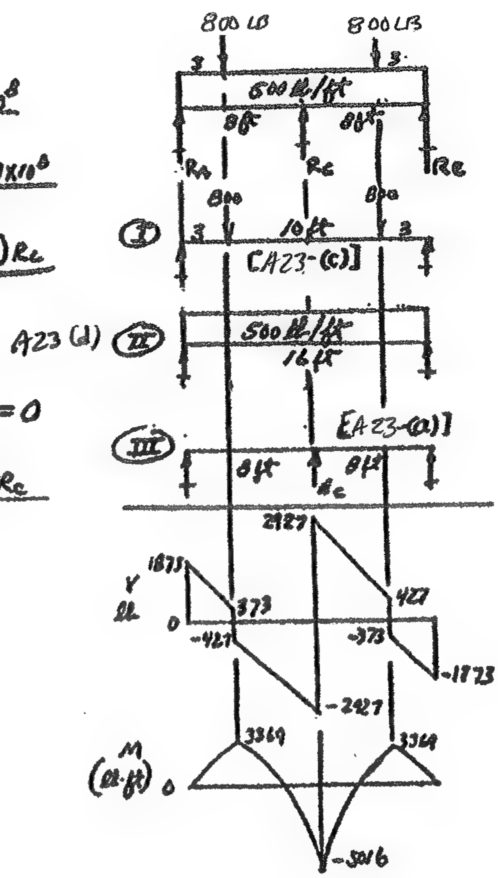
$$y_{c3}=\frac{+Pl^3}{48EI}=\frac{R_c(192)^3}{48EI}=\frac{(1.475\times10^5)R_c}{EI}$$

THEN $y_{c1}+y_{c2}+y_{c3}=0$

$$\frac{-1.265\times10^8}{EI}-\frac{7.37\times10^8}{EI}+\frac{R_c(1.475\times10^5)}{EI}=0$$

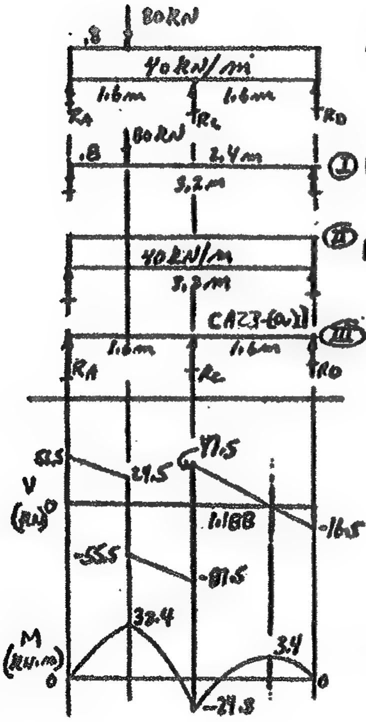
$$R_c=\frac{1.265\times10^8+7.37\times10^8}{1.475\times10^5}=5854 \text{ lb}=R_c$$

THEN $R_A=R_E=1873$ lb



9-68

FIGURE P9-68



$$y_{C1} = \frac{-(80\times10^3)(800)(1600)}{6EI(3200)}\left[3200^2 - 1600^2 - 800^2\right] = \frac{-3.755\times10^{13}}{EI}$$

$$y_{C2} = \frac{-5wL^3}{384EI} = \frac{-5(128\times10^3)(3200)^3}{384EI} = \frac{-5.461\times10^{13}}{EI}$$

$$y_{C3} = \frac{+PL^3}{48EI} = \frac{R_C(3200)^3}{48EI} = \frac{R_C(6.827\times10^8)}{EI}$$

THEN $y_{C1} + y_{C2} + y_{C3} = 0$

$$\frac{-3.755\times10^{13}}{EI} - \frac{5.461\times10^{13}}{EI} + \frac{R_C(6.827\times10^8)}{EI} = 0$$

$$R_C = \frac{+3.755\times10^{13} + 5.461\times10^{13}}{6.827\times10^8} = \underline{135\text{ kN} = R_C}$$

$\Sigma M_A = 0 = 80(.8) + 128(1.6) - 135(1.6) - R_D(3.2)$

$\underline{R_D = 16.5\text{ kN}}$

$\Sigma M_D = 0 = 80(2.4) + 128(1.6) - 135(1.6) - R_A(3.2)$

$\underline{R_A = 56.5\text{ kN}}$

9-69

FIG. P9-69

$W = w\cdot L = (40\text{ kN/m})(3.6\text{ m}) = 144\text{ kN}$

$$y_{C1} = \frac{-WL^3}{8EI} = \frac{-144(3600)^3}{8EI} = -8.398\times10^{11}/EI \quad [A24-(c)]$$

$$y_{C2} = \frac{-Pa^2}{6EI}(3L-a) = \frac{-60(2000)^2}{6EI}\left[3(3600) - 2000\right] \quad [A24-(b)]$$

$$y_{C2} = 3.52\times10^{11}/EI$$

$$y_{C3} = \frac{+R_C(3600)^3}{3EI} = R_C(1.555\times10^{10})/EI \quad [A24-(a)]$$

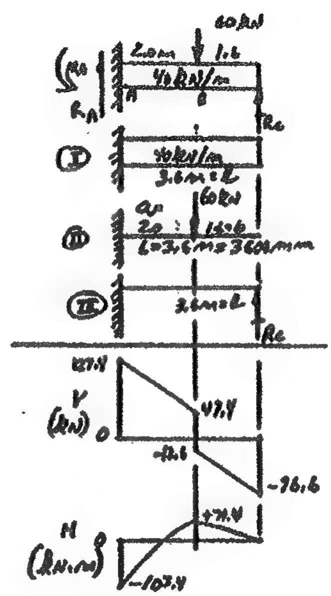
$y_{C1} + y_{C2} + y_{C3} = 0$

$-8.398\times10^{11}/EI - 3.52\times10^{11}/EI + R_C(1.555\times10^{10})/EI = 0$

$\underline{R_C = 76.6\text{ kN}}$

$R_A = 60\text{ kN} + 144\text{ kN} - 76.6\text{ kN} = \underline{127.4\text{ kN} = R_A}$

$M_A = 60(2.0) + 144(1.8) - R_C(3.6) = 103.44\text{ kN·m}$ (NEGATIVE)



9-70

$$y_{D1} = \frac{-wL^3}{8EI} = \frac{-4000(120)^3}{8EI} = -8.64\times10^8/EI \quad [A24\text{-}(c)]$$

$$y_{D2} = \frac{-Pa^2}{6EI}[3(L)-a] = \frac{-1200(36)^2}{6EI}[3(120)-36] \quad [A24\text{-}(b)]$$

$$y_{D2} = 8.398\times10^7/EI$$

$$y_{D3} = \frac{-800(72)^2}{6EI}[3(120)-72] = 1.991\times10^8/EI \quad [A24\,(b)]$$

$$y_{D4} = \frac{+R_D(120)^3}{3EI} = 5.76\times10^5(R_D)/EI \quad [A24\text{-}(a)]$$

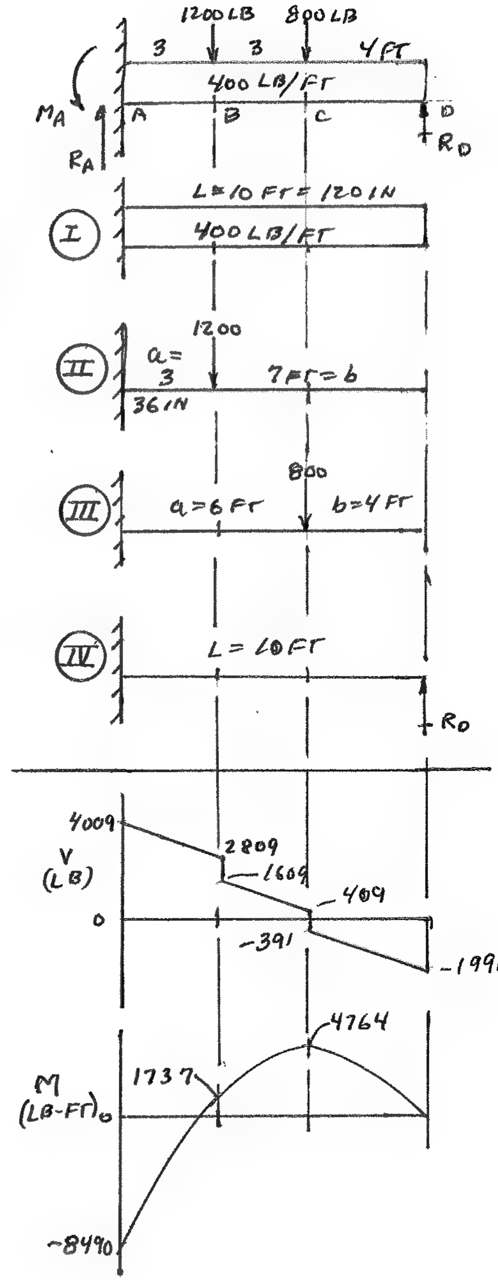
$$y_{D1} + y_{D2} + y_{D3} + y_{D4} = 0$$

$$\frac{-8.64\times10^8}{EI} - \frac{8.398\times10^7}{EI} - \frac{1.991\times10^8}{EI} + \frac{R_D(5.76\times10^5)}{EI} = 0$$

$\underline{R_D = 1991\text{ lb}}$

$R_A = 1200+800+4000-1991 = \underline{4009\text{ lb}} = R_A$

$M_A = 1200(3)+800(6)+4000(5)-1991(10) = 8490\text{ lb}\cdot\text{ft}$ (NEGATIVE)



Design Problems – Stress and Deflection Limits

9-71

APPENDIX A-25-(f)   APP. A-19 (WOOD)

V_{MAX} AND M_{MAX} OCCUR AT SECOND SUPPORT

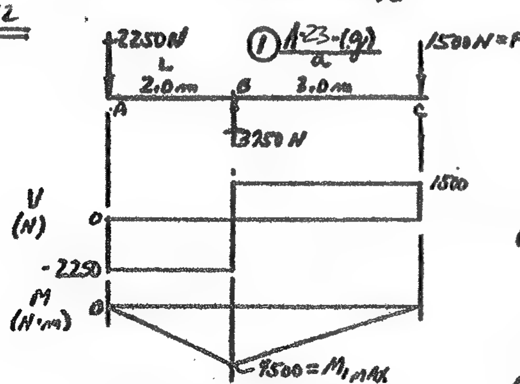
$V_{MAX} = 0.607\,wL = 0.607(100\,LB/FT)(2\,FT) = 121.4\,LB$

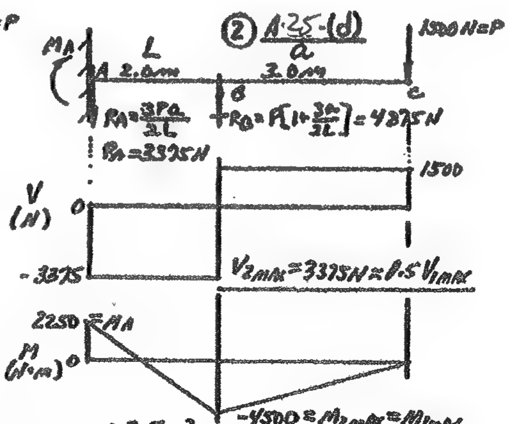
$\tau_{MAX} = \frac{3V}{2A} = \frac{3(121.4\,LB)}{2(1.50)(5.50)\,IN^2} = 22.07\,PSI$: $\tau_{ALLOW} = 70\,PSI$ – OK

$M_{MAX} = 0.107\,wL^2 = 0.107(100\,LB/FT)(2.0\,FT)^2 = 42.8\,LB\cdot FT \times 12\,IN/FT = 513.6\,LB\cdot IN$

$\sigma = \frac{M}{S} = \frac{513.6\,LB\cdot IN}{(5.50)(1.50)^2/6} = 249\,PSI$: $\sigma_{ALLOW} = 1000\,PSI$ OK

9-72





$y_{C1} = \frac{-Pa^2}{3EI}(L+a) = \frac{-1500(3)^2}{3EI}(2+3)$

$y_{C1} = \frac{22\,500}{EI}$

$y_{C2} = \frac{-PL^3}{EI}\left[\frac{a^2}{4L^2} + \frac{a^3}{3L^3}\right]$

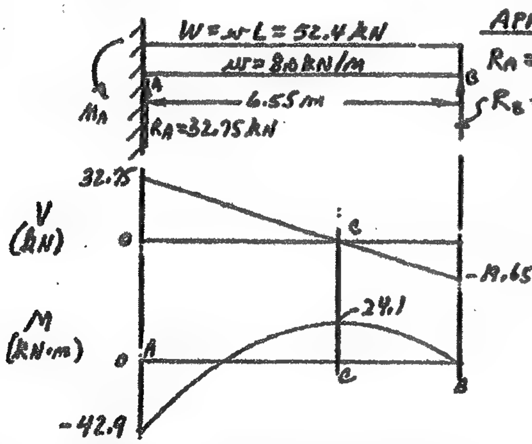
$y_{C2} = \frac{-1500(2)^3}{EI}\left[\frac{3^2}{4(2)^2} + \frac{3^3}{3(2)^3}\right] = \frac{20250}{EI} = 0.9\,y_{C1}$

NO MAJOR DIFFERENCE BETWEEN DESIGNS. ② IS 10% STIFFER. ② HAS 50% HIGHER SHEARING FORCE.

9-73

MULTIPLE DESIGNS POSSIBLE: USE V_{MAX} AND M_{MAX} FROM PROB. 9-72. SPECIFY MATERIAL, DESIGN STRESS, DESIGN FACTOR, SHAPE OF CROSS SECTION, DIMENSIONS. BECAUSE BENDING MOMENT IS EQUAL FOR BOTH DESIGNS, BOTH MAY BE THE SAME. BUT CHECK SHEAR.

9-74



APP. A-25-(c)

$R_A = \frac{5}{8}W = 32.75\,kN$

$R_B = \frac{3}{8}W = 19.65\,kN$

$M_A = 0.125\,WL = 0.125(52.4)(6.55)$

$M_A = -42.9\,kN\cdot m$

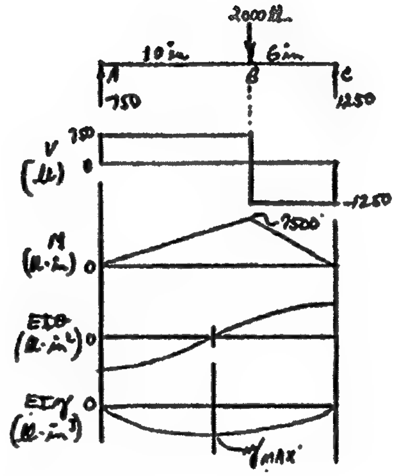
$M_C = 0.0703\,WL = 24.1\,kN\cdot m$

9-75

MULTIPLE DESIGNS POSSIBLE

Successive Integration Method

9-76



$I = bh^3/12 = 1(2)^3/12 = 0.667\ in^4$

$\boxed{M_{AB} = 750x}$

$M_{BC} = -1250x + C$

at $x = 16$, $M = 0$

$0 = -1250(16) + C \quad \therefore C = 20\,000$

$\boxed{M_{BC} = -1250x + 20\,000}$

$\boxed{EI\theta_{AB} = \int 750x\,dx = 375x^2 + C_1}$

$\boxed{EI\theta_{BC} = \int(-1250x + 20\,000)\,dx = -625x^2 + 20\,000x + C_2}$

$EIy_{AB} = \int(375x^2 + C_1)\,dx = \boxed{125x^3 + C_1x + C_3 = EIy_{AB}}$

$EIy_{BC} = \int(-625x^2 + 20\,000x + C_2)\,dx = \boxed{-208.3x^3 + 10\,000x^2 + C_2x + C_4 = EIy_{BC}}$

BOUNDARY CONDITIONS

① at $x = 0$, $EIy_{AB} = 0$ ③ at $x = 10$, $EI\theta_{AB} = EI\theta_{BC}$

② at $x = 16$, $EIy_{BC} = 0$ ④ at $x = 10$, $EIy_{AB} = EIy_{BC}$

SUBSTITUTING:

① $0 = 0 + 0 + C_3 \quad \therefore C_3 = 0$

② $EIy_{BC} = 0 = -208.3(16)^3 + 10\,000(16)^2 + 16C_2 + C_4 = 1.707\times10^6 + 16C_2 + C_4 = 0$

③ $375(10)^2 + C_1 = -625(10)^2 + 20\,000(10) + C_2$

$C_1 - C_2 = 1.0\times10^5$

④ $125(10)^3 + 10C_1 + \cancel{C_3}^{\,0} = -208.3(10)^3 + 10\,000(10)^2 + 10C_2 + C_4$

$10C_1 - 10C_2 - C_4 = 6.667\times10^5$

IN ② AND ③ SOLVE FOR C_1 AND C_4 IN TERMS OF C_2

$C_1 = C_2 + 1.0\times10^5$

$C_4 = -16C_2 - 1.707\times10^6$

SUBSTITUTE INTO ④

$10[C_2 + 1.0\times10^5] - 10C_2 - (-16C_2 - 1.707\times10^6) = 6.667\times10^5$

$\cancel{10C_2} + 1.0\times10^6 \cancel{-10C_2} + 16C_2 + 1.707\times10^6 = 6.667\times10^5$

$C_2 = -2.040\times10^6/16 = -1.275\times10^5 = \underline{-127\,500 = C_2}$

$C_1 = C_2 + 1.0\times10^5 = -1.275\times10^5 + 1.0\times10^5 = -0.1275\times10^5 = \underline{-27\,500 = C_1}$

$C_4 = -16C_2 - 1.707\times10^6 = -16[-1.275\times10^5] - 1.707\times10^6 = \underline{333\,000 = C_4}$

9-76 CONTINUED

FINAL EQUATIONS

$$EI\theta_{AB} = 375x^2 - 27500$$

$$EI\theta_{BC} = -625x^2 + 20000x - 127500$$

$$EIy_{AB} = 125x^3 - 27500x$$

$$EIy_{BC} = -208.3x^3 + 10000x^2 - 127500x + 333\,000$$

MAX y OCCURS WHERE $EI\theta = 0$

SET $EI\theta_{AB} = 0 = 375x^2 - 27500$

$$x = \sqrt{27500/375} = 8.56\text{ in}$$

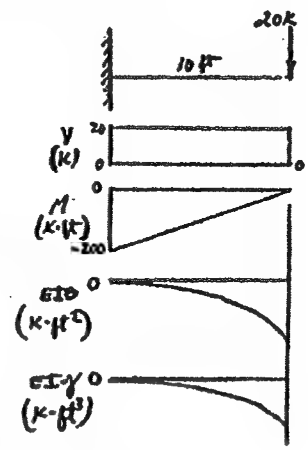
$$\text{MAX } EIy = (EIy_{AB})_{x=8.56} = 125(8.56)^3 - 27500(8.56) = -156\,997\text{ lb}\cdot\text{in}^3$$

$$\text{MAX } y = \frac{(EIy)_{MAX}}{EI} = \frac{-156997\text{ lb}\cdot\text{in}^3}{(30\times10^6)(0.667)(\text{lb/in}^2)(\text{in}^4)} = \underline{-0.0078\text{ in at } x = 8.56\text{ in}}$$

CHECK: $\sigma = \frac{Mc}{I} = \frac{(7500\text{ LB}\cdot\text{IN})(1.0\text{ IN.})}{0.667\text{ IN}^4} = 11244$ PSI OK FOR STEEL

9-77

$I = 890\text{ in}^4$ <u>W18X55</u>



$M = 20x - 200$

$EI\theta = \int(20x - 200)\,dx = 10x^2 - 200x + C_1$

at $x = 0$, $EI\theta = 0$ ∴ $C_1 = 0$

$EIy = \int(10x^2 - 200x)\,dx = 3.333x^3 - 100x^2 + C_2$

at $x = 0$, $EIy = 0$ ∴ $C_2 = 0$

THEN
$$\boxed{EIy = 3.333x^3 - 100x^2}$$

at $x = 10$ ft

$$EIy_{MAX} = 3.333(10^3) - 100(10)^2 = -6667\text{ K}\cdot\text{ft}^3$$

$$y_{MAX} = \frac{-6667\text{ K}\cdot\text{ft}^3}{(30\times10^6\text{ lb/in}^2)(890\text{ in}^4)} \times \frac{1000\text{ lb}}{\text{K}} \times \frac{1728\text{ in}^3}{\text{ft}^3} = \underline{-0.431\text{ in}}$$

CHECK: $\sigma = \frac{M}{S} = \frac{(200\text{ K}\cdot\text{FT})(1000\text{ LB/K})(12\text{ IN/FT})}{98.3\text{ IN}^3} = 24415$ PSI

$\sigma < S_y$
BUT HIGH FOR A36
STRUCTURAL STEEL.

9-78

$V = -4x + 4$ | $I = 1.530\ in^4 \times 4.162\times10^5 \frac{mm^4}{in^4} = 6.368\times10^5\ mm^4$

$M = -2x^2 + 4x + C_1 = -2x^2 + 4x - 2$

at $x = 1.0$, $M = 0$

$0 = -2 + 4 + C_1 \ \therefore\ C_1 = -2$

$EI\theta = \int(-2x^2+4x-2)dx = \frac{-2x^3}{3} + 2x^2 - 2x + \cancel{C_2}^{\,0}$

at $x=0$, $EI\theta = 0\ \therefore\ C_2 = 0$

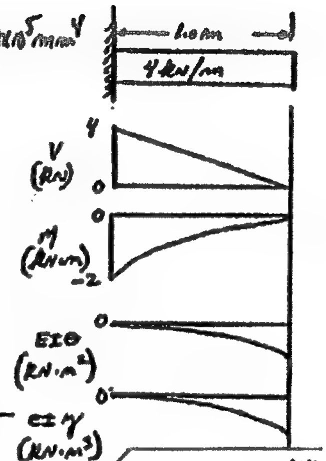
$$\boxed{EIy = \int EI\theta\, dx = -\frac{1}{6}x^4 + \frac{2x^3}{3} - x^2 + \cancel{C_3}^{\,0}}$$

at $x = 1.0\ m$

$EIy = -1/6 + 2/3 - 1 = -0.50\ kN\cdot m^3$

$$y = \frac{-0.50\ kN\cdot m^3\ (10^3\ N/kN)(10^{15}\ mm^5/m^5)}{(207\times10^9\ N/m^2)(6.368\times10^5\ mm^4)} = \underline{-3.79\ mm}$$

CHECK: $\sigma = M/S = 115\ MPa$ - OK.



9-79

W 24X26

$I = 2100\ in^4$

SHEAR: $V = -4x + 60$

MOMENT: $M = \int V dx = -2x^2 + 60x + C_1$

at $x = 0$, $M = -400\ k\text{-}ft\ \therefore\ C_1 = -400$

$M = -2x^2 + 60x - 400$

$EI\theta = \int M dx = \frac{-2x^3}{3} + 30x^2 - 400x + \cancel{C_2}^{\,0}$

at $x=0$, $EI\theta = 0\ \therefore\ C_2 = 0$

$$\boxed{EIy = \int EI\theta\, dx = \frac{-x^4}{6} + 10x^3 - 200x^2 + \cancel{C_3}^{\,0}}$$

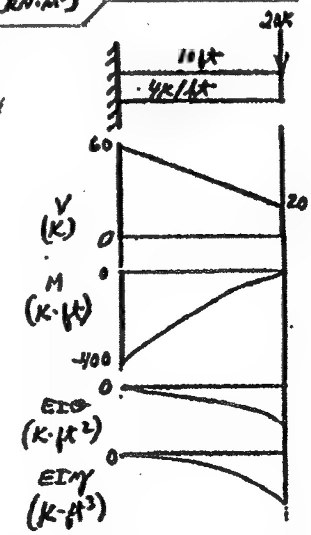
at $x=0$, $EIy = 0\ \therefore\ C_3 = 0$

at $x = 10\ ft$,

$EIy_{MAX} = \frac{-10^4}{6} + 10(10^3) - 20000 = -11667\ k\text{-}ft^3$

$$y_{max} = \frac{-11667\ k\text{-}ft^3\ (1000\ lb/k)}{(30\times10^6)\ lb/in^2\ (2100\ in^4)} \times \frac{(12\ in)^3}{ft^3} = \underline{-.320\ IN}$$

CHECK: $\sigma = \frac{M}{S} = 27273\ psi$ - HIGH



9-80

$V = -4x + 6$

$M = \int V dx = -2x^2 + 6x + C_1$

at $x=0$, $M = -4\ \therefore\ C_1 = -4$

$M = -2x^2 + 6x - 4$

$EI\theta = \int M dx = \frac{-2x^3}{3} + 3x^2 - 4x + \cancel{C_2}^{\,0}$

at $x=0$, $EI\theta = 0\ \therefore\ C_2 = 0$

$$\boxed{EIy = \int EI\theta\, dx = \frac{-x^4}{6} + x^3 - 2x^2 + \cancel{C_3}^{\,0}}$$

at $x=0$, $EIy = 0\ \therefore\ C_3 = 0$

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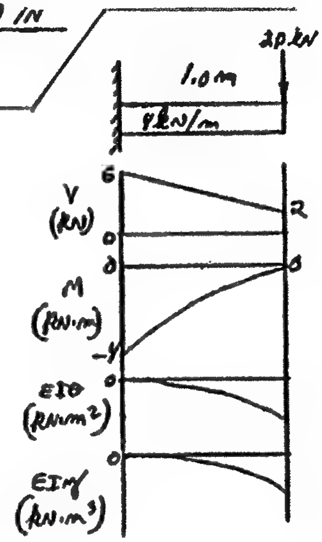


9-80 **CONTINUED**

at $x = 1.0\,m$,

$$EIy_{MAX} = -\tfrac{1}{6} + 1 - 2 = -1.167\ kN\cdot m^3$$

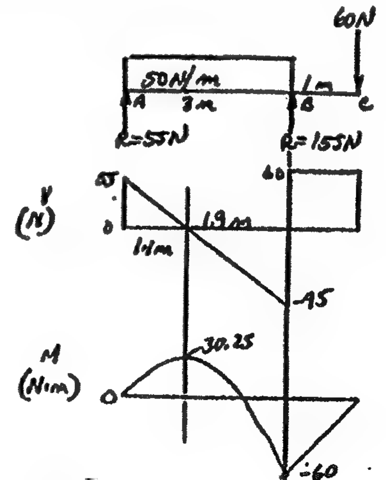
FOR $y_{MAX} = 5.0\,mm$

$$\text{RCQ'D } I = \frac{-1.167\ kN\cdot m^3}{E\,y_{MAX}} = \frac{+1.167\times10^3\ N\cdot m^3}{(207\times10^9\ N/m^2)(5.0\,mm)}\times\frac{(10^3\,mm)^5}{m^5} = +1.127\times10^6\,mm^4$$

$$I = \pi D^4/64 \ \therefore\ D = \sqrt[4]{64\,I/\pi} = \left[64(1.127\times10^6)/\pi\right]^{1/4} = \underline{69.2\,mm}$$

$$\text{CHECK: } \sigma = \frac{Mc}{I} = \frac{(4.0\,kN\cdot m)(34.6\,mm)}{1.127\times10^6\,mm^4}\cdot\frac{10^6\,N\cdot mm}{kN\cdot m} = 122.8\,MPa \text{ - OK FOR STEEL}$$

9-81



$V_{AB} = -50x + 55$

$\underline{V_{BC} = 60}$

$M_{AB} = \int V_{AB}\,dx = -25x^2 + 55x + \cancel{C}^{\,0}$

$M_{BC} = \int V_{BC}\,dx = 60x + C$

at $x = 4, M = 0$

$0 = 60(4) + C = 240 + C \ \therefore C = -240$

$\underline{M_{BC} = 60x - 240}$

$EI\theta_{AB} = \int M_{AB}\,dx = -(25/3)x^3 + 27.5x^2 + C_1$

$EI\theta_{BC} = \int M_{BC}\,dx = 30x^2 - 240x + C_2$

$EIy_{AB} = \int EI\theta_{AB}\,dx = -\left(\frac{25}{12}\right)x^4 + \left(\frac{55}{6}\right)x^3 + C_1x + \cancel{C_3}^{\,0}$

$\underline{EIy_{BC} = \int EI\theta_{BC}\,dx = 10x^3 - 120x^2 + C_2x + C_4}$

BOUNDARY CONDITIONS

① @ $x = 0$, $EIy_{AB} = 0$ ③ at $x = 3$, $EIy_{BC} = 0$

② @ $x = 3$, $EIy_{AB} = 0$ ④ at $x = 3$, $EI\theta_{AB} = EI\theta_{BC}$

SUBSTITUTING:

① $\boxed{C_3 = 0}$

③ $0 = 10(3)^3 - 120(3)^2 + 3C_2 + C_4$

$3C_2 + C_4 = 810$

② $0 = -(25/12)(3)^4 + (55/6)(3)^3 + 3C_1$

$C_1 = -78.75/3 = \boxed{-26.25 = C_1}$

④ $-(25/3)(3)^3 + 27.5(3)^2 - 26.25 = 30(3)^2 - 240(3) + C_2$

$\boxed{C_2 = 446.25}$

FROM ③: $C_4 = 810 - 3C_2 = 810 - 3(446.25) = \boxed{-528.75 = C_4}$

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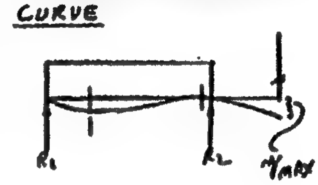
9-81 CONTINUED

FINAL EQUATIONS

$$\boxed{\begin{aligned} EI\theta_{AB} &= -(25/3)x^3 + 27.5x^2 - 26.25 \\ EI\theta_{BC} &= 30x^2 - 240x + 446.25 \\ EIy_{AB} &= -(25/12)x^4 + 55/6\,x^3 - 26.25x \\ EIy_{BC} &= 10x^3 - 120x^2 + 446.25 - 528.75 \end{aligned}}$$

SHAPE OF DEFLECTION CURVE



y_{MAX} OCCURS WHERE $EI\theta = 0$ OR AT RIGHT END OF OVERHANG.

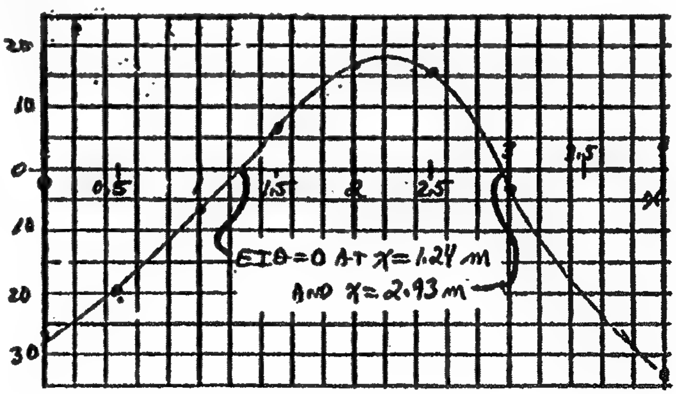
EVALUATE $EI\theta_{AB}$ AT SEVERAL POINTS + USE SOLVER.

$x=$	$EI\theta=$
0	−26.25
0.5	−20.417
1.0	−7.083
1.5	7.50
2.0	17.083
2.5	15.417
3.0	−3.75

ROOTS FOR $EI\theta_{AB}$:

$x_1 = 1.2351\ m$

$x_2 = 2.9341\ m$



EVALUATE EIy AT $x = 1.24\ m$, $x = 2.93\ m$, AND $x = 4.0\ m$.

$$\left[EIy_{AB}\right]_{x=1.24m} = -2.083(1.24)^4 + 9.167(1.24)^3 - 26.26(1.24) = -20.0\ N\cdot m^3$$

$$\left[EIy_{AB}\right]_{x=2.93m} = -2.083(2.93)^4 + 9.167(2.93)^3 - 26.26(2.93) = +0.121\ N\cdot m^3$$

$$\left[EIy_{BC}\right]_{x=4} = 10(4)^3 - 120(4)^2 + 446.25 - 528.7 = -23.75\ N\cdot m^3 \ \underline{MAX}$$

$$\text{REQ'D } I = \frac{-23.75\ N\cdot m^3}{E\,y_{MAX}} = \frac{-23.75\ N\cdot m^3}{(207\times10^9\ N/m^2)(-1\ mm)} \times \frac{(10^3\ mm)^5}{m^5} = \underline{1.15\times10^5\ mm^4}$$

CONVERT $I = 1.15\times10^5\ mm^4 \times 2.403\times10^{-6}\ in^4/mm^4 = \underline{0.276\ in^4}$

POSSIBLE BEAM DESIGNS:

	METRIC
1. $1\frac{1}{2}$ IN. SCH 40 PIPE : $I = 0.3099\ IN^4$	PIPE 38 STD
2. C3×6 CHANNEL : $I_y = 0.300\ IN^4$	C80 × 8.9
3. 2×2×1/4 HOLLOW STEEL TUBE : $I = 0.747\ IN^4$	HSS 51×51×6.4

CHECK: $\sigma = M/S$ – OK FOR ALL DESIGNS.

4. MECHANICAL TUBING: 2.0 IN O.D × 0.134 IN WALL $I = 0.344\ IN^4$

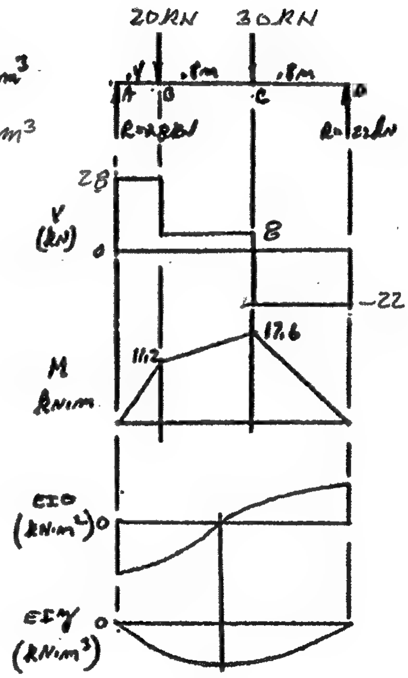
METRIC: 50.8 mm OD × 3.404 mm WALL; $I = 1.43\times10^5\ mm^4$

9-82

SELECT ALUMINUM I-BEAM ; $\sigma_a = 120\,MPa$

$$S = \frac{M}{\sigma_a} = \frac{17.6\times10^3\,N\cdot m}{120\,N/mm^2}\times\frac{10^3\,mm}{1m} = 1.467\times10^5\,mm^3$$

USE $I\,178\times 8.630$; $S = 2.01\times10^5\,mm^3$
$I = 1.79\times10^7\,mm^4$
SAME AS $I\,7\times5.800$



MOMENT –

$M_{AB} = 28X$

$M_{BC} = 8X + C$ BUT AT $X = .4$, $M = 11.2$

$11.2 = 8(.4) + C$

$C = 11.2 - 3.2 = 8.0$

$M_{BC} = 8X + 8$

$M_{CD} = -22X + C$ BUT AT $X = 2.0$, $M = 0$

$0 = -22(2.0) + C$

$C = 44$

$M_{CD} = -22X + 44$

SLOPE –

$EI\theta_{AB} = \int M_{AB}\,dx = 14X^2 + C_1$

$EI\theta_{BC} = \int M_{BC}\,dx = 4X^2 + 8X + C_2$

$EI\theta_{CD} = \int M_{CD}\,dx = -11X^2 + 44X + C_3$

DEFLECTION –

$EIy_{AB} = \int EI\theta_{AB}\,dx = (14/3)X^3 + C_1X + C_4$

$EIy_{BC} = \int EI\theta_{BC}\,dx = (4/3)X^3 + 4X^2 + C_2X + C_5$

$EIy_{CD} = \int EI\theta_{CD}\,dx = -(11/3)X^3 + 22X^2 + C_3X + C_6$

BOUNDARY CONDITIONS

① at $X=0$, $EIy_{AB} = 0$ ③ at $X=0.4$, $EI\theta_{AB} = EI\theta_{BC}$ ⑤ at $X=1.2$, $EI\theta_{BC} = EI\theta_{CD}$

② at $X=2.0$, $EIy_{CD} = 0$ ④ at $X=0.4$, $EIy_{AB} = EIy_{BC}$ ⑥ at $X=1.2$, $EIy_{BC} = EIy_{CD}$

SUBSTITUTING

① $0 = C_4$

② $EIy_{CD} = 0 = -(11/3)(2)^3 + 22(2)^2 + 2C_3 + C_6$

$2C_3 + C_6 = -58.66$

(CONTINUED NEXT PAGE)

9-82 CONTINUED

(3) $14(.4)^2+C_1 = 4(0.4)^2+8(0.4)+C_2$

$C_1 - C_2 = 1.60$

(4) $(14/3)(0.4)^3 + 0.4C_1 + \cancel{C_4}^{0} = (4/3)(0.4)^3 + 4(0.4)^2 + 0.4C_2 + C_5$

$0.4C_1 - 0.4C_2 - C_5 = 0.4266$

(5) $4(1.2)^2 + 1.2(8) + C_2 = -11(1.2)^2 + 44(1.2) + C_3$

$C_2 - C_3 = 21.6$

(6) $(4/3)(1.2)^3 + 4(1.2)^2 + 1.2C_2 + C_5 = -(11/3)(1.2)^3 + 22(1.2)^2 + 1.2C_3 + C_6$

$1.2C_2 + C_5 - 1.2C_3 - C_6 = 17.28$

SOLVE SIMULTANEOUSLY

$C_1 = -10.56$	$C_4 = 0$
$C_2 = -12.16$	$C_5 = 0.21\overline{33}$
$C_3 = -33.76$	$C_6 = 8.85\overline{33}$

FINAL EQUATIONS

$EI\theta_{AB} = 14x^2 - 10.56$	$EIy_{AB} = 4.667x^3 - 10.56x$
$EI\theta_{BC} = 4x^2 + 8x - 12.16$	$EIy_{BC} = (4/3)x^3 + 4x^2 - 12.16x + 0.21\overline{33}$
$EI\theta_{CD} = -11x^2 + 44x - 33.76$	$EIy_{CD} = -(11/3)x^3 + 22x^2 - 33.76x + 8.85\overline{3}$

SET $EI\theta_{BC} = 0 = 4x^2 + 8x - 12.16 = x^2 + 2x - 3.04$

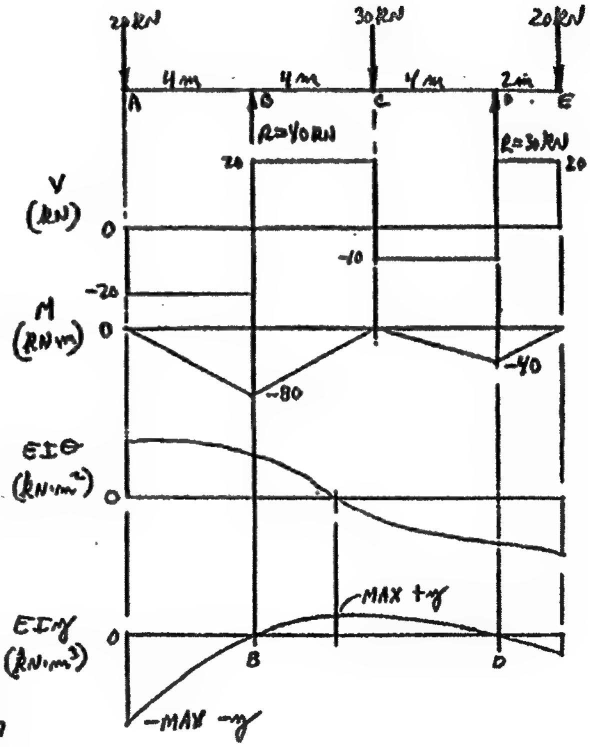
$x = \frac{-2}{2} \pm \frac{1}{2}\sqrt{4 - 4(-3.04)} = -1 \pm 2.01 = +1.01\,m$ — VALID POINT

THEN MAX y OCCURS AT $x = 1.01\,m$

$[EIy_{BC}]_{x=1.01} = (4/3)(1.01)^3 + 4(1.01)^2 - 12.16(1.01) + 0.213 = -6.615\,kN\cdot m^3$

$y_{MAX} = \frac{-6.615\,kN\cdot m^3}{EI} = \frac{-6.615 \times 10^3\,N\cdot m^3}{(69 \times 10^9\,N/m^2)(1.79 \times 10^7\,mm^4)} \frac{(10^3\,mm)^5}{m^5} = \underline{-5.37\,mm}$

9-83



$M_{AB} = -20x$

$M_{BC} = 20x - 160$

$M_{CD} = -10x + 80$

$M_{DE} = 20x - 280$

$EI\theta_{AB} = -10x^2 + C_1$

$EI\theta_{BC} = 10x^2 - 160x + C_2$

$EI\theta_{CD} = -5x^2 + 80x + C_3$

$EI\theta_{DE} = 10x^2 - 280x + C_4$

$EIy_{AB} = -(10/3)x^3 + C_1x + C_5$

$EIy_{BC} = (10/3)x^3 - 80x^2 + C_2x + C_6$

$EIy_{CD} = -(5/3)x^3 + 40x^2 + C_3x + C_7$

$EIy_{DE} = (10/3)x^3 - 140x^2 + C_4x + C_8$

BOUNDARY AND CONTINUITY CONDITIONS

① at $x=4$; $EIy_{AB}=0$ ⑤ at $x=4$, $EI\theta_{AB} = EI\theta_{BC}$

② at $x=4$, $EIy_{BC}=0$ ⑥ at $x=8$, $EI\theta_{BC} = EI\theta_{CD}$

③ at $x=12$, $EIy_{CD}=0$ ⑦ at $x=8$, $EIy_{BC} = EIy_{CD}$

④ at $x=12$, $EIy_{DE}=0$ ⑧ at $x=12$, $EI\theta_{CD} = EI\theta_{DE}$

CONSTANTS OF INTEGRATION FROM SIMULTANEOUS SOLUTION OF ① THRU ⑧

$C_1 = 306.\overline{66}$ $C_5 = -1013.\overline{33}$

$C_2 = 626.\overline{66}$ $C_6 = -1440$

$C_3 = -333.\overline{33}$ $C_7 = 1120$

$C_4 = 1826.\overline{66}$ $C_8 = -7520$

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9-83 CONTINUED

FINAL EQUATIONS

$EI\theta_{AB} = -10x^2 + 306.\overline{66}$	$EIy_{AB} = -(10/3)x^3 + 30\overline{6}x - 1013.\overline{33}$
$EI\theta_{BC} = 10x^2 - 160x + 626.\overline{66}$	$EIy_{BC} = (10/3)x^3 - 80x^2 + 62\overline{6}x - 1440$
$EI\theta_{CD} = -5x^2 + 80x - 333.\overline{33}$	$EIy_{CD} = -(5/3)x^3 + 40x^2 - 33\overline{3}x + 1120$
$EI\theta_{DE} = 10x^2 - 280x + 1826.\overline{66}$	$EIy_{DE} = (10/3)x^3 - 140x^2 + 182\overline{6}x - 7540$

SET $EI\theta_{BC} = 0 = 10x^2 - 160x + 626 = x^2 - 16x + 62.6$

$$x = \frac{16 \pm \sqrt{16^2 - 4(62.6)}}{2} = 8 \pm \tfrac{1}{2}(2.37) = 9.16 \text{ OR } \boxed{6.84\,m}$$

INVALID - NOT IN SEGMENT BC

MAX y AT $x = 0$ OR $x = 6.84\,m$ OR $x = 14\,m$

at $x=0$; $EIy_{AB} = -101\overline{3}\,kN\cdot m^3$ MAX NEG. y

at $x = 6.84\,m$; $EIy_{BC} = +170\,kN\cdot m^3$ MAX POS. y

at $x = 14$; $EIy_{DE} = -240\,kN\cdot m^3$

$E = 207 \times 10^9\,N/m^2$ W360X39 STEEL BEAM

$I = 1.02 \times 10^8\,mm^4$ APP. A7(SI) $S = 5.79 \times 10^5\,mm^3$

$$\text{at } x=0;\ y = \frac{-1013 \times 10^3\,N\cdot m^3\ (10^{15}\,mm^5/m^5)}{(207\times10^9\,N/m^2)(1.020\times10^8\,mm^4)} = -48.0\,mm$$

$$\text{at } x = 6.84\,m;\ y = -48.0 \times \frac{170}{-1013} = +8.07\,mm$$

$$\text{at } x = 14\,m;\ y = -48.0\,mm \times \frac{-240}{-1013} = -11.37\,mm$$

CHECK BENDING STRESS $\sigma = M/S$, $M_{MAX} = 80\,kN\cdot m$

$$\sigma = \frac{80\,kN\cdot m}{5.79\times10^5\,mm^3} \times \frac{10^3\,N}{kN} \times \frac{10^3\,mm}{m} = 138\,MPa$$

ASTM A992 STRUCTURAL STEEL $S_y = 345\,MPa$

$\sigma_d = 0.66\,S_y = 0.66(345) = 228\,MPa$ OK

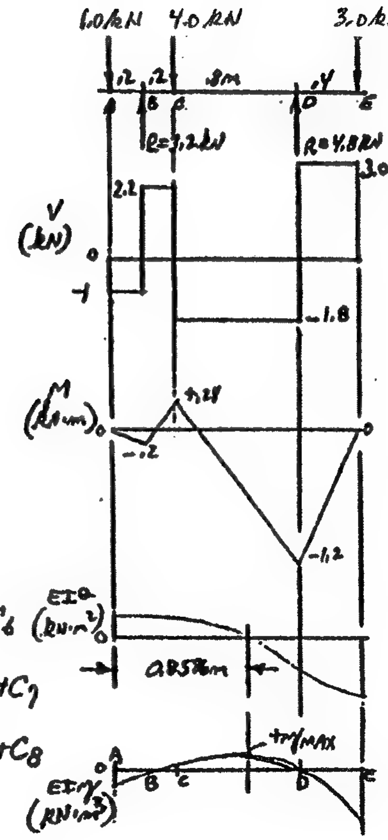
9-84

MOMENT: $M_{AB} = -x$

$M_{BC} = 2.2x - 0.64$

$M_{CD} = -1.8x + 0.96$

$M_{DE} = 3.0x - 4.8$



$EI\theta_{AB} = \int M_{AB}\,dx = -0.5x^2 + C_1$

$EI\theta_{BC} = \int M_{BC}\,dx = 1.1x^2 - 0.64x + C_2$

$EI\theta_{CD} = \int M_{CD}\,dx = -0.9x^2 + 0.96x + C_3$

$EI\theta_{DE} = \int M_{DE}\,dx = 1.5x^2 - 4.8x + C_4$

$EIy_{AB} = \int EI\theta_{AB}\,dx = -0.16\bar{6}x^3 + C_1x + C_5$

$EIy_{BC} = \int EI\theta_{BC}\,dx = 0.36\bar{6}x^3 - 0.32x^2 + C_2x + C_6$

$EIy_{CD} = \int EI\theta_{CD}\,dx = -0.3x^3 + 0.48x^2 + C_3x + C_7$

$EIy_{DE} = \int EI\theta_{DE}\,dx = 0.5x^3 - 2.4x^2 + C_4x + C_8$

BOUNDARY CONDITIONS

① at $x = 0.2$, $EIy_{AB} = 0$
② at $x = 0.2$, $EIy_{BC} = 0$
③ at $x = 1.2$, $EIy_{CD} = 0$
④ at $x = 1.2$, $EIy_{DE} = 0$
⑤ at $x = 0.2$, $EI\theta_{AB} = EI\theta_{BC}$
⑥ at $x = 0.4$, $EI\theta_{BC} = EI\theta_{CD}$
⑦ at $x = 1.2$, $EI\theta_{CD} = EI\theta_{DE}$
⑧ at $x = 0.4$, $EIy_{BC} = EIy_{CD}$

CONSTANTS - FROM SIMULTANEOUS SOLUTION OF ① THROUGH ⑧

$C_1 = 0.490\bar{6}$ $\quad C_5 = -0.0968$

$C_2 = 0.158\bar{6}$ $\quad C_6 = -0.0218\bar{6}$

$C_3 = -0.161\bar{3}$ $\quad C_7 = 0.0208$

$C_4 = 3.294\bar{6}$ $\quad C_8 = -1.316$

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9-84 CONTINUED

FINAL EQUATIONS

$EI\theta_{AB} = -0.5x^2 + .409\bar{6}$	$EIy_{AB} = -0.1\bar{6}\bar{6}x^3 + .490\bar{6}x - 0.0968$
$EI\theta_{BC} = 1.1x^2 - 0.64x + 0.158\bar{6}$	$EIy_{BC} = 0.36\bar{6}x^3 - 0.32x^2 + 0.158\bar{6}x - 0.02$
$EI\theta_{CD} = -0.9x^2 + 0.96x - 0.161\bar{3}$	$EIy_{CD} = -0.3x^3 + 0.48x^2 - 0.161x + 0.0208$
$EI\theta_{DE} = 1.5x^2 - 4.8x + 3.294\bar{6}$	$EIy_{DE} = 0.5x^3 - 2.4x^2 + 3.294x - 1.3616$

SET $EI\theta_{CD} = 0 = -0.9x^2 + 0.96x - 0.161\bar{3} = x^2 - 1.06\bar{6}x + 0.1792$

$$x = \frac{1.06 \pm \sqrt{1.06\bar{6}^2 - (4)(0.1792)}}{2} = 0.5335 \pm 0.325 = \underline{0.8577\,m} \text{ or } 0.208\,m$$

OUTSIDE CD

DEFLECTION AT $x = 0.8577\,m$ IN SEGMENT CD

$$EIy_{CD} = -0.3(0.8586)^3 + 0.48(0.8586)^2 - 0.161(0.8586) + 0.0208$$
$$= 0.0462\,kN \cdot m^3$$

AT $x=0$

$$EIy_{AB} = -0.16\bar{6}(0) + 0.4906(0) - 0.0968 = -0.0968\,kN \cdot m^3$$

AT $x = 1.6\,m$

$$EIy_{DE} = 0.5(1.6)^3 - 2.4(1.6)^2 + 3.295(1.6) - 1.362 = -0.1862\,kN \cdot m^3$$

MAXIMUM

FOR $y_{MAX} = -0.13\,mm$; $E = 207 \times 10^3\,N/mm^2$

$$I = \frac{EIy}{Ey} = \frac{-0.186 \times 10^3\,N \cdot m^3}{(207 \times 10^3\,N/mm^2)(-0.13\,mm)} \times \frac{(10^3\,mm)^3}{m^3} = 6.91 \times 10^6\,mm^4$$

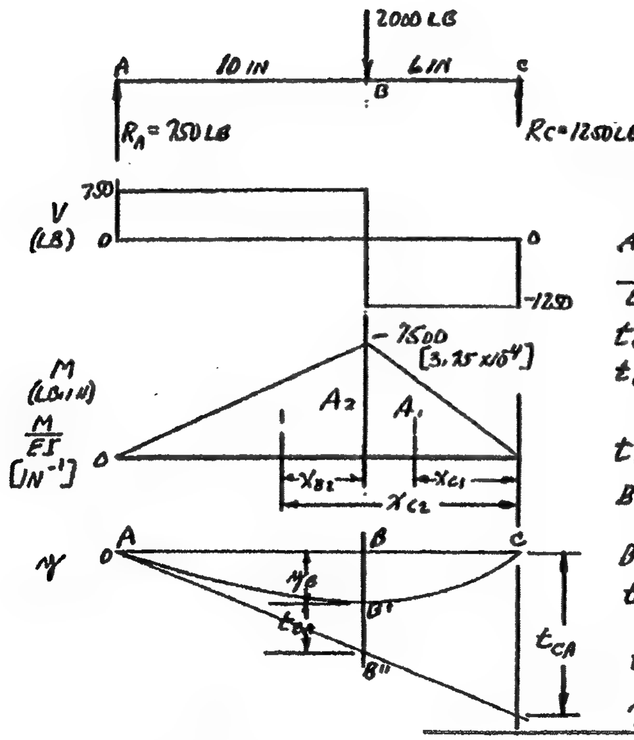
$$I = \pi D^4/64$$

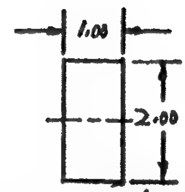
$$D = [64 I/\pi]^{1/4} = \left[\frac{64(6.91 \times 10^6)}{\pi}\right]^{1/4} = \underline{109\,mm}$$

CHECK: $\sigma = \frac{Mc}{I} = \frac{(1.2\,kN \cdot m)(109/2\,mm)}{6.91 \times 10^6\,mm^4} \times \frac{(10^3\,N)(10^3\,mm)}{kN \cdot m} = 9.46\,MPa$ - $\underline{OK}$

Moment-Area Method

9-85





$I = \frac{bh^3}{12} = \frac{1(2)^3}{12}$

$I = 0.666\ IN^4$

$EI = 30\times10^6 \frac{LB}{IN^2} \times 0.666\ IN^4$

$EI = 2.0\times10^7\ LB\cdot IN^2$

AT B:

$\frac{M}{EI} = \frac{7500\ LB\cdot IN}{2.0\times10^7\ LB\cdot IN^2} = 3.75\times10^{-4}\ IN^{-1}$

$t_{CA} = A_1 x_{C1} + A_2 x_{C2}$

$t_{CA} = (\frac{1}{2})(3.75\times10^{-4})(6)(4) + (\frac{1}{2})(3.75\times10^{-4})(10)(9.333)$

$t_{CA} = 0.022\ IN$

$BB'' = t_{CA}\cdot\frac{AB}{AC} = 0.022\cdot\frac{10}{16}$

$BB'' = 0.01375\ IN$

$t_{BA} = A_2 x_{B2}$

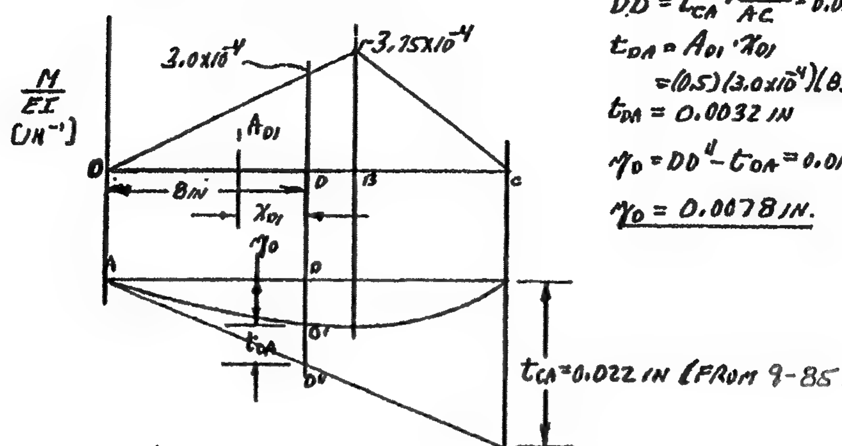
$= (.5)(3.75\times10^{-4})(10)(3.333)$

$t_{BA} = 0.00625\ IN$

$y_B = BB'' - t_{BA} = \underline{0.0075\ IN}$

9-86

SAME M/EI AS 9-85



$DD'' = t_{CA}\cdot\frac{AD}{AC} = 0.022\cdot\frac{8}{16} = 0.011\ IN$

$t_{DA} = A_{D1}\cdot x_{D1}$

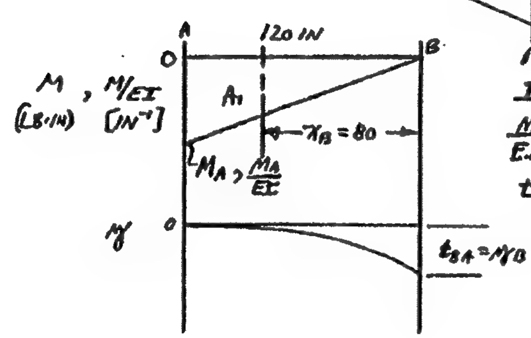
$= (0.5)(3.0\times10^{-4})(8)(2.667)$

$t_{DA} = 0.0032\ IN$

$y_D = DD'' - t_{DA} = 0.011 - 0.0032$

$\underline{y_D = 0.0078\ IN.}$

9-87



$M_A = 2.4\times10^{+6}\ LB\cdot IN$

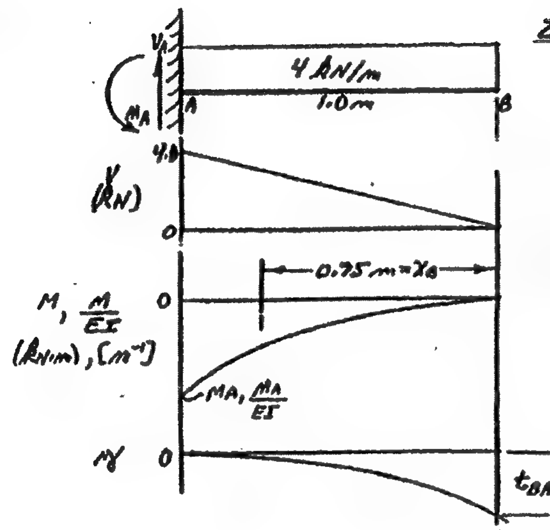
$I = 890\ IN^4$ (W18×55)

$\frac{M_A}{EI} = \frac{2.4\times10^{+6}\ LB\cdot IN}{(30\times10^6\ LB/IN^2)(890\ IN^4)} = 89.9\times10^{-6}\ IN^{-1}$

$t_{BA} = A_1 x_B = (0.5)(89.9\times10^{-6})(120)(80)$

$\underline{y_B = 0.432\ IN}$

9-88



2½ IN SCH 40 PIPE: PIPE 64 STD

$I = 1.530\,IN^4 \cdot \frac{(25.4\,mm)^4}{(IN)^4} \cdot \frac{1\,m^4}{(10^3\,mm)^4}$

$I = 6.37\times10^{-7}\,m^4 = 6.37\times10^{5}\,mm^4$

$M_A = 4.0\,kN \cdot 0.5\,m = 2.0\,kN\cdot m$

$\frac{M_A}{EI} = \frac{2.0\times10^3\,N\cdot m}{(207\times10^9\,N/m^2)(6.37\times10^{-7}\,m^4)}$

$\frac{M_A}{EI} = 0.0152\,m^{-1}$

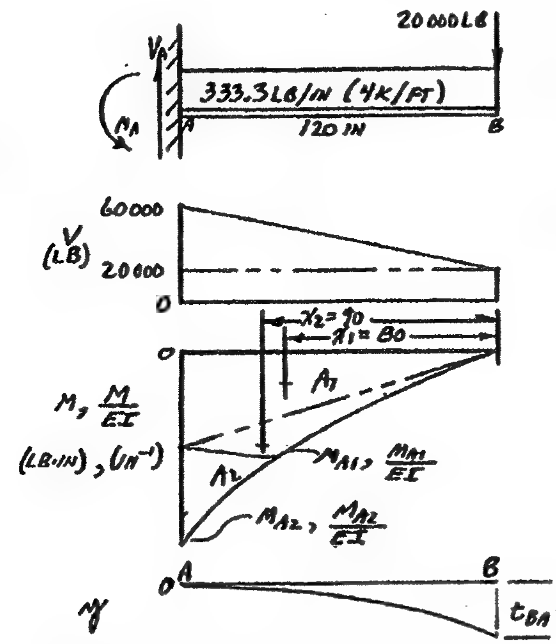
$t_{BA} = y_B = A_1 \cdot x_B$

$= (\frac{1}{3})(0.0152)(1.0)(0.75) =$

$t_{BA} = 0.00379\,m = 3.79\times10^{-3}\,m$

$\underline{y_B = 3.79\,mm}$

9-89



W24×76: $I = 2100\,IN^4$

$M_{A1} = (20000\,LB)(120\,IN) = 2.4\times10^6\,LB\cdot IN$

$M_{A2} = M_{A1} + (\frac{1}{2})(40000\,LB)(120\,IN)$

$M_{A2} = 2.4\times10^6 + 2.4\times10^6 = 4.8\times10^6\,LB\cdot IN$

$\frac{M_{A1}}{EI} = \frac{2.4\times10^6\,LB\cdot IN}{(30\times10^6\,LB/IN^2)(2100\,IN^4)} = 3.810\times10^{-5}\,IN^{-1}$

$\frac{M_{A2}}{EI} = 2\,\frac{M_{A1}}{EI} = 7.62\times10^{-5}\,IN^{-1}$

$t_{BA} = A_1 x_1 + A_2 x_2$

$= \frac{1}{2}(3.810\times10^{-5})(120)(80) +$

$\frac{1}{3}(3.810\times10^{-5})(120)(90)$

$t_{BA} = 0.1829 + 0.1371 = \underline{0.320\,IN = y_B}$

9-90

DIAGRAMS SAME FORM AS 9-89 ; DIA. OF BAR = 100 mm = 0.10 m

$I = \pi(0.1)^4/64 = 4.909\times10^{-6}\,m^4$; $EI = (69\times10^9\,N/m^2)(4.909\times10^{-6}\,m^4) = 3.39\times10^5\,N\cdot m^2$

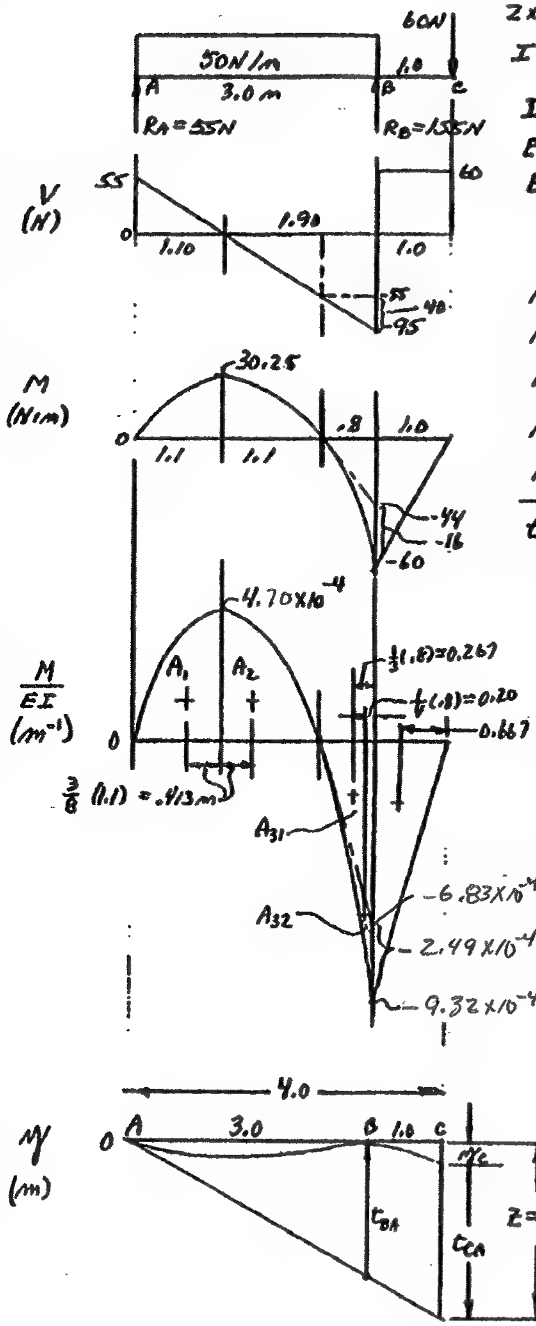
$M_{A1} = (2000\,N)(1.0\,m) = 2000\,N\cdot m$; $\frac{M_{A1}}{EI} = \frac{2000\,N\cdot m}{3.39\times10^5\,N\cdot m^2} = 5.91\times10^{-3}\,m^{-1}$

$M_{A2} = \frac{1}{2}(4000\,N)(1.0\,m) + M_{A1} = 4000\,N\cdot m$; $\frac{M_{A2}}{EI} = 1.18\times10^{-2}\,m^{-1}$

$t_{BA} = A_1 x_1 + A_2 x_2 = \frac{1}{2}(5.91\times10^{-3})(1.0)(0.667) + \frac{1}{3}(5.91\times10^{-3})(1.0)(0.75)$

$t_{BA} = 1.968\times10^{-3} + 1.476\times10^{-3} = 3.445\times10^{-3}\,m = \underline{3.445\,mm = y_B}$

9-91



HSS 51×51×6.4 SQ.TUBE $I = 3.11\times10^5\ mm^4$

2×2×¼ SQ.TUBE: $I = 0.747\ in^4$

$I = (0.747\ in^4)\times\frac{(25.4\ mm)^4}{1\ in^4}\times\frac{1\ m^4}{(10^3\ mm)^4}$

$I = 3.11\times10^{-7}\ m^4$

$EI = (207\times10^9\ N/m^2)(3.11\times10^{-7}\ m^4) =$

$EI = 64.4\times10^3\ N\cdot m^2$

$A_1 = \frac{2}{3}(1.1)(4.70\times10^{-4}) = 3.45\times10^{-4}$

$A_2 = A_1 = 3.45\times10^{-4}$

$A_{31} = \frac{1}{2}(0.8)(6.83\times10^{-4}) = 2.73\times10^{-4}$ ⊖ NEGATIVE

$A_{32} = \frac{1}{3}(0.8)(2.49\times10^{-4}) = 0.664\times10^{-4}$ ⊖

$A_4 = \frac{1}{2}(1.0)(9.32\times10^{-4}) = 4.66\ 10^{-4}$ ⊖

$t_{BA} = A_1 x_{B1} + A_2 x_{B2} + A_{31} x_{B31} + A_{32} x_{B32}$

$x_{B1} = 1.9 + 0.413 = 2.313\ m$

$x_{B2} = 1.9 - 0.413 = 1.487\ m$

$x_{B31} = 0.267\ m$

$x_{B32} = 0.20\ m$

THEN $t_{BA} = 1.225\times10^{-3}\ m$

$z = t_{BA}\cdot\frac{AC}{AB} = t_{BA}\cdot\frac{4.0}{3.0}$

$z = \frac{4}{3}(1.225\times10^{-3}) = 1.633\times10^{-3}\ m$

$t_{CA} = A_1 x_{C1} + A_2 x_{C2} + A_{31} x_{C31} + A_{32} x_{C32} + A_4 x_{C4}$

$x_{C1} = 2.9 + 0.413 = 3.313\ m$

$x_{C2} = 2.9 - 0.413 = 2.487\ m$

$x_{C31} = 1.0 + 0.267 = 1.267\ m$

$x_{C32} = 1.0 + 0.20 = 1.200\ m$

$x_{C4} = 0.667\ m$

THEN $t_{CA} = 1.265\times10^{-3}\ m$

$y_C = z - t_{CA}$

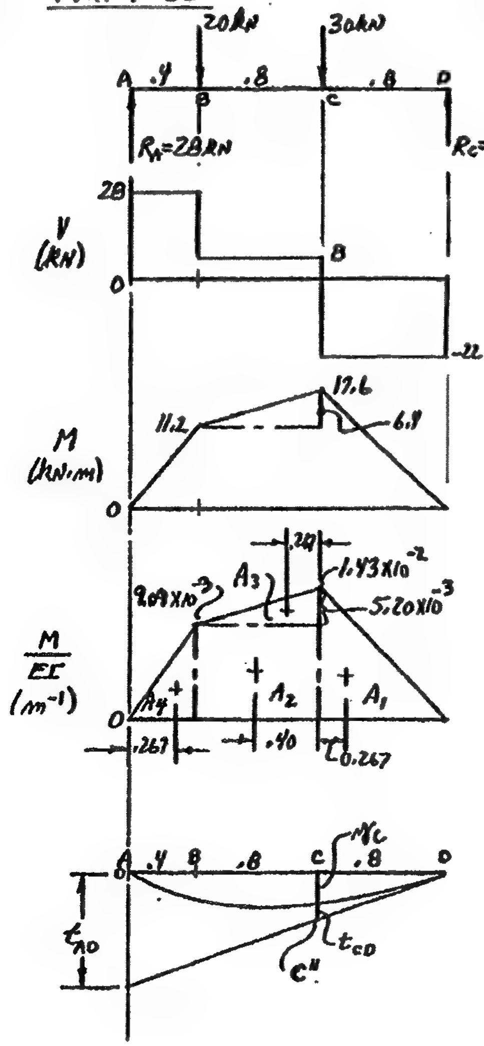
$= 1.633\times10^{-3} - 1.265\times10^{-3}$

$y_C = 0.368\times10^{-3}\ m = 0.368\ mm$

9-92

ALUM. I 178 × 8.630 $\quad I = 1.79 \times 10^7\ mm^4$

FIG. 9-82



ALUM $I\ 7 \times 5.800$: $I = 42.89\ in^4$

$I = (42.89\ in^4)\frac{(25.4\ mm)^4}{in^4}\frac{1\ m^4}{(10^3\ mm)^4}$

$I = 1.785 \times 10^{-5}\ m^4$

$EI = (69\times10^9)(1.785\times10^{-5}) = 1.23\times10^6\ N\cdot m^2$

$\frac{M_B}{EI} = \frac{11200\ N\cdot m}{1.23\times10^6\ N\cdot m^2} = 9.09\times10^{-3}\ m^{-1}$

SIMILARLY,

$\frac{M_C}{EI} = 1.43\times10^{-2}\ m^{-1}$

$A_1 = \frac{1}{2}(0.8)(1.43\times10^{-2}) = 5.72\times10^{-3}$

$A_2 = (0.8)(9.09\times10^{-3}) = 7.27\times10^{-3}$

$A_3 = \frac{1}{2}(0.8)(5.20\times10^{-3}) = 2.08\times10^{-3}$

$A_4 = \frac{1}{2}(0.4)(9.09\times10^{-3}) = 1.82\times10^{-3}$

$t_{AD} = A_1 x_{A1} + A_2 x_{A2} + A_3 x_{A3} + A_4 x_{A4}$

$x_{A1} = 0.4 + 0.8 + 0.267 = 1.467\ m$

$x_{A2} = 0.4 + 0.4 = 0.80\ m$

$x_{A3} = 0.4 + 0.8 - 0.267 = 0.933\ m$

$x_{A4} = 0.267\ m$

THEN $t_{AD} = 16.63\times10^{-3}\ m$

$CC'' = t_{AD}\cdot\frac{CD}{AD} = t_{AD}\cdot\frac{.8}{2.0} = 6.65\times10^{-3}\ m$

$t_{CD} = A_1 x_{C1} = (5.72\times10^{-3})(0.267m)$

$t_{CD} = 1.52\times10^{-3}\ m$

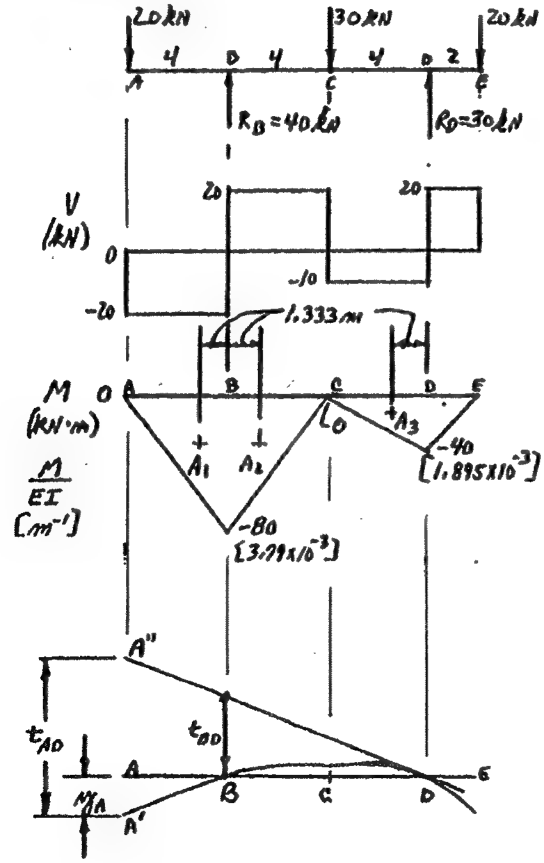
$y_C = CC'' - t_{CD} = (6.65 - 1.52)10^{-3}\ m$

$y_C = 5.13\times10^{-3}\ m = 5.13\ mm$

9-93

FIGURE 9-B3

STEEL W360 X 39 $I = 1.02 \times 10^{8}\ mm^4$



W14X26 : $I = 245\ IN^4$

$I = (245\ IN^4)\frac{(25.4\ mm)^4}{IN^4}\frac{1\ m^4}{(10^3\ mm)^4}$

$I = 1.02 \times 10^{-4}\ m^4$

$EI = (207 \times 10^9\ N/m^2)(1.02 \times 10^{-4}\ m^4)$

$EI = 21.1 \times 10^6\ N \cdot m^2$

$\frac{M_B}{EI} = \frac{80000\ N \cdot m}{21.1 \times 10^6\ N \cdot m^2} = 3.79 \times 10^{-3}\ m^{-1}$

$\frac{M_D}{EI} = \frac{1}{2}\frac{M_B}{EI} = 1.895 \times 10^{-3}\ m^{-1}$

$t_{BD} = A_2 x_{B2} + A_3 x_{B3}$

$A_1 = \frac{1}{2}(3.79 \times 10^{-3})(4) = 7.58 \times 10^{-3}$

$A_2 = A_1$

$A_3 = \frac{1}{2}(1.895 \times 10^{-3})(4) = 3.79 \times 10^{-3}$

$x_{B2} = 1.333\ m$

$x_{B3} = 8.0 - 1.333 = 6.667\ m$

THEN $t_{BD} = 35.4 \times 10^{-3}\ m$

$AA'' = t_{BD} \cdot \frac{AD}{BD} = t_{BD} \cdot \frac{12}{8}$

$AA'' = 53.1 \times 10^{-3}\ m$

$t_{AD} = A_1 x_{A1} + A_2 x_{A2} + A_3 x_{A3}$

$x_{A1} = 4.00 - 1.333 = 2.667\ m$

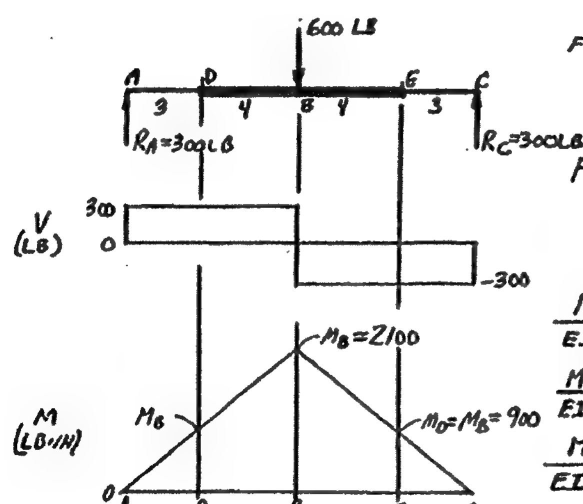
$x_{A2} = 4.00 + 1.333 = 5.333\ m$

$x_{A3} = 12.00 - 1.333 = 10.667\ m$

THEN $t_{AD} = 101.1 \times 10^{-3}\ m$

$y_A = t_{AD} - AA'' = (101.1 - 53.1)(10^{-3})\ m = 48.0 \times 10^{-3}\ m = 48.0\ mm = y_A$

9-94



FOR AD AND EC:

$I_1 = \frac{\pi (0.75)^4}{64} = 0.0155\ IN^4$

$EI_1 = (30 \times 10^6)(I_1) = 4.66 \times 10^5\ LB\text{-}IN^2$

FOR DE:

$I_2 = \frac{\pi (1.40)^4}{64} = 0.1886\ IN^4$

$EI_2 = (30 \times 10^6)(I_2) = 5.66 \times 10^6\ LB\text{-}IN^2$

$\frac{M_D}{EI_1} = \frac{M_E}{EI_1} = \frac{900\ LB\text{-}IN}{4.66 \times 10^5\ LB\text{-}IN^2} = 1.93 \times 10^{-3}\ IN^{-1}$

$\frac{M_D}{EI_2} = \frac{M_E}{EI_2} = \frac{900}{5.66 \times 10^6} = 0.159 \times 10^{-3}\ IN^{-1}$

$\frac{M_B}{EI_2} = \frac{2100}{5.66 \times 10^6} = 0.371 \times 10^{-3}\ IN^{-1}$

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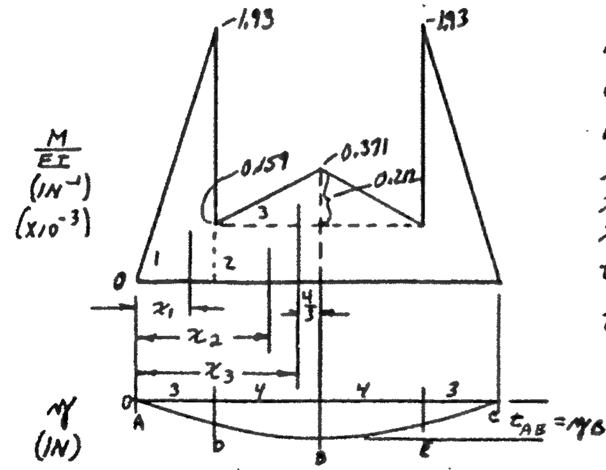
9-94 CONTINUED



$t_{AB} = y_B = A_1 x_1 \quad A_2 x_2 \quad A_3 x_3$

$A_1 = \frac{1}{2}(1.93 \times 10^{-3}\,IN^{-1})(2\,IN) = 2.90 \times 10^{-3}$

$A_2 = (0.159 \times 10^{-3})(4) = 0.636 \times 10^{-3}$

$A_3 = \frac{1}{2}(0.212 \times 10^{-3})(4) = 0.424 \times 10^{-3}$

$x_1 = (2/3)(3) = 2.000\,IN$

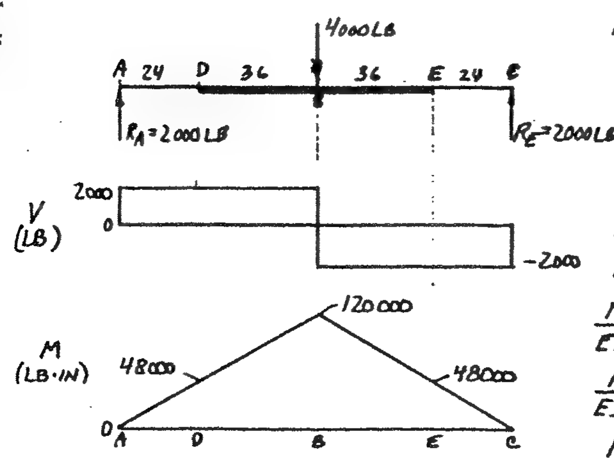
$x_2 = 3 + 2 = 5.000\,IN$

$x_3 = 7 - 4/3 = 5.667\,IN$

$t_{AB} = 0.00580 + 0.00318 + 0.00240$

$t_{AB} = y_B = 0.01138\,IN$

9-95



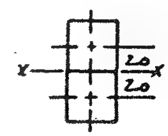
FOR AD AND EC: $4 \times 4 \times 1/4$

$I_1 = 7.80\,IN^4$

$EI_1 = (30 \times 10^6)(I_1) = 234.0 \times 10^6\,LB \cdot IN^2$

$I_2 = 2[I_1 + Ad^2]$

$= 2[7.80 + (3.37)(2)^2]$



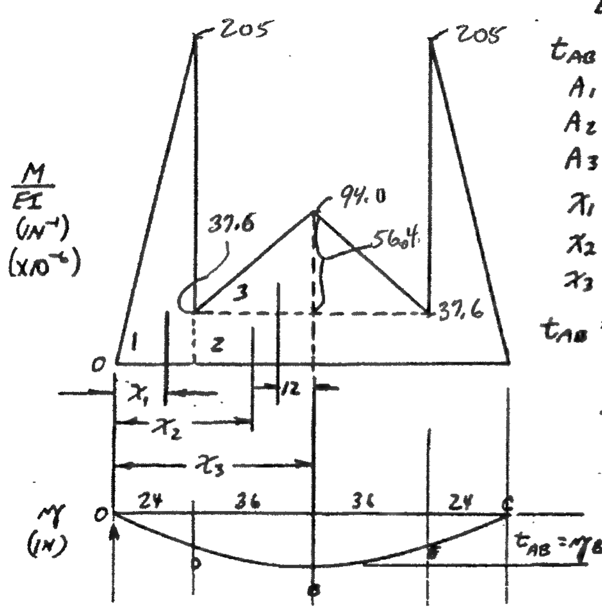
$I_2 = 42.56\,IN^4$

$EI_2 = (30 \times 10^6)(I_2) = 1277 \times 10^6\,LB \cdot IN^2$

$\frac{M_D}{EI_1} = \frac{48000}{234.0 \times 10^6} = 205 \times 10^{-6}\,IN^{-1}$

$\frac{M_D}{EI_2} = \frac{48000}{1277 \times 10^6} = 37.6 \times 10^{-6}\,IN^{-1}$

$\frac{M_B}{EI_2} = \frac{120000}{1277 \times 10^6} = 94.0 \times 10^{-6}\,IN^{-1}$



$t_{AB} = y_B = A_1 x_1 + A_2 x_2 + A_3 x_3$

$A_1 = \frac{1}{2}(205 \times 10^{-6})(24) = 2.46 \times 10^{-3}$

$A_2 = (37.6 \times 10^{-6})(36) = 1.354 \times 10^{-3}$

$A_3 = \frac{1}{2}(56.4 \times 10^{-6})(36) = 1.015 \times 10^{-3}$

$x_1 = (2/3)(24) = 16.0\,IN$

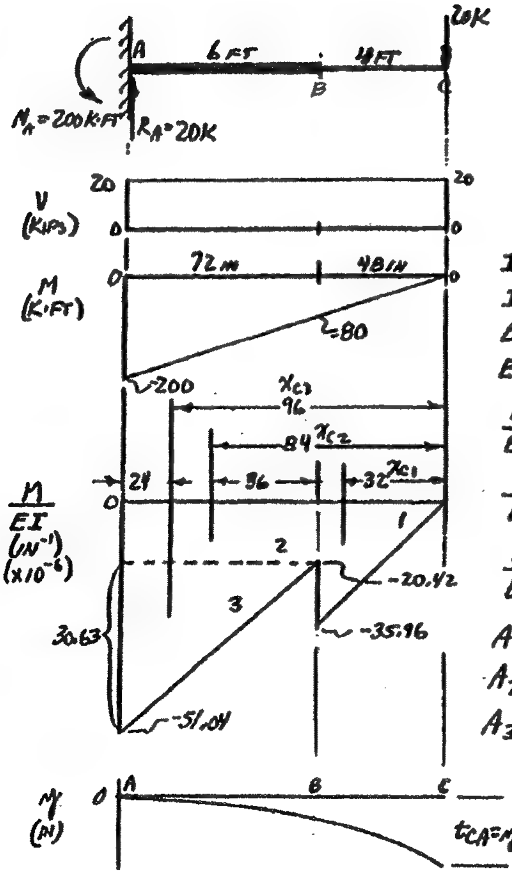
$x_2 = 24 + 18 = 42.0\,IN$

$x_3 = 24 + (2/3)(36) = 48.0\,IN$

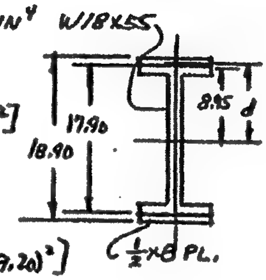
$t_{AB} = y_B = 0.0394 + 0.0569 + 0.0487$

$y_B = 0.145\,IN.$

9-96



FOR BC: $I_2 = 890\ IN^4$ W18X55



$d = 8.95 + 0.25$

$d = 9.20\ IN$

$I_1 = I_2 + 2[I_{PL} + Ad^2]$

$I_{PL} = \frac{8(0.50)^3}{12}$

$I_{PL} = 0.0833\ IN^4$

$I_1 = 890 + 2[0.083 + (4.0)(9.20)^2]$

$I_1 = 1567\ IN^4$

$EI_1 = (30\times10^6)(I_1) = 4.70\times10^{10}\ LB\cdot IN^2$

$EI_2 = (30\times10^6)(I_2) = 2.67\times10^{10}\ LB\cdot IN^2$

$\frac{M_A}{EI_1} = \frac{(200\,000)(12)\,LB\cdot IN}{4.70\times10^{10}\,LB\cdot IN^2} = 51.04\times10^{-6}\ IN^{-1}$

$\frac{M_B}{EI_1} = \frac{(80000)(12)}{4.71\times10^{10}} = 20.42\times10^{-6}\ IN^{-1}$

$\frac{M_B}{EI_2} = \frac{(80000)(12)}{2.67\times10^{10}} = 35.96\times10^{-6}\ IN^{-1}$

$A_1 = \frac{1}{2}(35.96\times10^{-6})(48) = 0.8629\times10^{-3}$

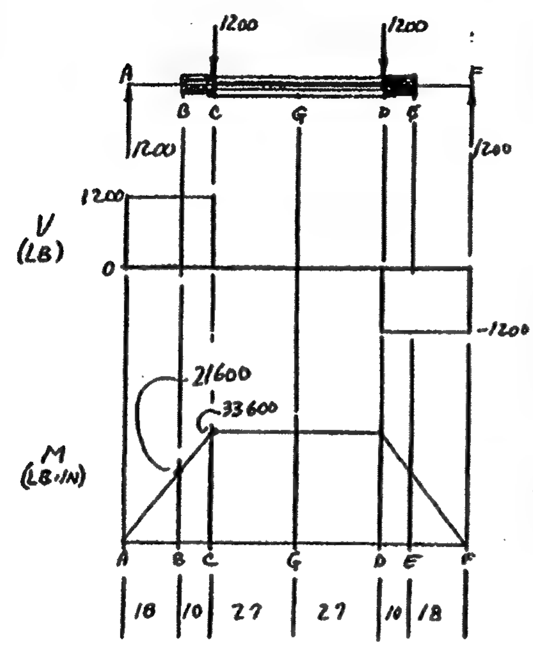
$A_2 = (20.42\times10^{-6})(72) = 1.470\times10^{-3}$

$A_3 = \frac{1}{2}(30.63\times10^{-6})(72) = 1.103\times10^{-3}$

$t_{CA} = y_C = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3}$

$t_{CA} = y_C$ $\quad y_C = 0.0276 + 0.1235 + 0.1058 = \underline{\underline{0.2569\ IN}}$

9-97



FOR AB AND EF: 3x3x 1/4; $I_1 = 3.02\ IN^4$

FOR CD: 4x4x 1/2; $I_2 = 11.9\ IN^4$

FOR BC AND DE:

$I_3 = I_1 + I_2 = 14.92\ IN^4$

$EI_1 = (30\times10^6)(I_1) = 90.6\times10^6\ LB\cdot IN^2$

$EI_2 = 447.6\times10^6\ LB\cdot IN^2$

$EI_3 = 357\times10^6\ LB\cdot IN^2$

$\frac{M_B}{EI_1} = \frac{21600\ LB\cdot IN}{90.6\times10^6\ LB\cdot IN^2} = 238.4\times10^{-6}\ IN^{-1}$

$\frac{M_B}{EI_2} = \frac{21600}{447.6\times10^6} = 48.26\times10^{-6}\ IN^{-1}$

$\frac{M_C}{EI_2} = \frac{33600}{447.6\times10^6} = 75.07\times10^{-6}\ IN^{-1}$

$\frac{M_C}{EI_3} = \frac{33600}{357\times10^6} = 94.12\times10^{-6}\ IN^{-1}$

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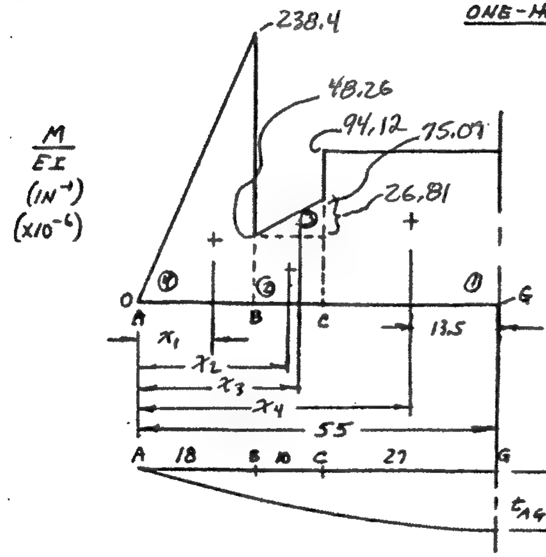
9-97 CONTINUED



ONE-HALF OF DIAGRAMS DRAWN TWICE SIZE

$A_1 = (94.12 \times 10^{-6})(27) = 2.541 \times 10^{-3}$

$A_2 = (48.26 \times 10^{-6})(10) = .4826 \times 10^{-3}$

$A_3 = \frac{1}{2}(26.81 \times 10^{-6})(10) = .1341 \times 10^{-3}$

$A_4 = \frac{1}{2}(238.4 \times 10^{-6})(18) = 2.146 \times 10^{-3}$

$t_{AG} = y_G = A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4$

$x_1 = 55 - 13.5 = 41.5$ IN.

$x_2 = 18 + 5 = 23.0$

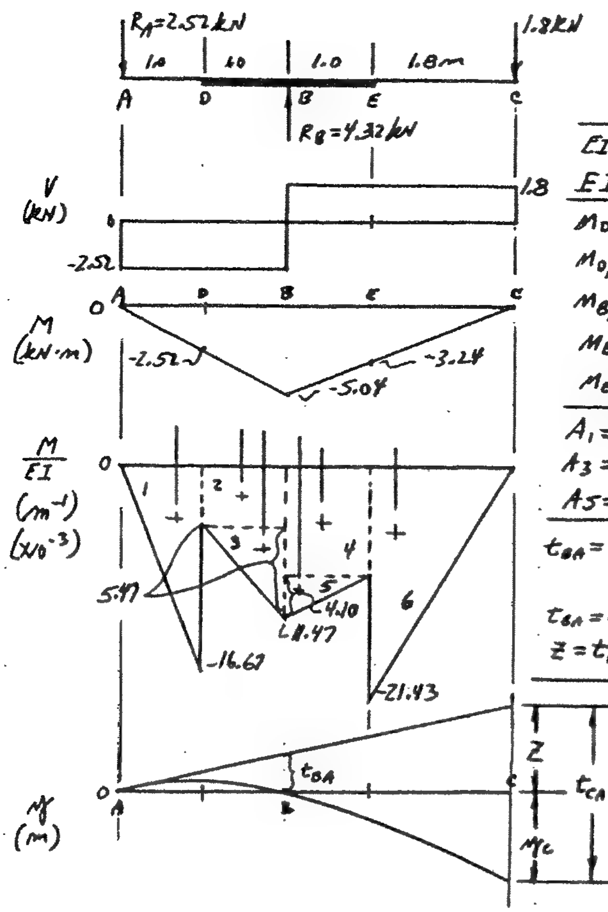
$x_3 = 18 + (2/3)(10) = 24.667$ IN

$x_4 = (2/3)(18) = 12.0$ IN

$t_{AG} = y_G = 0.1055 + 0.0111 + 0.0033 + 0.0257$

$y_G = 0.1456$ IN.

9-98



FOR AD AND EC:

$I_1 = (0.286)(0.089)^3/12 = 16.8 \times 10^{-6}\ m^4$

FOR DB AND BE:

$I_2 = (0.286)(0.127)^3/12 = 48.82 \times 10^{-6}\ m^4$

$EI_1 = (9 \times 10^9\ N/m^2)(16.8 \times 10^{-6}\ m^4) = 1.51 \times 10^5\ N\cdot m^2$

$EI_2 = 4.39 \times 10^5\ N\cdot m^2$

$M_D/EI_1 = 16.67 \times 10^{-3}\ m^{-1}$

$M_D/EI_2 = 5.74 \times 10^{-3}\ m^{-1}$

$M_B/EI_2 = 11.47 \times 10^{-3}\ m^{-1}$

$M_E/EI_2 = 7.37 \times 10^{-3}\ m^{-1}$

$M_E/EI_1 = 21.43 \times 10^{-3}\ m^{-1}$

$A_1 = 8.33 \times 10^{-3}$; $A_2 = 5.74 \times 10^{-3}$

$A_3 = 2.87 \times 10^{-3}$; $A_4 = 7.37 \times 10^{-3}$

$A_5 = 2.05 \times 10^{-3}$; $A_6 = 19.28 \times 10^{-3}$

$t_{BA} = A_1 \underbrace{\bar{x}_{B1}}_{1.333} + A_2 \underbrace{\bar{x}_{B2}}_{0.50} + A_3 \underbrace{\bar{x}_{B3}}_{0.333}$

$t_{BA} = 0.0111 + 0.00287 + 0.00096 = 0.01493$ m

$z = t_{BA} \cdot \frac{AC}{AB} = 0.01493 \cdot \frac{4.8}{2.0} = 0.03584$ m

$t_{CA} = A_1 \overbrace{\bar{x}_{C1}}^{4.133} + A_2 \overbrace{\bar{x}_{C2}}^{3.30} + A_3 \overbrace{\bar{x}_{C3}}^{3.133}$

$+ A_4 \overbrace{\bar{x}_{C4}}^{2.30} + A_5 \overbrace{\bar{x}_{C5}}^{2.467} + A_6 \overbrace{\bar{x}_{C6}}^{1.20}$

$t_{CA} = 0.0344 + 0.0189 + 0.0090$
$+ 0.0170 + 0.0051 + 0.0231$

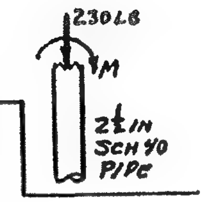
$t_{CA} = 0.1075$ m

$y_C = t_{CA} - z = 0.1075\ m - 0.0358\ m = 0.0717\ m = 71.7 \times 10^{-3}\ m$

$y_C = 71.7$ mm

Combined Normal Stresses

10-1 $M = (230\,LB)(48\,IN) = 11040\,LB\cdot IN$; $A = 1.704\,IN^2$; $S = 1.064\,IN^3$



$$\sigma = \frac{-F}{A} - \frac{M}{S} = \frac{-230\,LB}{1.704\,IN^2} - \frac{11040\,LB\cdot IN}{1.064\,IN^3} = \underline{-10510\,PSI}$$

10-2

$F_h = F\sin 30° = 1.70\,kN$

$F = 3.4\,kN$

$F_V = F\cos 30° = 2.94\,kN$

$M = F_V \cdot 350\,mm = 1.029\times10^6\,N\cdot mm$

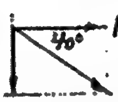
$AREA = bh = (18)(75) = 1350\,mm^2$

$S = bh^2/6 = 16875\,mm^3$

$\underline{\sigma_{MAX}\ \text{OCCURS ON TOP OF BRACKET AT WALL}}$

$$\sigma_{MAX} = \frac{F_h}{A} + \frac{M}{S} = \frac{1700\,N}{1350\,mm^2} + \frac{1.029\times10^6\,N\cdot mm}{16875\,mm^3} = \underline{62.2\,MPa}$$

10-3



$F_h = F\cos 40° = 4596\,LB$

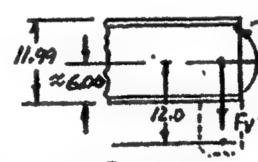
$F = 6000\,LB$

$F_V = F\sin 40° = 3857\,LB$

MOMENT DUE TO F_V: $M_2 = F_V \cdot 52\,IN$

$M_2 = 2.006\times10^5\,LB\cdot IN$

$\underline{W12\times16\ BEAM}$: $A = 4.71\,IN^2$; $S = 17.1\,IN^3$



$$\text{AT N: } \sigma_N = \frac{F_h}{A} + \frac{M_2}{S} - \frac{M_1}{S} = \frac{4596}{4.71} + \frac{2.006\times10^5}{17.1} - \frac{55150}{17.1} = \underline{9480\,PSI}$$

$$\text{AT M: } \sigma_M = \frac{F_h}{A} - \frac{M_2}{S} + \frac{M_1}{S} = \underline{-7530\,PSI}$$

10-4 $M = 6000\,LB \cdot 52\,IN = 3.12\times10^5\,LB\cdot IN$

$$\sigma = \frac{M}{S} = \frac{3.12\times10^5\,LB\cdot IN}{17.1\,IN^3} = \underline{18246\,PSI}$$ TENSION AT N; COMPR. AT M

10-5 LOADING IS SAME AS 11-3 EXCEPT F_h ACTS TO LEFT AND M_1 IS OPPOSITE DIRECTION.

$$\text{AT N: } \sigma_N = \frac{-F_h}{A} + \frac{M_2}{S} + \frac{M_1}{S} = \frac{-4596}{4.71} + \frac{2.006\times10^5}{17.1} + \frac{55150}{17.1} = \underline{13980\,PSI}$$

$$\text{AT M: } \sigma_M = \frac{-F_h}{A} - \frac{M_2}{S} - \frac{M_1}{S} = -975 - 11731 - 3225 = \underline{-15931\,PSI}$$

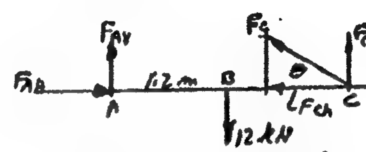
10-6 $M = (125\,N)(145\,mm) = 18125\,N\cdot mm$; $S = \frac{bh^2}{6} = \frac{(4)(10^2)}{6} = 66.67\,mm^3$

$$\sigma = \frac{-F}{A} - \frac{M}{S} = \frac{-125\,N}{40\,mm^2} - \frac{18125\,N\cdot mm}{66.67\,mm^3} = -3.125 - 271.9 = \underline{-275\,MPa}$$

10-7 $F_{AV} = F_{CV} = 12\,kN/2 = 6.0\,kN$

$\tan\theta = 1.5/2.4 = 0.625$

$$F_{Ch} = F_{Ah} = \frac{F_{CV}}{\tan\theta} = \frac{6.0\,kN}{0.625} = 9.6\,kN$$



$M = F_{AV}\cdot 1.2\,m = 7.2\,kN\cdot m$ $\underline{\text{AT B}}$

W6×12: $A = 3.55\,IN^2$

$A = 2290\,mm^2$

$S = 7.31\,IN^3$

$S = 1.20\times10^5\,mm^3$

$$\sigma = \frac{-F_h}{A} - \frac{M}{S} = \frac{-9600\,N}{2290\,mm^2} - \frac{7.2\times10^6\,N\cdot mm}{1.20\times10^5\,mm^3}$$

$\sigma = -4.19 - 60.1 = \underline{-64.3\,MPa}$

COMPRESSION ON TOP OF BEAM

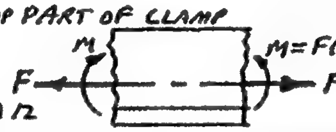
10-8 $M = 160\,N \cdot 80\,mm = 12\,800\,N \cdot mm$; $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(12^2 - 10^2) = 34.6\,mm^2$

$S = \frac{\pi}{32}\frac{(D^4 - d^4)}{D} = \frac{\pi(12^4 - 10^4)}{32(12)} = 87.8\,mm^3$

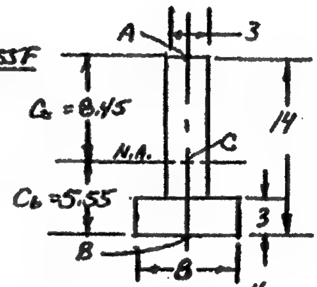
AT POINT A: $\sigma = \frac{-F}{A} - \frac{M}{S}$

$\sigma = \frac{-160\,N}{34.6\,mm^2} - \frac{12800\,N \cdot mm}{87.8\,mm^3} = \underline{150.4\,MPa}$

10-9 F.B.D. OF TOP PART OF CLAMP



CAST ZINC ZA 12



$I = 1032\,mm^4$
$A = 57\,mm^2$

DESIGN STRESS: $\sigma_d = \frac{S_u}{4} = \frac{400\,MPa}{4} = 100\,MPa$

AT A: $\sigma_A = \frac{F}{A} - \frac{Mc_t}{I} = \underbrace{\frac{F}{57} - \frac{31.55F(8.45)}{1032}}$

LET $\sigma_A = \sigma_d = 100\,MPa = -0.2408\,F$ COMPR.

$F = 100/0.2408 = \underline{415\,N}$

AT B: $\sigma_B = \frac{F}{A} + \frac{Mc_b}{I} = \frac{F}{57} + \frac{31.55F(5.55)}{1032} = 0.1872\,F$ TENSION

LET $\sigma_B = \sigma_d = 100\,MPa = 0.1872\,F$

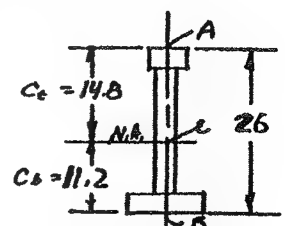
$F = 100/0.1872 = \underline{534\,N}$

LIMITING VALUE OF $\underline{F = 415\,N}$ IF S_u IS EQUAL IN TENSION AND COMPRESSION. NOTE THAT $S_{uc} > S_{ut}$ FOR SOME CAST ALLOYS.

10-10 ASTM A220 45008: $S_{ut} = 448\,MPa$, $S_{uc} = 1650\,MPa$

$\sigma_{dt} = 448/4 = 112\,MPa$: $\sigma_{dc} = 1650/4 = 412\,MPa$

$M = F(42 + 11.2) = 53.2\,F$



$A = 114\,mm^2$
$I = 8806\,mm^4$

AT A: $\sigma_A = \frac{F}{A} - \frac{Mc_t}{I}$

$\sigma_A = \frac{F}{114} - \frac{53.2F(14.8)}{8806}$

$\sigma_A = -0.0806\,F = \sigma_{dc} = 412\,MPa$

$F = 412/0.0806 = 5109\,N$

AT B: $\sigma_B = \frac{F}{A} + \frac{Mc_b}{I} = \frac{F}{114} + \frac{53.2F(11.2)}{8806} = 0.0764\,F = \sigma_{dt} = 112\,MPa$

$F = 112/0.0764 = \underline{1465\,N}$ LIMITING VALUE

10-11 FOR M16×2 THREADED ROD: STRESS AREA $= 157\,mm^2$ (APP A3)

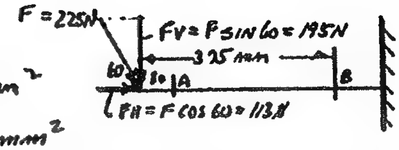
COMPUTE SECTION MODULUS BASED ON D_{ROOT}: $S = \frac{\pi D_R^3}{32} = \frac{\pi(13.2)^3}{32} = 226\,mm^3$

$\sigma = K_t\left[\frac{F}{A} + \frac{M}{S}\right] = 3.0\left[\frac{1200}{157} + \frac{(1200)(44)}{226}\right] = \underline{724\,MPa}$ TENSION

FOR $\sigma_d = S_y/2$; $S_y = 2\sigma = 2(724) = 1448\,MPa$

USE $\underline{\text{AISI 4140 OQT 700}}$, $S_y = 1462\,MPa$ - ONE POSSIBLE CHOICE

10-12 $M_A = F_V(30) = 5850\ N\cdot mm$



$M_B = F_V(375) = 73125\ N\cdot mm$

$S_A = \frac{bh^2}{6} = \frac{10(14)^2}{6} = 327\ mm^3;\ A_A = bh = 140\ mm^2$

$S_B = \frac{10(25)^2}{6} = 1042\ mm^3;\ A_B = bh = 250\ mm^2$

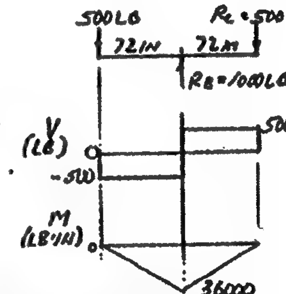
ASSUME $S_{ut} \ll S_{uc}$ – TENSION GOVERNS DESIGN FACTOR, $S_{ut} = 552\ MPa$

AT A: $\sigma_A = \frac{-F_h}{A_A} + \frac{M_A}{S_A} = \frac{-113}{140} + \frac{5850}{327} = 17.08\ MPa$

AT B: $\sigma_B = \frac{-F_h}{A_B} + \frac{M_B}{S_B} = \frac{-113}{250} + \frac{73125}{1042} = \underline{69.7\ MPa}$

$N = \frac{S_{ut}}{\sigma_B} = \frac{552\ MPa}{69.7\ MPa} = \underline{7.9}$

10-13 FOR S3×5.7: $A = 1.67\ IN^2$; $S = 1.68\ IN^3$



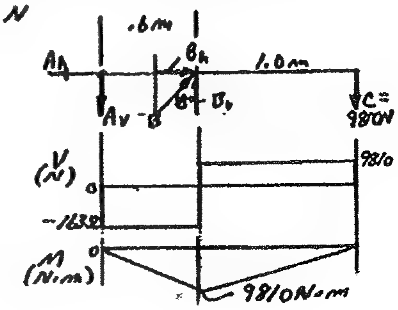
AT B-TOP: $\sigma = \frac{F}{A} + \frac{M}{S} = \frac{4600}{1.66} + \frac{36000}{1.67} = \underline{24328\ PSI}$ TENSION

AT B-BOTTOM:

$\sigma = \frac{F}{A} - \frac{M}{S} = \frac{4600}{1.66} - \frac{36000}{1.67} = \underline{-18785\ PSI}$ COMPRESSION

10-14 FOR TUBE: $A = 2170\ mm^2$; $I = 5.45\times10^6\ mm^4$; $S = \frac{I}{c} = 7.16\times10^4\ mm^3$

LOAD: $w = m\cdot g = 1000\ kg \cdot 9.81\ m/s^2 = 9810\ N$



$\Sigma M_A = 0 = 9810(1.6) - B_V(0.6)$

$B_V = 26160\ N = B_h = A_h$

$A_V = B_V - C_V = 26160 - 9810 = 16350\ N$

NEAR B – TO LEFT:

$\sigma = \frac{B_h}{A} + \frac{M_B}{S} = \frac{26160}{2170} + \frac{9810\times10^3}{7.16\times10^4}$

$\underline{\sigma = 137\ MPa}$

10-15 σ_{MAX} TO LEFT OF B (SEE 11-14).

$M_B = (1.0\ m)(F) = (1000\ mm)F$

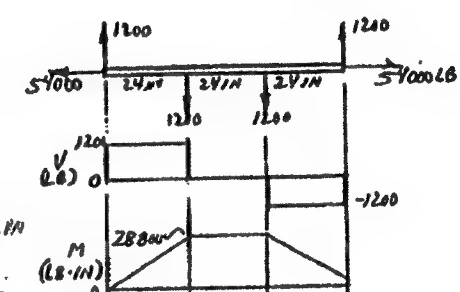
$B_V = B_H = \frac{1.6F}{0.6} = 2.67F$

$\sigma_d = \frac{S_y}{3} = \frac{414\ MPa}{3} = 138\ MPa$

$\sigma = \frac{B_H}{A} + \frac{M_B}{S} = \frac{2.67F}{2170} + \frac{1000\ F}{7.16\times10^4} = 0.0152\ F = 138\ MPa = \sigma_d$

$F = 138/0.0152 = \underline{9081\ N}$: $m = \frac{F}{g} = \frac{9081\ kg\,m/s^2}{9.81\ m/s^2} = \underline{926\ kg}$

10-16



$\sigma_d = 0.6S_y = 0.6(36000) = 21600\ psi$

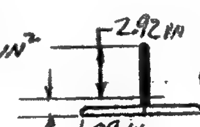
$\sigma = \frac{P}{A} + \frac{M}{S}$ USE TRIAL AND ERROR

FOR BENDING ONLY:

$S = \frac{M}{\sigma_d} = \frac{28800}{21600} = 1.33\ IN^3$

FOR AXIAL TENSION:

$A = \frac{P}{\sigma_d} = \frac{54000}{21600} = 2.50\ IN^2$



TRY L4×4×1/4: (TWO)

$A = 2(1.93) = 3.86\ IN^2$

$I = 2I_x = 2(3.00) = 6.00\ IN^4$; $C_b = y = 1.08\ IN$; $C_t = 4.0 - 1.08 = 2.92\ IN$

AT BOTTOM: $\sigma = \frac{P}{A} + \frac{MC_b}{I} = \frac{54000}{3.86} + \frac{28800(1.08)}{6.00} = \underline{19102\ psi}$

OK

10-17 FROM PROBLEM P5-77 : $M = 120\ N\cdot m$: $P = 800N$ AXIAL TENSION

$$\sigma = \frac{P}{A} + \frac{M}{S} = \frac{P}{a^2} + \frac{M}{a^3/6} = \frac{P}{a^2} + \frac{6M}{a^3}$$

MULTIPLY BY a^3

$\sigma a^3 = Pa + 6M$: $\sigma a^3 - Pa - 6M = 0$: LET $\sigma = 42\ MPa = 42\ N/mm^2$

$42a^3 - 800a - 6(120000) = 0$: THEN $a = 26\ mm$

10-18 FROM P5-78 : $M = 344\ LB\cdot IN$: $P = 250\ LB$ AXIAL COMPR.

$$\sigma = \frac{-P}{A} - \frac{M}{S} = \frac{-P}{a^2} - \frac{M}{a^3/6} = \frac{-P}{a^2} - \frac{6M}{a^3}$$

$\sigma a^3 = -Pa - 6M$: $\sigma a^3 + Pa + 6M = 0$: LET $\sigma = -6000\ PSI$ COMPR.

$-6000a^3 + 250a + 6(344) = 0$: THEN $a = 0.720\ IN$

10-19 FROM P5-79 : $M = 42200\ N\cdot mm$: $P = 1200\ N$ AXIAL TENSION

FROM 10-17 : $\sigma a^3 - Pa - 6M = 0$: $42a^3 - 1200a - 6(42200) = 0$: $a = 18.7\ mm$

10-20 FROM P5-80 : $M = 520\ LB\cdot IN$: $P = 400\ LB$ AXIAL TENSION

FROM 10-17 : $\sigma a^3 - Pa - 6M = 0$: $6000a^3 - 400a - 6(520) = 0$: $a = 0.832\ IN$

Combined Normal and Shear Stresses

10-21 $A = \pi(40)^2/4 = 1257\ mm^2$: $Z_p = \pi(40)^3/16 = 12566\ mm^3$

$$\sigma = \frac{P}{A} = \frac{150000\ N}{1257\ mm^2} = 119\ MPa \ : \ \tau = \frac{T}{Z_p} = \frac{500000\ N\cdot mm}{12566\ mm^3} = 39.8\ MPa$$

$$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{119}{2}\right)^2 + 39.8^2} = 71.6\ MPa$$

10-22 $A = \pi(2.25)^2/4 = 3.98\ IN^2$: $Z_p = \pi(2.25)^3/16 = 2.24\ IN^3$

$$\sigma = \frac{P}{A} = \frac{47000\ LB}{3.98\ IN^2} = 11809\ psi \ : \ \tau = \frac{T}{Z_p} = \frac{8500\ LB\cdot IN}{2.24\ IN^3} = 3795\ PSI$$

$$\tau_{MAX} = \sqrt{\left(\frac{11809}{2}\right)^2 + 3795^2} = 7019\ psi$$

10-23 $A = \pi(4.00)^2/4 = 12.57\ IN^2$: $Z_p = \pi(4.00)^3/16 = 12.57\ IN^3$

$$\sigma = \frac{-P}{A} = \frac{-40000\ LB}{12.57\ IN^2} = 3183\ psi \ : \ \tau = \frac{T}{Z_p} = \frac{25000\ LB\cdot IN}{12.57\ IN^3} = 1989\ psi$$

$$\tau_{MAX} = \sqrt{\left(\frac{3183}{2}\right)^2 + 1989^2} = 2548\ psi$$

10-24 FOR 12 IN PIPE : $A = 15.74\ IN^2$: $Z_p = 94.18\ IN^3$

$$\sigma = \frac{-P}{A} = \frac{-250000\ LB}{15.74\ IN^2} = -15883\ psi \ : \ \tau = \frac{T}{Z_p} = \frac{180000\ LB\cdot IN}{94.18\ IN^3} = 1911\ psi$$

$$\tau_{MAX} = \sqrt{\left(\frac{15883}{2}\right)^2 + 1911^2} = 8168\ psi$$

10-25 FOR 3 IN PIPE : $A = 2.228\ IN^2$: $Z_p = 3.448\ IN^3$

$$\sigma = \frac{-P}{A} = \frac{-25000\ LB}{2.228\ IN^2} = -11221\ psi \ : \ \tau = \frac{T}{Z_p} = \frac{15500\ LB\cdot IN}{3.448\ IN^3} = 4495\ psi$$

$$\tau_{MAX} = \sqrt{\left(\frac{11221}{2}\right)^2 + 4495^2} = 7189\ psi$$

10-26 $T = (20\ LB)(8\ FT)(12\ IN/FT) = 1920\ LB\cdot IN$: $M = (20\ LB)(15\ FT)(12\ IN/FT) = 3600\ LB\cdot IN$

$$T_e = \sqrt{T^2 + M^2} = \sqrt{1920^2 + 3600^2} = 4080\ LB\cdot IN$$

$$Z_p = \frac{\pi}{16}\frac{D^4 - d^4}{D} = \frac{\pi(1.50^4 - 1.375^4)}{16(1.50)} = 0.195\ IN^3$$

$$\tau = \frac{T_e}{Z_p} = \frac{4080\ LB\cdot IN}{0.195\ IN^3} = 20920\ psi$$

LET $\tau_d = S_y/2N$: $N = \frac{S_y}{2\tau} = \frac{40000}{2(20920)} = 0.96$ UNSAFE

10-27 NEAR SUPPORT: $T = 300F = 300(1200) = 3.6 \times 10^5\ N\cdot mm$

$M = 450F = 450(1200) = 5.4 \times 10^5\ N\cdot mm$

$T_e = \sqrt{T^2 + M^2} = 6.49 \times 10^5\ N\cdot mm$; $Z_p = \frac{\pi}{16}(40)^3 = 12566\ mm^3$

$\tau = \frac{T_e}{Z_p} = \frac{6.49 \times 10^5\ N\cdot mm}{12566\ mm^3} = \underline{51.6\ MPa}$

10-28 TOTAL DOWNWARD LOAD $= 200 + 200 + 300 + 600 = 1300\ LB$

$M = (1300\ LB)(36\ IN) = 46\,800\ LB\cdot IN$ AT SUPPORT

NET TORQUE $= 600(40) + 300(20) - 200(20) - 200(20) = 18000\ LB\cdot IN$

$T_e = \sqrt{T^2 + M^2} = \sqrt{18000^2 + 46800^2} = 50\,142\ LB\cdot IN$

REQD $Z_p = \frac{T_e}{\tau_d} = \frac{50142\ LB\cdot IN}{8000\ LB/IN^2} = 6.27\ IN^3$ <u>USE 4 IN SCH 40 PIPE</u>

Rotating Shafts – Combined Torsional Shear and Bending Stresses

10-29 $M_B = (2400\ N)(150\ mm) = 360000\ N\cdot mm$; $S = \frac{\pi D^3}{32} = \frac{\pi(20)^3}{32} = 785\ mm^3$

$Z_p = \frac{\pi D^3}{16} = \frac{\pi(20)^3}{16} = 1571\ mm^3$; $\tau = \frac{T}{Z_p} = \frac{1500000\ N\cdot mm}{1571\ mm^3} = 955\ MPa$

$\sigma = \frac{M}{S} = \frac{360\,000\ N\cdot mm}{785\ mm^3} = 459\ MPa$

$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{459}{2}\right)^2 + 955^2} = \underline{982\ MPa}$

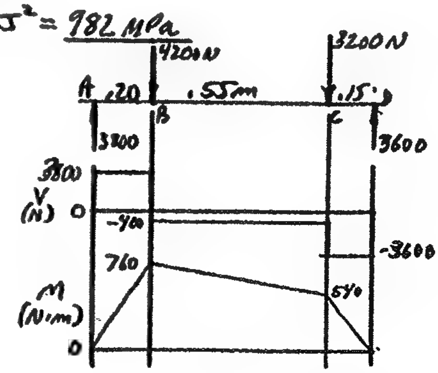
10-30 $S = \frac{\pi D^3}{32} = \frac{\pi(25)^3}{32} = 1534\ mm^3$

$Z_p = \frac{\pi D^3}{16} = 2S = 3068\ mm^3$

$\sigma = \frac{M}{S} = \frac{760000\ N\cdot mm}{1534\ mm^3} = 495\ MPa$

$\tau = \frac{T}{Z_p} = \frac{4500000}{3068} = 1467\ MPa$

$\tau_{max} = \sqrt{\left(\frac{495}{2}\right)^2 + 1467^2} = \underline{1488\ MPa}$



10-31 $T = \frac{63000(P)}{n} = \frac{63000(25)}{1150} = 1370\ LB\cdot IN$

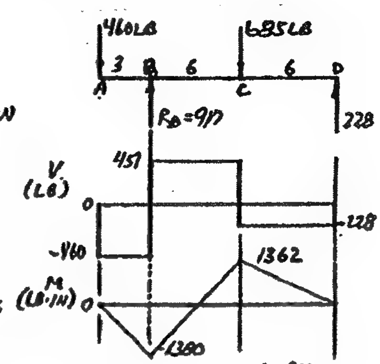
<u>AT B:</u> $T_e = \sqrt{T^2 + M^2} = \sqrt{1370^2 + 1380^2} = 1945\ LB\cdot IN$

$Z_p = \frac{\pi D^3}{16} = \frac{\pi(1.0)^3}{16} = 0.196\ IN^3$

$\tau = \frac{T_e}{Z_p} = \frac{1945}{0.196} = 9923\ psi = \tau_d = \frac{S_y}{2N}$

REQD $S_y = 2N\tau = 2(6)(9923) = 119\,000\ psi$

<u>AISI 1141 OQT 900</u> $S_y = 129\ ksi$, 15% ELONG



10-32 (a)

$T_A = (450 - 90)(6) = 2160\ LB\cdot IN$ CCW $= T_{AC}$

$T_C = (1200 - 240)(4) = 3840\ LB\cdot IN$ CW

$T_E = (1050 - 210)(2) = \underline{1680\ LB\cdot IN\ CCW = T_{CE}}$

(b) $Z_p = \frac{\pi D^3}{16} = \frac{\pi(1.75)^3}{16} = 1.05\ IN^3$; $S = \frac{\pi}{32}D^3 = 0.526\ IN^3$

<u>AT C:</u> $\tau = \frac{2160(1.6)}{1.05} = 3291\ psi$

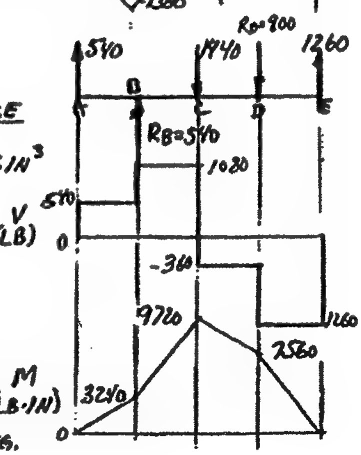
$\sigma = \frac{9720\ (1.6)}{0.526} = 29\,567\ psi$

(c) $\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = 15145\ psi = \tau_d = \frac{S_y}{2N}$

LET $N = 4$

REQD $S_y = 2N\tau = 2(4)(15145) = 121\,161\ psi$

<u>AISI 1141 OQT 900, $S_y = 129\ ksi$, 15% ELONG.</u>



10-33

$T_B = (1500 - 300)(50) = 60000\ N\cdot mm$

$T_C = (600 - 120)(125) = 60000\ N\cdot mm$

AT B: $Z_p = \frac{\pi(28)^3}{16} = 4310\ mm^3$; $A = \frac{\pi(28)^2}{4} = 616\ mm^2$

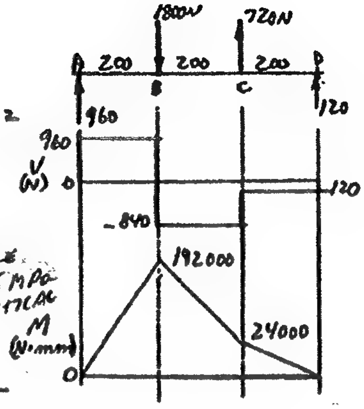
$S = \frac{\pi(28)^3}{32} = 2155\ mm^3$

$\tau = T/Z_p = [60000/4310]\,1.6 = 22.3\ MPa$

$\sigma_b = \pm M/S = [\pm 192000/2155]\,1.6 = 142.6\ MPa$ } NET TENSILE $= 126.5\ MPa$ IS CRITICAL

$\sigma_c = -P/A = [-6200/616]\,1.6 = -16.1\ MPa$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{126.5}{2}\right)^2 + (22.3)^2} = 67.1\ MPa$



10-34 REQD $S_y = 2N(67.1) = \underline{537\ MPa}$ – USE AISI 1040 1100

Combined Axial Tension and Direct Shear Stresses

10-35 8-32 UNC THREAD: $D_m = 0.164\ IN$: $A_t = 0.0140\ IN^2$

IN THREADS: $P = \sigma A_t = (15000\ LB/IN^2)(0.0140\ IN^2) = 210\ LB$

FOR D_m: $A_s = A_m = \pi D_m^2/4 = \pi(0.164)^2/4 = 0.0211\ IN^2$

$\sigma = P/A_m = 210/0.0211 = 9941\ PSI$; $\tau = F_s/A_s = 120/0.0211 = 5681\ PSI$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{9941}{2}\right)^2 + 5681^2} = \underline{7548\ PSI}$

10-36 FOR 1/4-20 UNC THREAD: $D_m = 0.250\ IN$; $A_t = 0.0318\ IN^2$

$P = \sigma A_t = (15000)(0.0318) = 477\ LB$

FOR D_m: $0.250\ IN$: $A_R = A_s = \pi(0.25)^2/4 = 0.0491\ IN^2$

$\sigma = P/A_R = 477/0.0491 = 9717\ PSI$; $\tau = F_s/A_s = 775/0.0491 = 15788\ PSI$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{9717}{2}\right)^2 + 15788^2} = \underline{16519\ PSI}$

10-37 FOR 4-48 UNF THREAD: $D_m = 0.112\ IN$: $A_t = 0.00661\ IN^2$

$P = \sigma A_t = 15000(0.00661) = 99.2\ LB$

FOR $D_m = 0.112\ IN$: $A_R = A_s = \pi(0.112)^2/4 = 0.00985\ IN^2$

$\sigma = P/A_R = 99.2/0.00985 = 10069\ PSI$; $\tau = F_s/A_s = 50/0.00985 = 5075\ PSI$

$\tau_{MAX} = \sqrt{\left(\frac{10069}{2}\right)^2 + 5075^2} = \underline{7149\ PSI}$

10-38 FOR 1 1/4-12 THREAD: $D_m = 1.250\ IN$: $A_t = 1.073\ IN^2$

$P = \sigma A_t = 15000(1.073) = 16095\ LB$

FOR D_m: $A_R = A_s = \pi(1.25)^2/4 = 1.227\ IN^2$

$\sigma = P/A_R = 16095/1.227 = 13115\ PSI$; $\tau = F_s/A_s = 2500/1.227 = 2037\ PSI$

$\tau_{MAX} = \sqrt{\left(\frac{13115}{2}\right)^2 + 2037^2} = \underline{6867\ PSI}$

10-39 FOR M16×2: $D_m = 16\ mm$; $A_t = 157\ mm^2$

$P = \sigma A_t = (120\ N/mm^2)(157\ mm^2) = 18840\ N$

FOR $D_m = 16\ mm$: $A_R = A_s = \pi(16)^2/4 = 201\ mm^2$

$\sigma = P/A_R = 18840\ N/201\ mm^2 = 93.7\ MPa$; $\tau = F_s/A_s = 8000\ N/201\ mm^2 = 39.8\ MPa$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{93.7}{2}\right)^2 + 39.8^2} = \underline{61.5\ MPa}$

10-40 FOR M48×5: $D_m = 48\ mm$; $A_t = 1473\ mm^2$

$P = \sigma A_t = (120\ N/mm^2)(1473\ mm^2) = 1.77\times10^5\ N$

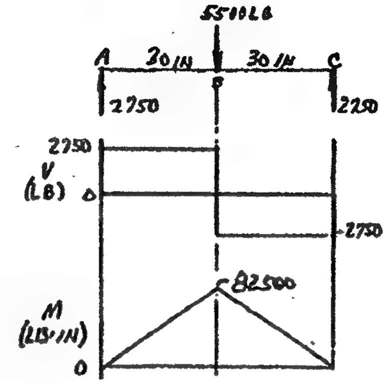
FOR $D_m = 48\ mm$: $A_R = A_s = \pi(48)^2/4 = 1810\ mm^2$

$\sigma = P/A_R = \frac{1.77\times10^5 N}{1810\ mm^2} = 97.7\ MPa$; $\tau = \frac{F_s}{A_s} = \frac{80000\ N}{1810\ mm^2} = 44.2\ MPa$

$\tau_{MAX} = \sqrt{\left(\frac{97.7}{2}\right)^2 + 44.2^2} = \underline{65.9\ MPa}$

10-41

AT B: $V = 2750$ LB : $M = 82500$ LB·IN



$\tau_{MAX} = \sqrt{(\sigma/2)^2 + \tau^2}$

FOR BEAM CROSS SECTION:

$A = bh = (2)(6) = 12.00\ IN^2$

$S = bh^2/6 = (2)(6)^2/6 = 12.00\ IN^3$

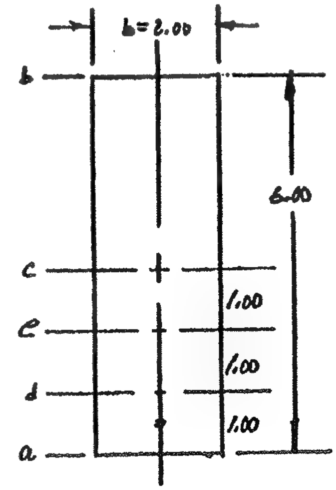
$I = bh^3/12 = (2)(6)^3/12 = 36.00\ IN^4$

AT a: $\tau = 0$: $\sigma = M/S$

$\sigma = \dfrac{82500\ LB \cdot IN}{12.00\ IN^2} = 6875\ PSI$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{\sigma}{2} = 3438\ PSI$

AT b: SAME AS (a)



AT c: $\sigma = 0$: $\tau = \tau_{MAX} = 3V/2A$

$\tau_{MAX} = \dfrac{3(2750)}{2(12)} = 344\ PSI$

AT d: $\sigma = \dfrac{My}{I} = \dfrac{(82500)(2.0)}{36.0} = 4583\ PSI$

$\tau = \dfrac{VQ}{Ib} = \dfrac{(2750)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 191\ PSI$

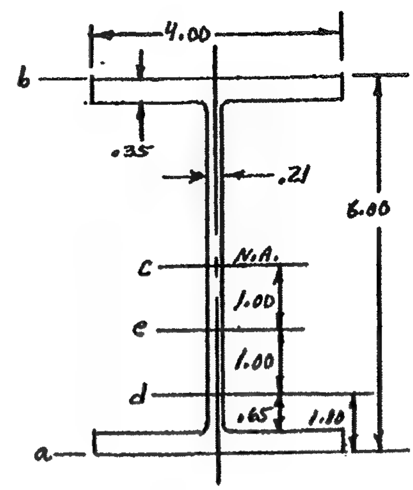
$\tau_{MAX} = \sqrt{\left(\frac{4583}{2}\right)^2 + 191^2} = 2300\ PSI$

AT e: $\sigma = \dfrac{My}{I} = \dfrac{(82500)(1.0)}{36.0} = 2292\ PSI$

$\tau = \dfrac{VQ}{Ib} = \dfrac{(2750)(2.0)(2.0)(2.0)}{(36.0)(2.0)} = 306\ PSI$

$\tau_{MAX} = \sqrt{\left(\frac{2292}{2}\right)^2 + 306^2} = 1186\ PSI$

10-42 FROM 10-41: $V = 2750$ LB : $M = 82500$ LB·IN



FOR ALUM I 6×4.692 : $A = 3.990\ IN^2$

$S = 8.50\ IN^3$: $I = 25.50\ IN^4$

AT a AND b: $\tau = 0$, $\sigma = M/S$

$\sigma = \dfrac{82500\ LB \cdot IN}{8.50\ IN^3} = 9706\ PSI$

$\tau_{MAX} = \sigma/2 = 4853\ PSI$

AT c: $\sigma = 0$: $\tau = \tau_{MAX} = \dfrac{VQ}{It}$

$Q = (.35)(4.00)(2.825) + (.21)(2.65)(1.325)$

$Q = 3.955 + 0.737 = 4.692\ IN^3$

$\tau_{MAX} = \dfrac{(2750)(4.692)}{(25.50)(.21)} = 2410\ PSI$

AT d:

$\sigma = \dfrac{My}{I} = \dfrac{(82500)(2.0)}{25.50} = 6471\ PSI$

$\tau = \dfrac{VQ}{It} = \dfrac{(2750)(3.999)}{(25.50)(0.21)} = 8213\ PSI$

$Q = 3.955 + (.21)(0.65)(2.325) = 3.999\ IN^3$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{6471}{2}\right)^2 + 8213^2} = 8827\ PSI$

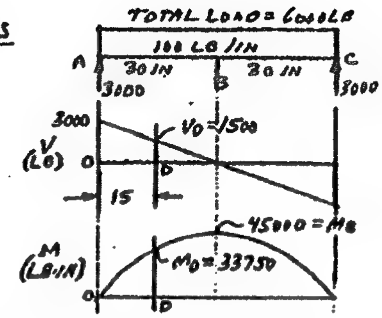
AT e:

$\sigma = \dfrac{My}{I} = \dfrac{82500(1.0)}{25.50} = 3235\ PSI$: $Q = 3.955 + (0.21)(1.65)(0.825) = 4.241\ IN^3$

$\tau = \dfrac{VQ}{It} = \dfrac{(2750)(4.241)}{(25.50)(0.21)} = 2178\ PSI$

$\tau_{MAX} = \sqrt{\left(\frac{3235}{2}\right)^2 + 2178^2} = 2713\ PSI$

10-43 SEE 10-41 FOR BEAM SECTION PROPERTIES



AT MIDDLE OF BEAM - B:

AT a AND b: $\sigma = \frac{M}{S} = \frac{45000}{12.00} = 3750$ psi

$\tau = 0$: $\tau_{max} = \frac{\sigma}{2} = 1875$ psi

AT c: $\sigma = 0$: $\tau = 0$: $\tau_{max} = 0$

AT d: $\sigma = \frac{Mc}{I} = \frac{45000(2.0)}{36.0} = 2500$ psi

$\tau = 0$: $\tau_{max} = \sigma/2 = 1250$ psi

AT e: $\sigma = \frac{Mc}{I} = \frac{45000(1.0)}{36.0} = 1250$ psi : $\tau = 0$: $\tau_{max} = \frac{\sigma}{2} = 625$ psi

AT SUPPORTS A AND C: $V = 3000$ LB : $M = 0$: $\sigma = 0$

AT a AND b: $\tau = 0$: $\tau_{max} = 0$

AT c: $\tau = 3V/2A = 3(3000)/(2)(12) = 375$ psi $= \tau_{max}$

AT d: $\tau = \tau_{max} = \frac{VQ}{It} = \frac{(3000)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 208$ psi

AT e: $\tau = \tau_{max} = \frac{VQ}{It} = \frac{(3000)(2.0)(2.0)(2.0)}{(36.0)(2.0)} = 333$ psi

AT D - 15 IN FROM A: $V = 1500$ LB : $M = 33750$ LB·IN

AT a AND b: $\tau = 0$: $\sigma = \frac{M}{S} = \frac{33750}{12.0} = 2813$ psi : $\tau_{max} = \frac{\sigma}{2} = 1406$ psi

AT c: $\sigma = 0$: $\tau = \frac{3V}{2A} = \frac{3(1500)}{2(12)} = 188$ psi $= \tau_{max}$

AT d: $\sigma = \frac{Mc}{I} = \frac{(33750)(2.0)}{36.0} = 1875$ psi

$\tau = \frac{VQ}{It} = \frac{(1500)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 104$ psi

$\tau_{max} = \sqrt{\left(\frac{1875}{2}\right)^2 + 104^2} = 943$ psi

AT e: $\sigma = \frac{Mc}{I} = \frac{(33750)(1.0)}{36.0} = 938$ psi

$\tau = \frac{VQ}{It} = \frac{(1500)(2.0)(2.0)(2.0)}{(36.0)(2.0)} = 167$ psi

$\tau_{max} = \sqrt{\left(\frac{938}{2}\right)^2 + 167^2} = 498$ psi

Noncircular Sections – Combined Normal and Torsional Shear Stresses

10-44 FROM FIG 4-27: $Q = 0.208a^3 = 0.208(25)^3 = 3250\ mm^3$

$\tau = T/Q = \frac{245\,000\ N\cdot mm}{3250\ mm^3} = 75.4$ MPa : $\sigma = \frac{P}{A} = \frac{75\,000\ N}{(25)^2\ mm^2} = 120$ MPa

$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{120}{2}\right)^2 + 75.4^2} = 96.4$ MPa

10-45 FROM FIG. 4-27: $Q = \frac{bh^2}{3 + 1.8(h/b)} = \frac{50(30)^2}{3 + 1.8(30/50)} = 11\,029\ mm^3$

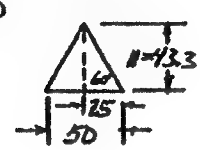
$\tau = \frac{T}{Q} = \frac{525\,000\ N\cdot mm}{11\,029\ mm^3} = 47.6$ MPa : $\sigma = \frac{P}{A} = \frac{175000}{(30)(50)} = 116.7$ MPa

$\tau_{max} = \sqrt{\left(\frac{116.7}{2}\right)^2 + 47.6^2} = 75.3$ MPa

10-46 FROM FIG. 4-27: $Q = 0.050\,a^3 = 0.050(50)^3 = 6250\ mm^3$

$\tau = \frac{T}{Q} = \frac{775\,000\ N\cdot mm}{6250\ mm^3} = 124\ MPa$; $\sigma = \frac{P}{A} = \frac{115\,000}{\frac{1}{2}(50)(43.3)} = 106\ MPa$

$\tau_{MAX} = \sqrt{\left(\frac{106}{2}\right)^2 + 124^2} = \underline{135\ MPa}$



10-47 $S_y = 50\ ksi$; $\sigma_d = \frac{S_y}{3} = \frac{50000}{3} = 16667\ psi = P/A$

(a) $P = \sigma \cdot A = (16667\ LB/IN^2)(2.44\ IN^2) = 40667\ LB$

$\tau_{MAX} = \frac{\sigma}{2} = \underline{8333\ PSI}$ $a = b = 3.00\ IN$; $t = 0.233\ IN$

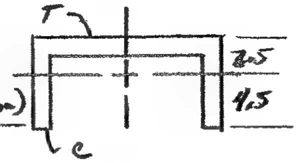
(b) FROM FIG. 4-27: $Q = 2t(a-t)(b-t) = 2(0.233)(2.767)(2.767) = 3.568\ IN^3$

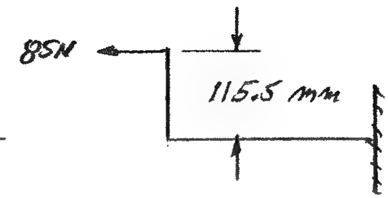
$\tau = \frac{T}{Q} = \frac{(950)(12)(18\cdot IN)}{3.568\ IN^3} = 3195\ psi$

$\tau_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{16667}{2}\right)^2 + 3195^2} = \underline{8925\ psi} = \frac{S_y}{2N}$

$N = \frac{S_y}{2\tau_{MAX}} = \frac{50000}{2(8925)} = \underline{2.80}$

10-48

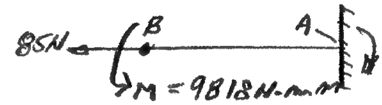




TENSILE: TOP FROM B TO A.

$$\sigma = \frac{85N}{33.0\,mm^2} + \frac{(9818\,N{\cdot}mm)(2.5\,mm)}{128\,mm^4}$$

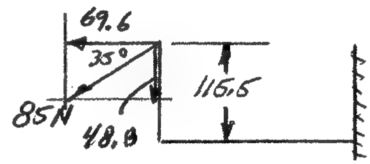
$$\sigma = 2.58 + 191.7 = 194.3\ N/mm^2 = 194\ MPa$$



COMPRESSIVE: BOTTOM FROM B TO A

$$\sigma = 2.58 - \frac{(9818\,N{\cdot}mm)(4.5\,mm)}{128\,mm^4} = -343\,MPa$$

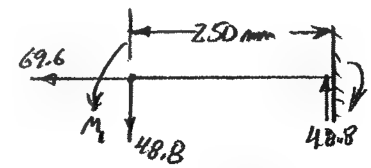
10-49 TENSILE: TOP AT A



$$\sigma = \frac{69.6N}{33.0\,mm^2} + \frac{(20239\,N{\cdot}mm)(2.5\,mm)}{128\,mm^2} = 397\,MPa$$

COMPRESSIVE: BOTTOM

$$\sigma = 2.11\,MPa - \frac{(20239)(4.5)}{128} = -709\,MPa$$

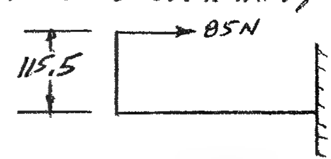


$M_1 = 69.6(115.5) = 8039\ N{\cdot}mm$

$M_2 = 48.8(250) = 12200\ N{\cdot}mm$

$M_1 + M_2 = 20239\ N{\cdot}mm$

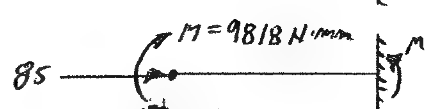
10-50 TENSION: BOTTOM



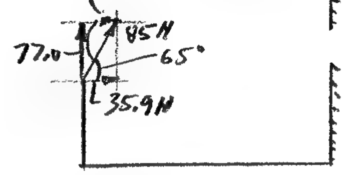
$$\sigma = \frac{-85N}{33.0\,mm^2} + \frac{(9818\,N{\cdot}mm)(4.5\,mm)}{128\,mm^4} = 343\,MPa$$

COMPRESSIVE: TOP

$$\sigma = \frac{-85N}{33.0\,mm^2} - \frac{(9818)(2.5)}{128} = -194\,MPa$$



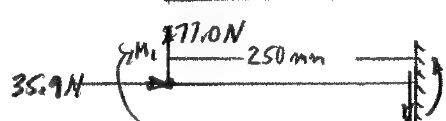
10-51 TENSION: BOTTOM



$$\sigma = \frac{-35.9N}{33.0\,mm^2} + \frac{(23396\,N{\cdot}mm)(4.5\,mm)}{128\,mm^4}$$

$$\sigma = -1.09 + 822.5 = 821\,MPa$$

COMPRESSIVE: TOP



$$\sigma = \frac{-35.9}{33.0} - \frac{(23396)(2.5)}{128} = -1.09 - 457.0$$

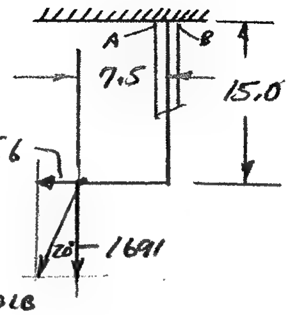
$$\sigma = -458\ MPa$$

$M_1 = (35.9)(115.5) = 4146\ N{\cdot}mm$

$M_2 = (77.0)(250) = 19250\ N{\cdot}mm$

$M_1 + M_2 = 23396\ N{\cdot}mm$

10-52 3×3×1/4 STEEL TUBING, $S = 2.01\ IN^3$, $A = 2.44\ IN^2$



$M_1 = (615.6\ LB)(15.0\ IN) = 9234\ LB \cdot IN$

$M_2 = (1691\ LB)(7.5\ IN) = 12\,683\ LB \cdot IN$

AXIAL LOAD = 1691 LB ↓ TENSION

NET BENDING MOMENT = $M_2 - M_1$

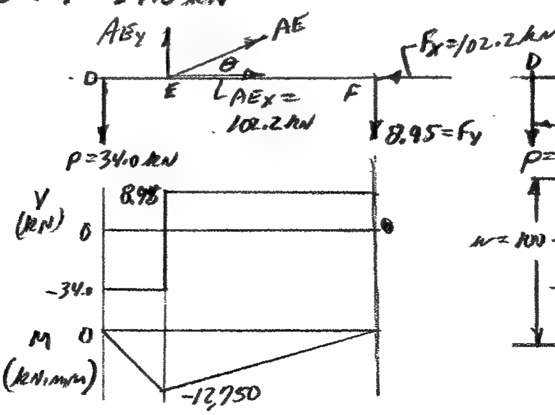
$M_{NET} = 12\,683 - 9234 = 3449\ LB \cdot IN$

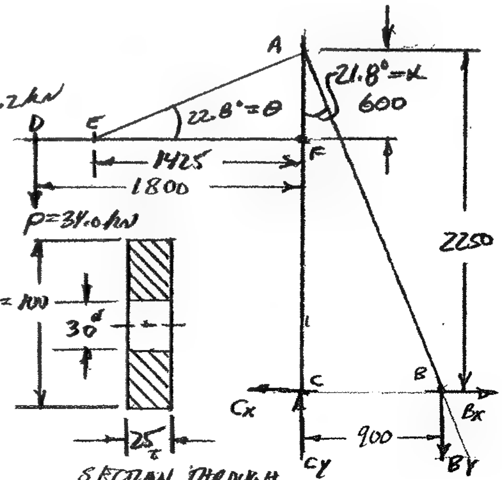
PRODUCES TENSION AT A, COMPR. AT B

$\sigma_A = \dfrac{(1691\ LB)}{2.44\ IN^2} + \dfrac{(3449\ LB \cdot IN)}{2.01\ IN^3} = 693 + 1716 = \underline{2409\ PSI\ TENSION}$

$\sigma_B = 693 - 1716 = \underline{-1023\ PSI\ COMPRESSION}$

10-53 LOAD = P = 34.0 kN





$\Sigma M_F = 0 = (34.0\ kN)(1800\ mm) - AE_y(1425\ mm)$

$AE_y = 42.95\ kN$

$\Sigma F_y = 0 = 42.95 - 34.0 - F_y$; $F_y = 8.95\ kN$ ↓

$AE = AE_y / \sin\theta = 42.95\ kN / \sin 22.8° = 110.8\ kN$

$AE_x = AE \cos\theta = (110.8\ kN)\cos 22.8° = 102.2\ kN$ ←

$\Sigma M_C = 0 = P(1800) - B_y(900)$

$B_y = 34.0\ kN(2) = 68\ kN$

$\Sigma F_y = 0 = C_y - 34.0 - 68.0$

$C_y = 102\ kN$

$\tan\alpha = B_x / B_y$

$B_x = B_y \tan\alpha = 68 \tan 21.8° = 27.2\ kN$

$C_x = B_x$

$AB = B_y / \cos\alpha = 68 / \cos 21.8° = 73.2\ kN$

AT E: FLAT PLATE WITH CENTRAL HOLE
APP. A-21-4, CURVE C. – BENDING

$\sigma_b = \dfrac{M K_t C}{I_{NET}} = \dfrac{K_t M w}{(w^3 - d^3)(t)} = \dfrac{(12\,750\ kN \cdot mm)(100\ mm)}{(100^3 - 30^3)(25)\ mm^4} \times \dfrac{1000\ N}{kN} = 52.4\ MPa$ BENDING

$d/w = 30/100 = 0.30 \Rightarrow K_t = 1.0$

TENSION-TOP; COMPR.-BOTTOM

DIRECT COMPRESSION $\quad \sigma_c = \dfrac{K_t AE_x}{A_{NET}} = \dfrac{(3.70)(102.2\ kN)}{(100 - 30)(25)\ mm^2} \times \dfrac{1000\ N}{kN} = 216\ MPa$ NEAR HOLE

$K_t = 3.70$ FOR CURVE B

COMBINATION OF σ_b AND σ_c IS PROBLEMATIC BECAUSE $\sigma_{b\,MAX}$ OCCURS NEAR TOP OR BOTTOM OF SECTION WHILE $\sigma_{c\,MAX}$ OCCURS NEAR CENTERLINE NEAR HOLE.

σ_{MAX} TENSILE IS AT E TO LEFT WHERE $\sigma_b = 52.4\ MPa$ AND NO COMPR. STRESS.

σ_{MAX} COMPR. IS TO RIGHT OF E WITH A VALUE BETWEEN 216 MPa AND 216 + 52.4 = 268 MPa ON LOWER PART OF THE CROSS SECTION.

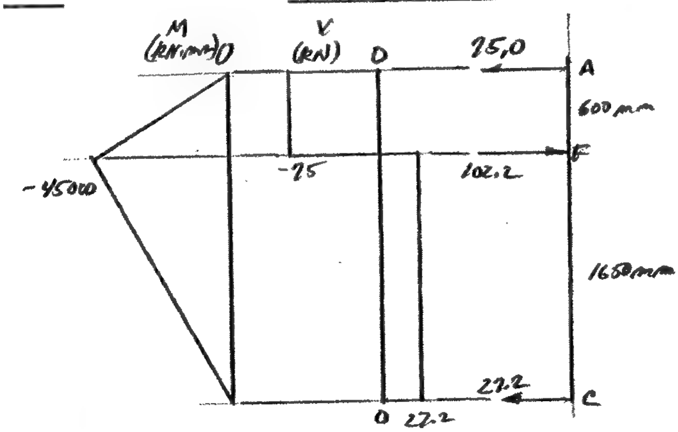
(NEXT PAGE)

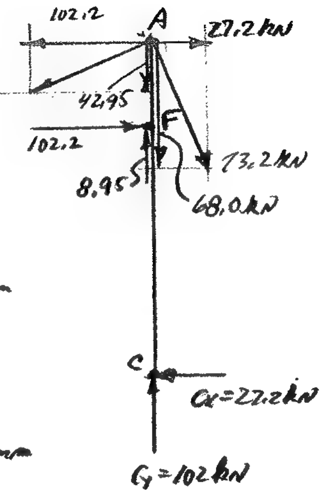
10-53 (CONTINUED)

MEMBER AFC - LOADS FROM FBD OF WHOLE STRUCTURE

AXIAL LOAD: AF = 42.95 + 68.0 = 110.95 kN COMPR.

CF = 110.95 − 8.95 = 102 kN COMPR.





BENDING STRESS AT F (SAME SECTION PROPERTIES AS AT E) ($K_t = 1.0$)

$$\sigma_{b_F} = \frac{M K_t C}{I_{NET}} = \frac{(75000\ kN\text{-}mm)\,w}{(w^3 - d^3)\,t} = \frac{(75000\ kN\text{-}mm)(100\ mm)}{(100^3 - 30^3)(25)\ mm^4} \times \frac{1000\ N}{kN}$$

$\sigma_{bF} = \pm 308\ MPa$ TENSILE ON RIGHT SIDE; COMPR. ON LEFT.

COMPRESSION NEAR F USE $P_F = 110.95$ kN ABOVE F. $K_t = 3.70$ FOR $d/w = 0.30$

$$\sigma_{CF} = \frac{-P_F K_t}{A_{NET}} = -\frac{(110.5\ kN)(3.70)}{(100-30)(25)\ mm^2} \times \frac{1000\ N}{kN} = -233.6\ MPa \text{ NEAR HOLE.}$$

COMBINED σ_b AND σ_c IS PROBLEMATIC AS AT E ON MEMBER DEF.
BECAUSE EFFECT OF K_t IS LOCALIZED NEAR HOLE, σ_c IS LIKELY TO BE MUCH LESS NEAR OUTSIDE SURFACES WHERE σ_b IS MAXIMUM.

ASSUME $\sigma_c = \frac{-P_F}{A_{NET}} = \frac{(-110.5\ kN)(1000\ N/kN)}{(100-30)(25)\ mm^2} = -63.1\ MPa$ NEAR OUTSIDE EDGE ABOVE F.

$$\sigma_c = \frac{(-102\ kN)(1000\ N/kN)}{(100-30)(25)\ mm^2} = -58.3\ MPa \text{ BELOW F.}$$

COMBINED STRESS − TENSION − BELOW F.

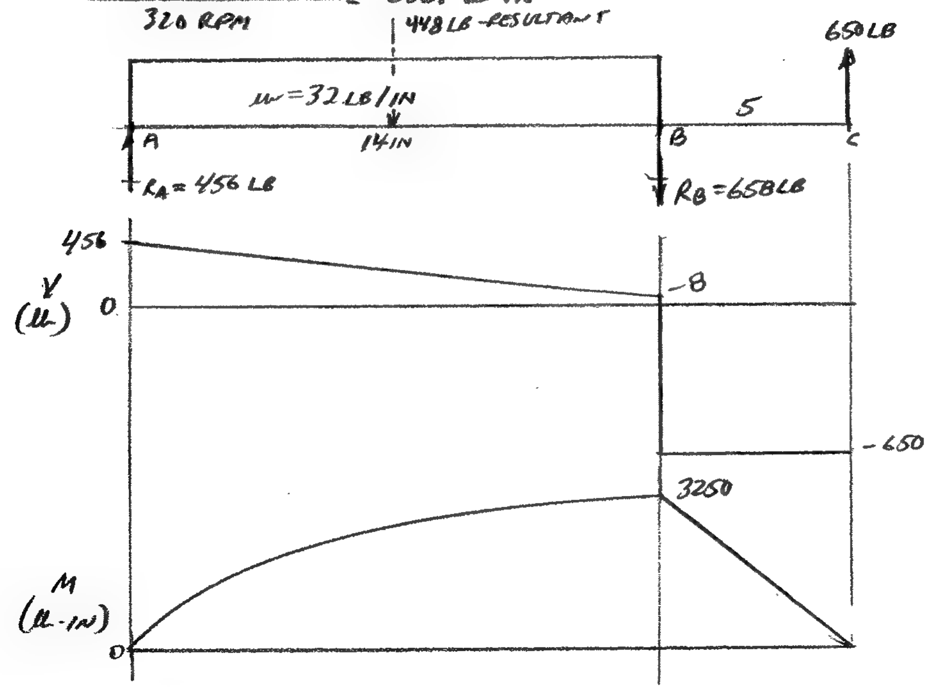
$$\sigma_T = +308\ MPa - 58.3 = 249.7\ MPa \text{ ON RIGHT SIDE.}$$

COMBINED STRESS − COMPRESSION − ABOVE F.

$$\sigma_c = -308\ MPa - 63.1 = -371\ MPa \text{ ON LEFT SIDE.}$$

10-54 SHAFT: POWER = 30.5 HP; n = 320 RPM

$$T = \frac{63\,000(30.5\ HP)}{320\ RPM} = 6064\ LB\text{-}IN$$



MAXIMUM BENDING AND TORSION OCCUR NEAR B.
USE EQUIVALENT TORQUE METHOD, EQ (10-8).
MAX STRESS OCCURS AT STEP IN 1.00 IN DIA. SHAFT

$$Z_P = \frac{\pi D^3}{16} = \frac{\pi(1.0\ IN)^3}{16} = 0.196\ IN^3$$

$$\frac{r}{d} = \frac{0.03}{1.0} = 0.03;\ \frac{D}{d} = \frac{1.25}{1.0} = 1.25;\ \overset{A21\text{-}9}{K_{t_{BEND}} = 2.40};\ \overset{A21\text{-}7}{K_{t_{TORS}} = 1.77}$$

$$\underline{T_e} = \sqrt{(K_{tB}M)^2 + (K_{tT}T)^2} = \sqrt{[2.40(3250)]^2 + [1.77(6064)]^2} = \underline{13\,268\ LB\text{-}IN}$$

$$\tau_{MAX} = \frac{T_e}{Z_P} = \frac{13\,268\ LB\cdot IN}{0.196\ IN^3} = 67\,694\ psi$$

LET N = 4; $\tau_d = S_y/2N = S_y/8$

REQ'D $S_y = 8\tau_{MAX} = = 8(67694) = 541{,}550\ psi = \underline{542\ ksi}$

SPECIFY AISI 1040 OQT 1100 S_y = 552 KSI, 24% ELONGATION
OTHER STEELS COULD BE USED WITH S_y > 542 KSI AND GOOD DUCTILITY.

10-55 FIND STRESS ON ELEMENTS M AND N. $P = 450\,N$

FORCE P ACTS 18 mm TO RIGHT OF CENTERLINE AND 8 mm ABOVE CL.

AT M:

$$\text{AXIAL STRESS} = \frac{P}{A} = \frac{450N}{(20)(28)\,mm^2} = 0.804\ MPa \text{ TENSION}$$

BENDING MOMENT: $M_1 = (450\,N)(8\,mm) = 3600\,N\cdot mm$

$$S = \frac{bh^2}{6} = \frac{(20)(28)^2}{6} = 2613\ mm^3$$

$$\sigma_{M_1} = \frac{M_1}{S} = \frac{3600\,N\cdot mm}{2613\,mm^3} = 1.377\,MPa \text{ TENSION AT M.}$$

$$\sigma_{M_{TOTAL}} = 0.804 + 1.377 = 2.181\,MPa \text{ TENSION AT M}$$

AT N:

AXIAL STRESS = 0.804 MPa AS AT M. (TENSION)

BENDING MOMENT: $M_2 = (450N)(18mm) = 8100\,N\cdot mm$

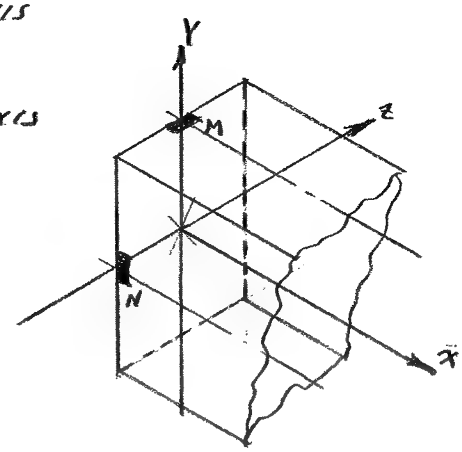
$$S = \frac{h(b)^2}{6} = \frac{(28)(20)^2}{6} = 1867\ mm^3$$

$$\sigma_{M_2} = \frac{M_2}{S} = \frac{8100\,N\cdot mm}{1867\,mm^3} = 4.339\,MPa \text{ COMPRESSION AT N}$$

$$\sigma_{N_{TOTAL}} = 0.804 - 4.339 = -3.535\,MPa \text{ COMPRESSION AT N.}$$

ELEMENT M IS ON NEUTRAL AXIS FOR BENDING ABOUT Y-AXIS.

ELEMENT N IS ON NEUTRAL AXIS FOR BENDING ABOUT Z-AXIS



Combined Stresses – Mohr's Circle

NOTE: The complete solutions for problems 10-56 – 10-105 require the construction of the complete Mohr's circle and the drawing of the principal stress element and the maximum shear stress element. Listed below are the significant numerical results. Following the list are representative examples of the complete solutions. Note that the problems fall into groups of similar forms as described below.

A. **Problems 10-56 to 10-59**: The x-axis on the Mohr's circle lies in the ***first quadrant.***

   Problems 10-60 to 10-63: The x-axis on the Mohr's circle lies in the ***second quadrant.***

   Problems 10-64 to 10-67: The x-axis on the Mohr's circle lies in the ***third quadrant.***

   Problems 10-68 to 10-71: The x-axis on the Mohr's circle lies in the ***fourth quadrant.***

   Problems 10-72 to 10-79: The x-axis on the Mohr's circle could lie in the ***any quadrant.***

   Problems 10-80 to 10-83: The x-axis on the Mohr's circle lies ***along the original X-axis*** and the ***principal stresses are the same as the normal stresses on the given element.***

B. **Problems 10-84 to 10-95:** The Mohr's circle from the given data ***results in both principal stresses having the same sign.*** For this class of problems, the supplementary circle is drawn using the procedures discussed in Section 10-11 of the text. The results include three principal stresses where $\sigma_1 > \sigma_2 > \sigma_3$. Also, the maximum shear stress is found from the radius of the circle containing σ_1 and σ_3 and is equal to $\sigma_1/2$ or $\sigma_3/2$ whichever has the greatest magnitude. Angles of rotation of the resulting elements are not requested.

C. **Problems 10-96 to 10-105:** The Mohr's circles from earlier problems are used to find the ***stress condition on the element at some specified angle of rotation.*** The listed results include the two normal stresses and the shear stress on the specified element.

Combined Uniaxial Normal and Shear Stresses

Problems 10-106 to 10-109: These use the same data as Problems 10-72 to 10-75 and each has a given uniaxial normal stress, σ_x, and a shear stress, τ_{xy}. For this special case, Equation 10-2 can be used to compute the maximum shear stress directly. The solution method is similar to that used in Problems 10-21 to 10-28.

$$\tau_{max} = \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} \qquad \text{(Equation 10-2)}$$

CHAPTER 10 – PROBLEMS 10-56 TO 10-83

Prob. No.	σ_1	σ_2	ϕ (deg)	τ_{max}	σ_{avg}	ϕ' (deg)
10-56	315.4 MPa	-115.4 MPa	10.9 cw	215.4 MPa	100.0 MPa	34.1 ccw
10-57	255.2 MPa	-55.2 MPa	7.5 cw	155.2 MPa	100.0 MPa	37.5 ccw
10-58	110.0 MPa	-40.0 MPa	26.6 cw	75.0 MPa	35.0 MPa	18.4 ccw
10-59	202.1 MPa	-42.1 MPa	27.5 cw	122.1 MPa	80.0 MPa	17.5 ccw
10-60	23.5 ksi	-8.5 ksi	19.3 ccw	16.0 ksi	7.5 ksi	64.3 ccw
10-61	42.8 ksi	-29.8 ksi	14.9 ccw	36.3 ksi	6.5 ksi	59.9 ccw
10-62	79.7 ksi	-9.7 ksi	31.7 ccw	44.7 ksi	35.0 ksi	76.7 ccw
10-63	36.6 ksi	-54.6 ksi	13.0 ccw	45.6 ksi	-9.0 ksi	58.0 ccw
10-64	677.6 kPa	-977.6 kPa	77.5 ccw	827.6 kPa	-150.0 kPa	57.5 cw
10-65	137.8 kPa	-587.8 kPa	84.0 ccw	362.8 kPa	-225.0 kPa	51.0 cw
10-66	327.0 kPa	-1202.0 kPa	60.9 ccw	764.5 kPa	-437.5 kPa	74.1 cw
10-67	79.9 kPa	-354.9 kPa	74.8 ccw	217.4 kPa	-137.5 kPa	60.2 cw
10-68	570.0 psi	-2070.0 psi	71.3 cw	1320.0 psi	-750.0 psi	26.3 cw
10-69	1676.1 psi	-6676.1 psi	81.7 cw	4176.1 psi	-2500.0 psi	36.7 cw
10-70	4180.0 psi	-5180.0 psi	71.6 cw	4680.0 psi	-500.0 psi	26.6 cw
10-71	8600.7 psi	-150.7 psi	89.5 cw	4375.7 psi	4225.0 psi	44.5 cw
10-72	360.2 MPa	-100.2 MPa	27.8 ccw	230.2 MPa	130.0 MPa	72.8 ccw
10-73	1827.1 kPa	-377.1 kPa	24.4 cw	1102.1 kPa	725.0 kPa	20.6 ccw
10-74	23.9 ksi	-1.9 ksi	15.9 cw	12.9 ksi	11.0 ksi	29.1 ccw
10-75	7971.2 psi	-1221.2 psi	21.4 ccw	4596.2 psi	3375.0 psi	66.4 ccw
10-76	4.4 ksi	-32.4 ksi	20.3 cw	18.4 ksi	-14.0 ksi	24.7 ccw
10-77	527.6 MPa	-87.6 MPa	67.8 cw	307.6 MPa	220.0 MPa	22.8 cw
10-78	321.0 MPa	-61.0 MPa	66.4 ccw	191.0 MPa	130.0 MPa	68.6 cw
10-79	344.5 kPa	-1904.5 kPa	23.0 ccw	1124.5 kPa	-780.0 kPa	68.0 ccw
10-80	225.0 MPa	-85.0 MPa	0.0	155.0 MPa	70.0 MPa	45.0 ccw
10-81	6250.0 psi	-875.0 psi	0.0	3562.5 psi	2687.5 psi	45.0 ccw
10-82	775.0 kPa	-145.0 kPa	0.0	460.0 kPa	315.0 kPa	45.0 ccw
10-83	38.6 ksi	-13.4 ksi	0.0	26.0 ksi	12.6 ksi	45.0 ccw

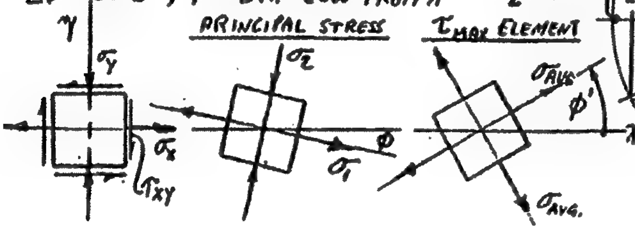
PROB. 10-56 DETAILED SOLUTION

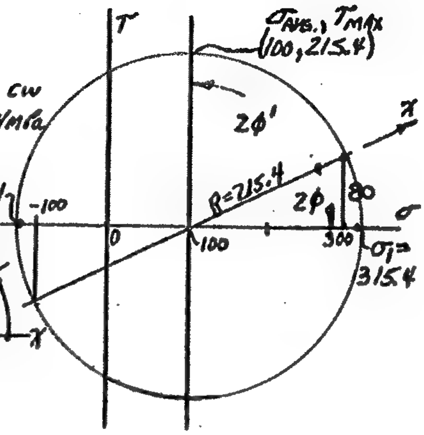
GIVEN: $\sigma_x = 300\,MPa$; $\sigma_y = -100\,MPa$; $\tau_{xy} = 80\,MPa$ cw

$\sigma_1 = 315.4\,MPa$; $\sigma_2 = -115.4\,MPa$; $\tau_{MAX} = 215.4\,MPa$

$\sigma_{AVG} = 100\,MPa$; $2\phi = 21.8°$; $\phi = 10.9°$ CW FROM x.

$2\phi' = 68.2°$; $\phi' = 34.1°$ CCW FROM x





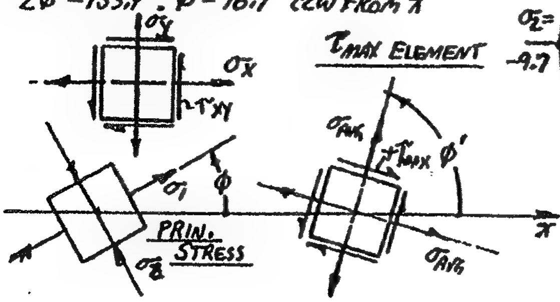
PROB 10-62 DETAILED SOLUTION

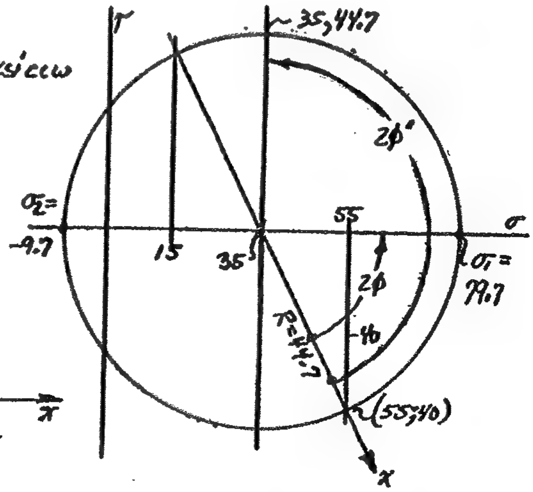
GIVEN: $\sigma_x = 55\,ksi$; $\sigma_y = 15\,ksi$; $\tau_{xy} = 40\,ksi$ ccw

$\sigma_1 = 79.7\,ksi$; $\sigma_2 = -9.7\,ksi$; $\tau_{MAX} = 44.7\,ksi$

$2\phi = 63.4°$; $\phi = 31.7°$ CCW FROM x

$2\phi' = 153.4°$; $\phi' = 76.7°$ CCW FROM x





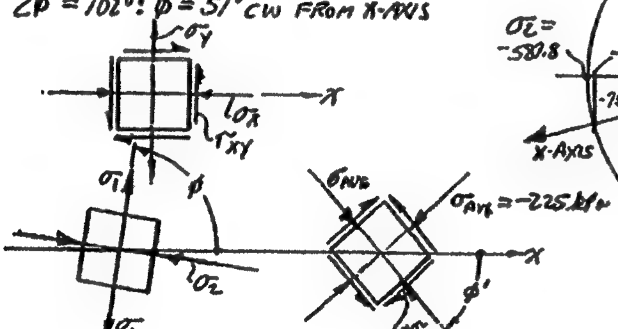
PROB. 10-65 DETAILED SOLUTION

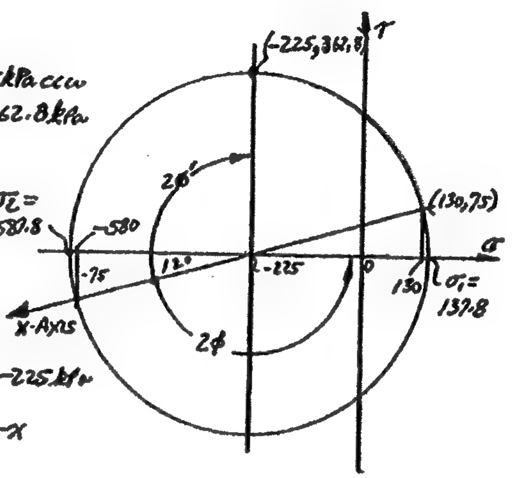
GIVEN: $\sigma_x = -580\,kPa$: $\sigma_y = 130\,kPa$: $\tau_{xy} = 75\,kPa$ ccw

$\sigma_1 = 137.8\,kPa$: $\sigma_2 = -587.8\,kPa$: $\tau_{MAX} = 362.8\,kPa$

$2\phi = 168°$: $\phi = 84°$ CCW FROM X-AXIS

$2\phi' = 102°$: $\phi' = 51°$ CW FROM X-AXIS





CHAPTER 10 – PROBLEMS 10-84 TO 10-95

Prob. No.	σ_1	σ_2	σ_3	τ_{max}
10-84	328.1 MPa	71.9 MPa	0.0 MPa	164.0 MPa
10-85	264.0 MPa	136.0 MPa	0.0 MPa	132.0 MPa
10-86	214.5 MPa	75.5 MPa	0.0 MPa	107.2 MPa
10-87	161.1 MPa	68.9 MPa	0.0 MPa	80.5 MPa
10-88	35.0 ksi	10.0 ksi	0.0 ksi	17.5 ksi
10-89	41.8 ksi	21.2 ksi	0.0 ksi	20.9 ksi
10-90	55.6 ksi	14.4 ksi	0.0 ksi	27.8 ksi
10-91	62.9 ksi	19.1 ksi	0.0 ksi	31.5 ksi
10-92	0.0 kPa	-307.9 kPa	-867.1 kPa	433.5 kPa
10-93	0.0 kPa	-37.5 kPa	-337.5 kPa	168.8 kPa
10-94	0.0 psi	-295.7 psi	-1804.3 psi	902.1 psi
10-95	0.0 psi	-2167.6 psi	-6832.4 psi	3416.2 psi

CHAPTER 10 – PROBLEMS 10-96 TO 10-105

Prob. No.	σ_A	$\sigma_{A'}$	τ_A
10-96	130.7 MPa	69.3 MPa	213.2 MPa cw
10-97	269.3 MPa	-69.3 MPa	133.2 MPa ccw
10-98	-37.9 MPa	197.9 MPa	31.6 MPa ccw
10-99	19.1 ksi	-6.1 ksi	34.0 ksi ccw
10-100	3.6 ksi	-21.6 ksi	43.9 ksi cw
10-101	-300.0 kPa	-150.0 kPa	355.0 kPa cw
10-102	-2010.3 psi	510.3 psi	392.6 psi cw
10-103	-765.5 psi	-234.5 psi	4672.5 psi cw
10-104	8363.5 psi	86.5 psi	1421.2 psi cw
10-105	894.8 kPa	555.2 kPa	1088.9 kPa ccw

CHAPTER 10 – PROBLEMS 10-106 TO 10-109

10-106 $\tau_{MAX} = \sqrt{(\sigma_X/2)^2 + (\tau_{XY})^2} = \sqrt{(260/2)^2 + (190)^2} = \underline{230.2\text{ MPa}}$

Using a similar technique:

10-107 $\tau_{MAX} = \underline{1102\text{ kPa}}$ 10-108 $\underline{12.9\text{ ksi}}$ 10-109 $\underline{4596\text{ psi}}$

PROB. 10-70 DETAILED SOLUTION

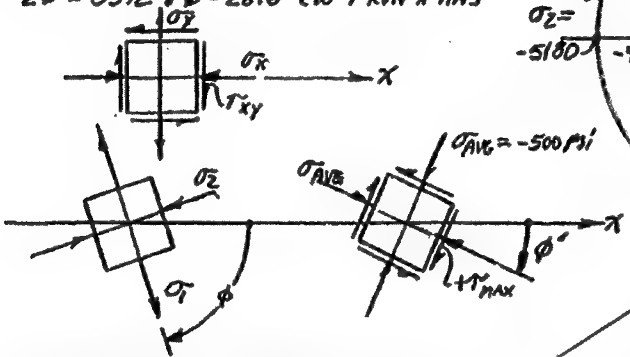
GIVEN: $\sigma_x = -4250$ psi; $\sigma_y = 3250$ psi

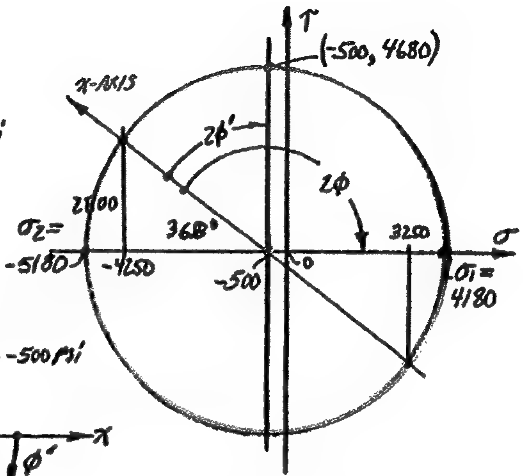
$\tau_{xy} = 2800$ psi CW

$\sigma_1 = 4180$ psi; $\sigma_2 = -5180$ psi; $\tau_{MAX} = 4680$ psi

$2\phi = 143.2^\circ$; $\phi = 71.6^\circ$ CW FROM X-AXIS

$2\phi' = 53.2^\circ$; $\phi' = 26.6^\circ$ CW FROM X-AXIS





PROB. 10-87 DETAILED SOLUTION

GIVEN: $\sigma_x = 150$ MPa; $\sigma_y = 80$ MPa

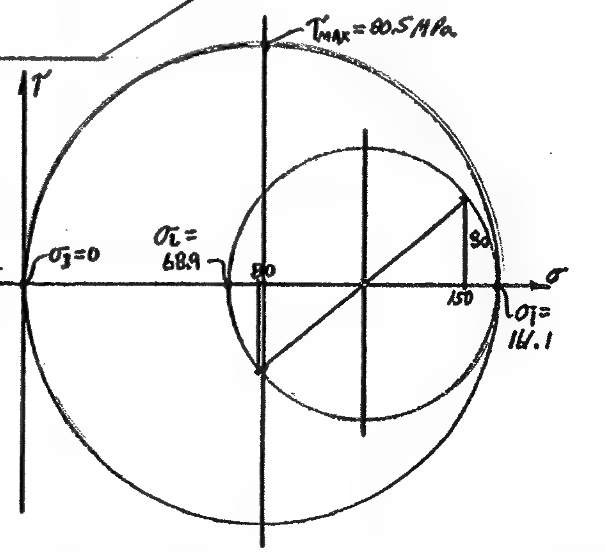
$\tau_{xy} = 30$ MPa CW

PRIMARY MOHR'S CIRCLE GIVES

$\sigma_1 = 161.1$ MPa, $\sigma_2 = 68.9$ MPa;

BECAUSE BOTH HAVE THE SAME SIGN, SECONDARY CIRCLE IS DRAWN WITH $\sigma_3 = 0$. THEN

$\tau_{MAX} = \sigma_1/2 = 80.5$ MPa



PROB 10-96 DETAILED SOLUTION

BASIC MOHR'S CIRCLE FROM PROB 10-56

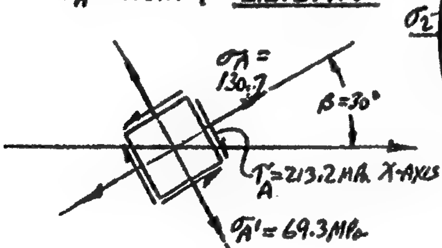
FOR $\beta = 30^\circ$ CCW FROM X-AXIS

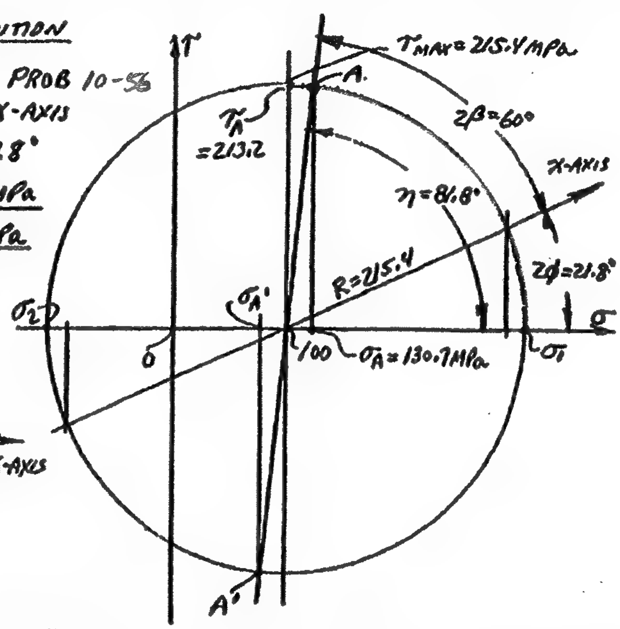
$\eta = 2\phi + 2\beta = 21.8 + 60 = 81.8^\circ$

$\sigma_A = 100 + R\cos\eta = \underline{130.7\text{ MPa}}$

$\sigma_{A'} = 100 - R\cos\eta = \underline{69.3\text{ MPa}}$

$\tau_A = R\sin\eta = \underline{213.2\text{ MPa}}$





SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.
Input data in shaded elements

Aluminum 6061-T6

Material Properties *SI Metric Units*

Modulus of Elasticity	69.0 x 10^9 Pa
Poisson's Ratio	0.33

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-110

Strain from Gage 1	1480 x10^{-6} m/m
Strain from Gage 2	165 x10^{-6} m/m
Strain from Gage 3	428 x10^{-6} m/m

Results:

Max Principal Strain	1902 x10^{-6} m/m
Min Principal Strain	6 x10^{-6} m/m
Angle β	-28.2 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	147 Mpa
Min Principal Stress	49.1 Mpa
Max Shear Strain	1897 radians [Dimensionless]
Max Shear Stress	49.2 MPa [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 73.7 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-118

Strain from Gage 1	1480 x10^{-6} m/m
Strain from Gage 2	165 x10^{-6} m/m
Strain from Gage 3	428 x10^{-6} m/m

Results:

Max Principal Strain	1494 x10^{-6} m/m
Min Principal Strain	-112 x10^{-6} m/m
Angle β	-5.4 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	113 Mpa
Min Principal Stress	29.5 MPa
Max Shear Strain	1607 radians [Dimensionless]
Max Shear Stress	41.7 MPa [in plane of initial element]

******Only when Max and Min principal stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 56.4 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.

Input data in shaded elements

Aluminum 7075-T6

Material Properties *SI Metric Units*

Modulus of Elasticity	71.7	$\times 10^9$ Pa
Poisson's Ratio	0.33	

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-111

Strain from Gage 1	853	$\times 10^{-6}$ m/m
Strain from Gage 2	406	$\times 10^{-6}$ m/m
Strain from Gage 3	641	$\times 10^{-6}$ m/m

Results:

Max Principal Strain	1104	$\times 10^{-6}$ m/m
Min Principal Strain	390	$\times 10^{-6}$ m/m
Angle β	-36.4	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	99.2	Mpa
Min Principal Stress	60.7	Mpa
Max Shear Strain	714	$\times 10^{-6}$ radians [Dimensionless]
Max Shear Stress	19.3	MPa [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 49.6 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-119

Strain from Gage 1	853	$\times 10^{-6}$ m/m
Strain from Gage 2	406	$\times 10^{-6}$ m/m
Strain from Gage 3	641	$\times 10^{-6}$ m/m

Results:

Max Principal Strain	892	$\times 10^{-6}$ m/m
Min Principal Strain	375	$\times 10^{-6}$ m/m
Angle β	-15.9	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	81.7	Mpa
Min Principal Stress	53.9	MPa
Max Shear Strain	516	$\times 10^{-6}$ radians [Dimensionless]
Max Shear Stress	13.9	MPa [in plane of initial element]

******Only when Max and Min principal stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 40.8 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.
Input data in shaded elements

AISI 1040 cold drawn steel

Material Properties *SI Metric Units*

Modulus of Elasticity 207.0 x 10^9 Pa
Poisson's Ratio 0.29

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-112

Strain from Gage 1 389 x10^{-6} m/m
Strain from Gage 2 737 x10^{-6} m/m
Strain from Gage 3 -290 x10^{-6} m/m

Results:

Max Principal Strain 816 x10^{-6} m/m
Min Principal Strain -717 x10^{-6} m/m

Angle β 31.9 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 137.5 Mpa
Min Principal Stress -108.6 Mpa

Max Shear Strain 1534 radians [Dimensionless]
Max Shear Stress 123.0 MPa [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 68.7 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-120

Strain from Gage 1 389 x10^{-6} m/m
Strain from Gage 2 737 x10^{-6} m/m
Strain from Gage 3 -290 x10^{-6} m/m

Results:

Max Principal Strain 882 x10^{-6} m/m
Min Principal Strain -324 x10^{-6} m/m

Angle β 39.7 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 178.0 Mpa
Min Principal Stress -15.5 MPa

Max Shear Strain 1206 radians [Dimensionless]
Max Shear Stress 96.8 MPa [in plane of initial element]

******Only when Max and Min principal stresses have the same sign******
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 89.0 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.
Input data in shaded elements

AISI 4140 OQT 900 steel

Material Properties *SI Metric Units*

Modulus of Elasticity	207.0 x 10^9 Pa
Poisson's Ratio	0.29

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-113

Strain from Gage 1	925 x10^{-6} m/m
Strain from Gage 2	-631 x10^{-6} m/m
Strain from Gage 3	552 x10^{-6} m/m

Results:

Max Principal Strain	2121 x10^{-6} m/m
Min Principal Strain	-644 x10^{-6} m/m
Angle β	-41.1 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	437.1 Mpa
Min Principal Stress	-6.5 Mpa
Max Shear Strain	2764 radians [Dimensionless]
Max Shear Stress	221.8 MPa [in plane of initial element]

*****Only when Max and Min Principal Stresses have the same sign*****

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 218.5 MPa

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-121

Strain from Gage 1	925 x10^{-6} m/m
Strain from Gage 2	-631 x10^{-6} m/m
Strain from Gage 3	552 x10^{-6} m/m

Results:

Max Principal Strain	1220 x10^{-6} m/m
Min Principal Strain	-656 x10^{-6} m/m
Angle β	-23.4 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	232.7 Mpa
Min Principal Stress	-68.3 MPa
Max Shear Strain	1876 radians [Dimensionless]
Max Shear Stress	150.5 MPa [in plane of initial element]

*****Only when Max and Min principal stresses have the same sign*****

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 116.4 MPa

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.
Input data in shaded elements

Copper C14500 hard

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity	17.0	x 10^6 psi
Poisson's Ratio	0.33	

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-114

Strain from Gage 1	169	x10^{-6} in/in
Strain from Gage 2	-266	x10^{-6} in/in
Strain from Gage 3	543	x10^{-6} in/in
Results:		
Max Principal Strain	1006	x10^{-6} in/in
Min Principal Strain	-294	x10^{-6} in/in
Angle β	36.6	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	17335	psi
Min Principal Stress	731	psi
Max Shear Strain	1299	radians [Dimensionless]
Max Shear Stress	8302	psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 8667 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-122

Strain from Gage 1	169	x10^{-6} in/in
Strain from Gage 2	-266	x10^{-6} in/in
Strain from Gage 3	543	x10^{-6} in/in
Results:		
Max Principal Strain	616	x10^{-6} in/in
Min Principal Strain	-319	x10^{-6} in/in
Angle β	-43.8	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	9748	psi
Min Principal Stress	-2204	psi
Max Shear Strain	935	radians [Dimensionless]
Max Shear Stress	5976	psi [in plane of initial element]

******Only when Max and Min principal stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 4874 psi

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.
Input data in shaded elements

Titanium Ti-6Al-4V, aged

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity	16.5	x 10^6 psi
Poisson's Ratio	0.3	

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-115

Strain from Gage 1	775	x10^{-6} in/in
Strain from Gage 2	369	x10^{-6} in/in
Strain from Gage 3	-318	x10^{-6} in/in
Results:		
Max Principal Strain	793	x10^{-6} in/in
Min Principal Strain	-336	x10^{-6} in/in
Angle β	7.2	degrees
		[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress	12548	psi
Min Principal Stress	-1776	psi
Max Shear Strain	1129	radians [Dimensionless]
Max Shear Stress	7162	psi [in plane of initial element]

*******Only when Max and Min Principal Stresses have the same sign*******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 6274 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-123

Strain from Gage 1	775	x10^{-6} in/in
Strain from Gage 2	369	x10^{-6} in/in
Strain from Gage 3	-318	x10^{-6} in/in
Results:		
Max Principal Strain	913	x10^{-6} in/in
Min Principal Strain	-363	x10^{-6} in/in
Angle β	19.2	degrees
		[From the axis of gage 1 to the nearer principal axis]
Max Principal Stress	14587	psi
Min Principal Stress	-1607	psi
Max Shear Strain	1276	radians [Dimensionless]
Max Shear Stress	8097	psi [in plane of initial element]

*******Only when Max and Min Principal Stresses have the same sign*******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 7294 psi

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.
Input data in shaded elements

Ductile Iron, ASTM A536, 80-55-6

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity	24.0	$\times 10^6$ psi
Poisson's Ratio	0.27	

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-116

Strain from Gage 1	389	$\times 10^{-6}$ in/in
Strain from Gage 2	737	$\times 10^{-6}$ in/in
Strain from Gage 3	-290	$\times 10^{-6}$ in/in

Results:

Max Principal Strain	816	$\times 10^{-6}$ in/in
Min Principal Strain	-717	$\times 10^{-6}$ in/in
Angle β	31.9	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	16117	psi
Min Principal Stress	-12863	psi
Max Shear Strain	1534	radians [Dimensionless]
Max Shear Stress	14490	psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 8059 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-124

Strain from Gage 1	389	$\times 10^{-6}$ in/in
Strain from Gage 2	737	$\times 10^{-6}$ in/in
Strain from Gage 3	-290	$\times 10^{-6}$ in/in

Results:

Max Principal Strain	882	$\times 10^{-6}$ in/in
Min Principal Strain	-324	$\times 10^{-6}$ in/in
Angle β	39.7	degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	20559	psi
Min Principal Stress	-2236	psi
Max Shear Strain	1206	radians [Dimensionless]
Max Shear Stress	11397	psi [in plane of initial element]

******Only when Max and Min Principal Stresses have the same sign******

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 10280 psi

SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 10-12 for method.
Input data in shaded elements

Stainless Steel, AISI 501 OQT 1000

Material Properties *U.S. Customary Unit System*

Modulus of Elasticity	29.0	x 10^6 psi
Poisson's Ratio	0.30	

Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 10-22 to 10-24]

Problem 10-117

Strain from Gage 1	1532	x10^{-6} in/in
Strain from Gage 2	-228	x10^{-6} in/in
Strain from Gage 3	893	x10^{-6} in/in

Results:

Max Principal Strain	2688	x10^{-6} in/in
Min Principal Strain	-263	x10^{-6} in/in

Angle β -38.7 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	83147	psi
Min Principal Stress	17317	psi

Very high stress: s_y = 135 ksi
N = 1.62 Low

Max Shear Strain 2951 x 10^{-6} radians [Dimensionless]
Max Shear Stress 32915 psi [in plane of initial element]

*****Only when Max and Min Principal Stresses have the same sign*****
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 41574 psi

Delta [0, 60,120 degree] Rosette Data [Uses Equations 10-25 to 10-27]

Problem 10-125

Strain from Gage 1	1532	x10^{-6} in/in
Strain from Gage 2	-228	x10^{-6} in/in
Strain from Gage 3	893	x10^{-6} in/in

Results:

Max Principal Strain	1761	x10^{-6} in/in
Min Principal Strain	-296	x10^{-6} in/in

Angle β -19.5 degrees
[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress	53289	psi
Min Principal Stress	7390	psi

Max Shear Strain 2058 x 10^{-6} radians [Dimensionless]
Max Shear Stress 22949 psi [in plane of initial element]

*****Only when Max and Min Principal Stresses have the same sign*****
[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 26644 psi

11-1 $\frac{Le}{r} = \frac{(1.0)(800)}{(20/4)} = 160$: FOR $S_y = 331$ MPa, $C_c = 110$: LONG COLUMN (FIG. 11-5)

USE EULER EQ. (11-4) : $A = \pi D^2/4 = \pi (20)^2/4 = 314\ mm^2$

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(207\times10^9 N/m^2)(314 mm^2)}{(160)^2} \times \frac{1 m^2}{10^6 mm^2} = \underline{25.1\ kN}$$

11-2 $Le/r = (1.0)(350)/(20/4) = 70$: $C_c = 110$ - SHORT COLUMN : EQ(11-6)

$$P_{CR} = AS_y\left[1 - \frac{S_y(Le/r)^2}{4\pi^2 E}\right] = (314 mm^2)(331 N/mm^2)\left[1 - \frac{331\times10^6 Pa(70)^2}{4\pi^2(207\times10^9 Pa)}\right]$$

$\underline{P_{CR} = 83.3\ kN}$

11-3 $Le/r = 160$: FOR ALUM, $S_y = 276$ MPa - $C_c = 70$: FIG 11-6 : LONG COLUMN

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(69\times10^9 N/m^2)(314 mm^2)}{(160)^2} \times \frac{1 m^2}{10^6 mm^2} = \underline{8.35\ kN}$$

11-4 FIXED ENDS : $Le/r = (0.65)(800)/(20/4) = 104$: $C_c = 110$: SHORT COLUMN

$$P_{CR} = (314)(331)\left[1 - \frac{(331\times10^6)(104)^2}{4\pi^2(207\times10^9)}\right] = \underline{58.4\ kN}$$

11-5 $A = 314\ mm^2 = b^2$: $b = \sqrt{314} = 17.7$ mm ; $r = b/\sqrt{12} = 5.12$ mm

$Le/r = (1.0)(800)/5.12 = 156$: $C_c = 138$: LONG COLUMN

$$P_{CR} = \frac{\pi^2(207\times10^9)(314)}{(156)^2(10^6)} = \underline{26.2\ kN}$$

11-6 1 IN SCH 40 PIPE : $r = 0.421$ IN $(25.4 mm/IN) = 10.69$ mm

$A = 0.494\ IN^2 \times (25.4)^2 mm^2/IN^2 = 318.7\ mm^2$

$S_y = 331$ MPa ; $C_c \approx 110$ - FIG 11-5

- PIPE 25 STD METRIC NAME APP A-12 (SI)

(a) FIXED ENDS : $Le/r = (0.65)(2050 mm)/10.69 mm = 124.6$ - LONG

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(207\times10^3 N/mm^2)(318.7 mm^2)}{(124.6)^2} = \underline{41.5\ kN}$$

(b) FIXED-PINNED : $Le/r = (0.80)(2050)/10.69 = 153$ - LONG

$$P_{CR} = \frac{\pi^2(200\times10^3 N/mm^2)(318.7 mm^2)}{(153)^2} = \underline{27.8\ kN}$$

(c) PINNED : $Le/r = 1.0(2050)/10.69 = 192$ - LONG

$$P_{CR} = \frac{\pi^2(207\times10^3)(318.7)}{(192)^2} = 17.7\ kN$$

(d) FIXED-FREE : $Le/r = (2.10)(2050)/10.69 = 403$ LONG

$$P_{CR} = \frac{\pi^2(207\times10^3)(318.7)}{(403)^2} = \underline{4.01\ kN}$$

11-7 $r_{MIN} = 12/\sqrt{12} = 3.46\,mm$; $Le/r = 1.0(210)/3.46 = 60.6$

FOR $S_y = 469\,MPa$ - STEEL : $C_c = 90$ — SHORT COLUMN

$A = (12)(25) = 300\,mm^2$

$$P_{CR} = (300\,mm^2)(469\,N/mm^2)\left[1 - \frac{(469\times10^6\,Pa)(60.6)^2}{4\pi^2(207\times10^9\,Pa)}\right] = \underline{111\,kN}$$

11-8 FOR A36 STRUCTURAL STEEL : $S_y = 248\,MPa$ — $C_c = 122$; $E = 200\times10^3\,N/mm^2$

FOR S6 X 12.5 : $r_{MIN} = \sqrt{\dfrac{I_Y}{A}} = \sqrt{\dfrac{1.82\,IN^4}{3.67\,IN^2}} = 0.704\,IN \times 25.4\,mm/IN = 17.9\,mm$

$Le/r = (0.65)(5450)/17.9 = 198 > C_c$ · LONG : $A = 3.67\,IN^2 \cdot \dfrac{(25.4)^2\,mm^2}{IN^2} = 2368\,mm^2$

$$P_a = \frac{\pi^2 EA}{1.92(Le/r)^2} = \frac{\pi^2(200\times10^3)(2368)}{1.92(198)^2} = \underline{62.1\,kN}$$ SEE ALSO APP. A-8 (SI)

11-9 3 IN SCH 40 PIPE : $r = 1.164\,IN$: $Le/r = (2.0)(8\,FT)(12\,IN/FT)/1.164\,IN = 173$

$S_y = 30\,KSI$: $C_c = 138$ - LONG COLUMN ; $E = 30\times10^6\,PSI$ FOR STEEL.

$$P_a = \frac{P_{CR}}{N} = \frac{\pi^2 EA}{N(Le/r)^2} = \frac{\pi^2(30\times10^6)(2.228)}{2(173)^2} = 7318\,LB/COLUMN$$

$$\text{No. OF COLUMNS} = \frac{\text{TOTAL LOAD}}{7318\,LB/COL.} = \frac{(75\,LB/FT^2)(20\times40)\,FT^2}{7318\,LB/COL.} = 8.20 - \underline{\text{USE } 9}$$

11-10 I 10 × 8.646 : $r_{MIN} = 1.42\,IN \times 25.4\,mm/IN = 36.1\,mm$

$A = 7.352\,IN^2 \times (25.4)^2\,mm^2/IN^2 = 4743\,mm^2$

$\boxed{\text{I 254 X 12.87 (SI) SEE APP A-11 (SI)}}$

$Le/r = 1.0(2800)/36.1 = 77.6$: FOR $S_y = 276\,MPa$ ALUM., $C_c = 70$ - LONG COLUMN

(EQ 11-18 b) $$P_a = \frac{352000(A)}{(Le/r)^2} = \frac{352000(4743)}{(77.6)^2} = \underline{277\,kN}$$

11-11 $Le/r = 1400/36.1 = 38.8$ — INTERMED. : (EQ 11-17 b)

$$P_a = A\left[139 - 0.869(Le/r)\right] = 4743\,mm^2\left[139 - 0.869(38.8)\right]N/mm^2 = \underline{499\,kN}$$

11-12 W 8 × 10 : $r_{MIN} = \sqrt{I_Y/A} = \sqrt{2.09/2.96} = 0.840\,IN$

$\boxed{\text{ASTM A992 — USE AISC METHOD}}$

$$SR = KL/r = \frac{0.8(12.5\,FT)(12\,IN/FT)}{0.840\,IN} = 142.9$$

$SR_t = 4.71\sqrt{E/S_y} = 4.71\sqrt{29\times10^6/50\,000} = 113$ - LONG COLUMN

$$S_{CR} = 0.877 S_e = \frac{0.877\,\pi^2 E}{(SR)^2} = \frac{0.877(\pi^2)(29\times10^6)}{(142.9)^2} = 12\,292\,psi$$

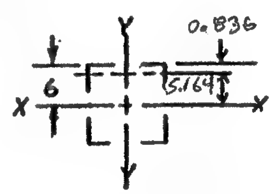
$$\underline{P_a} = \frac{P_n}{1.67} = \frac{(S_{CR})(A)}{1.67} = \frac{(12292)(2.96)}{1.67} = \underline{21787\,LB}$$

11-13

$$I_{XX} = I_{YY} = 4\left[1.23 + 1.44(5.164)^2\right] = 158.5\,in^4$$

$A = 4(1.44\,in^2) = 5.76\,in^2$

$\boxed{S_y = 36\,ksi \quad C_c = 126}$



$r = \sqrt{I/A} = \sqrt{158.5/5.76} = 5.246\,IN$

$Le/r = (1.0)(18.4\,ft)(12\,in/ft)/5.246\,IN = 42.1 < C_c \rightarrow$ JOHNSON EQ.

$$P_a = \frac{P_{CR}}{N} = \frac{(5.76)(36000)}{3}\left[1 - \frac{36000(42.1)^2}{4\pi^2(29\times10^6)}\right] = \underline{65\,300\,lb}$$

11-14

$r_{XX} = 2.44$ in

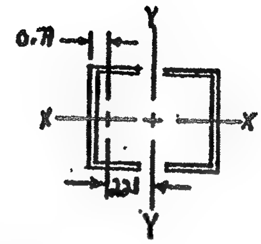
$I_{YY} = 2[1.53 + 2.41(2.21)^2] = 26.6$ in^4

$r_{YY} = \sqrt{I/A} = \sqrt{26.6/(2\times2.41)} = 2.35$ in

$Le = L = 10.5$ FT (12 IN/FT) $= 126$ IN.

$Le/r = 126/2.35 = 53.6$ (INT.)

EQ (11-11a) $P_a = 2(2.41)[20.2 - 0.126(53.6)] = 64.8$ kips $= \underline{64\,800 \text{ lb}}$



11-15

$F = mg = 1320\text{kg} \cdot 9.81\text{m/s}^2 = 12950\text{ N} = F_V$

$F_H = F/\tan\theta = 12950\text{N}/\tan 22.6 = 31110\text{ N}$

FOR S150 x 18.6 : $r = \sqrt{I_Y/A} = \sqrt{7.49\times10^5/2360} = 17.8$ mm

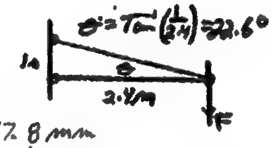
$Le/r = 2400/17.8 = 134.7$ – LONG [$C_c = 126$ FOR $S_y = 36$ KSI]

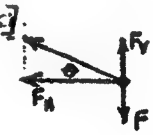
$$P_a = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(200\times10^3 \text{N/mm}^2)(2360\text{mm}^2)}{(134.7)^2}$$

$P_{cr} = 256,747$ N

IF $P_a = P_{cr}/N$

$$N = \frac{P_{cr}}{P} = \frac{256,747\text{ N}}{31110\text{ N}} = \underline{8.25 \text{ SAFE}}$$





11-16 $r_{MIN} = \frac{0.125}{\sqrt{12}} = 0.036\ IN$: $A = 0.25 \times 0.125 = 0.0313\ IN^2$

FOR 1040 CD: $S_y = 82\ ksi$ – $C_c = 83$: $\frac{Le}{r} = \frac{8.40}{0.036} = 233 > C_c$ – LONG COLUMN

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2 (30 \times 10^6)(0.0313)}{(233)^2} = \underline{171\ LB}$$

$$N = \frac{P_{CR}}{P} = \frac{171\ LB}{50\ LB} = \underline{3.41\ OK}$$

11-17 FOR AISI 1141 OQT 1300: $S_y = 469\ MPa$: $C_c = 95$: $A = \frac{\pi D^2}{4} = \frac{\pi (12)^2}{4} = 113\ mm^2$

$r = \frac{D}{4} = \frac{12}{4} = 3.0\ mm$: $\frac{Le}{r} = \frac{(0.8)(190)}{3.0} = 50.7 < C_c$ – SHIRT COLUMN

$$P_{CR} = (113\ mm^2)(469\ N/mm^2)\left[1 - \frac{469\ MPa\ (50.7)^2}{4\pi^2 (207 \times 10^3\ MPa)}\right] = \underline{45.2\ kN}$$

FOR $N=3$: $P_a = \frac{P_{CR}}{3} = \frac{45.2\ kN}{3} = \underline{15.1\ kN}$

11-18 FOR AISI 1020 HR; $S_y = 48\ ksi$; $C_c \approx 105$

$r = \frac{D}{4} = \frac{0.800\ IN}{4} = 0.200\ IN$: $\frac{Le}{r} = \frac{28.5}{0.20} = 142.5$ – LONG : $A = \frac{\pi (0.8)^2}{4} = 0.503\ IN^2$

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2 (30 \times 10^6)(0.503)}{(142.5)^2} = \underline{7334\ LB} : N = \frac{P_{CR}}{P} = \frac{7334}{1375} = \underline{5.33\ OK}$$

11-19 2 IN SCH. 40 PIPE: $r = 0.787\ IN$: $A = 1.075\ IN^2$: FIXED/PINNED – $k = 0.8$

$\frac{Le}{r} = \frac{(0.8)(14\ FT)(12\ IN/FT)}{0.787} = 171$: FOR AISI 1040 HR, $S_y = 48\ ksi$ – $C_c = 105$ – LONG

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2 (30 \times 10^6)(1.075)}{(171)^2} = \underline{10914\ LB}$$

EACH COLUMN CARRIES 5000 LB : $N = \frac{P_{CR}}{P} = \frac{10914\ LB}{5000\ LB} = \underline{2.18}$ – MARGINAL

11-20 LACK OF RESTRAINT AT TOP OF COLUMNS MAKE THEM <u>FREE</u>.

FOR FIXED-FREE: $\frac{Le}{r} = \frac{(2.10)(14\ FT)(12\ IN/FT)}{0.787\ IN} = 448$ – VERY LONG

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2 (30 \times 10^6)(1.075)}{(448)^2} = \underline{1584\ LB} : P_{ACTUAL} = 5000\ LB \text{ – FAILURE}$$

11-21

$r = 1.25\ in/\sqrt{12} = 0.361$; $A = (1.25\ in)^2 = 1.563\ in^2$

COLUMNS ARE FIXED–FREE ; $Le = 2.10 = 84\ in.$

$Le/r = 84/0.361 = 233$ – LONG

ALUMINUM 6061-T6 ; $S_y = 40\,000\ psi$; $C_c \approx 70$

$$\underline{P_{cr}} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2 (10 \times 10^6)(1.563)}{(233)^2} = \underline{2849\ lb}$$

EACH COLUMN CARRIES 1500 LB

$\underline{N} = P_{cr}/P = 2849/1500 = \underline{1.90}$ – LOW

11-22 ASSUME FIXED-PINNED ENDS: $r_{MIN} = \frac{2.00}{\sqrt{12}} = 0.577$ IN

$$\frac{Le}{r} = \frac{(0.80)(12.75FT)(12IN/FT)}{0.577 IN} = 212$$: AISI 1040 WQT 1100; $S_y = 80KSI$ – $C_c = 85$ LONG

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(30\times10^6 LB/IN^2)(6.0 IN^2)}{(212)^2} = 39525\ LB$$

$$N = \frac{P_{CR}}{P} = \frac{39525}{12500} = 3.16\ OK$$

THIS IS PROBABLY CONSERVATIVE BECAUSE PIN MAY PROVIDE SOME RESTRAINT AGAINST BUCKLING WITH RESPECT TO VERTICAL AXIS. THUS Le/r WOULD BE SMALLER; P_{CR} AND N WOULD BE LARGER.

11-23 ASSUME A LONG COLUMN: EQ. 11-5: $Le = (0.8)(12.75FT)(12IN/FT) = 122 IN$

$$REQD\ I = \frac{NPLe^2}{\pi^2 E} = \frac{(4.0)(12500LB)(122IN)^2}{\pi^2(30\times10^6 LB/IN^2)} = 2.53\ IN^4 = \frac{\pi D^4}{64}$$

$$D_{MIN} = \left[\frac{64(2.53)}{\pi}\right]^{1/4} = 2.68\ IN : r = \frac{D}{4} = \frac{2.68}{4} = 0.670 : \frac{Le}{r} = \frac{122}{0.67} = 182 - LONG$$

11-24 LET $E = 29\times10^6 PSI$ FOR STRUCTURAL STEEL: $S_y = 36KSI$: $C_c \approx 130$

ASSUME COLUMN IS LONG: $I_{MIN} = \frac{NPLe^2}{\pi^2 E} = \frac{(4.0)(12500)(122)^2}{\pi^2(29\times10^6)} = 2.60\ IN^4$

USE 3 IN SCH. 40: $I = 3.017\ IN^4$; $r = 1.164\ IN$; $\frac{Le}{r} = \frac{122}{1.164} = 105 < C_c$ – SHORT

$$P_a = \frac{AS_y}{N}\left[1 - \frac{S_y(Le/r)^2}{4\pi^2 E}\right] = \frac{(2.228IN^2)(36000LB/IN^2)}{4}\left[1 - \frac{(36000)(105)^2}{4\pi^2(29\times10^6)}\right] = 13114\ LB$$ OK

11-25 FOR BUCKLING ABOUT X-X AXIS – FIXED-PINNED: $Le = (0.8)(12.75FT)(12IN/FT) = 122 IN$

FOR Y-Y AXIS – FIXED-FIXED: $Le = 0.65(12.75)(12) = 99.5 IN$: ASSUME LONG COLUMN

$$I_{X MIN} = \frac{NPLe^2}{\pi^2 E} = \frac{4(12500)(122)^2}{\pi^2(10\times10^6)} = 7.54\ IN^4$$ — I 5×3.700 BEAM SHAPE REQD

$$I_{Y MIN} = \frac{4(12500)(99.5)^2}{\pi^2(10\times10^6)} = 5.02\ IN^4$$ — I 7×5.800 REQD.; $r_y = 1.08\ IN$

$$\frac{Le}{r_y} = \frac{99.5 IN}{1.08 IN} = 92.1$$: FOR 6061-T6: $S_y = 40KSI$; $C_c = 70$ – LONG COLUMN – OK

11-26 $Le = 0.8L = 0.8(16.5FT)(12IN/FT) = 158 IN$: 3×3×1/4 — $r = 1.11 IN$; $A = 2.44 IN^2$

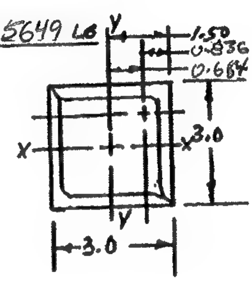
$Le/r = 158/1.11 = 142$: FOR ASTM A500, GRADE B: $S_y = 46KSI$ – $C_c = 110$ – LONG

$$P_{CR} = \frac{\pi^2 EA}{(Le/r)^2} = \frac{\pi^2(29\times10^6)(2.44)}{(142)^2} = 34635\ LB : P_a = \frac{P_{CR}}{N} = \frac{34635}{3} = 11545\ LB$$

11-27 $Le = 0.8L = 0.8(16.5)(12) = 158 IN$: FOR 4×2×1/4, $r_{MIN} = 0.779 IN$; $A = 2.44 IN^2$

$Le/r = 158/0.779 = 203$: $C_c = 110$ (PROB 11-26) – LONG

$$P_{CR} = \frac{\pi^2(29\times10^6)(2.44)}{(203)^2} = 16947\ LB : P_a = \frac{P_{CR}}{N} = \frac{16947}{3} = 5649\ LB$$



11-28 FROM 11-26: $Le = 158 IN$: $S_y = 36KSI$ $C_c = 130$

$$I_x = I_y = 2[I + Ad^2] = 2[1.23 + 1.44(0.664^2)] = 3.73\ IN^4$$

$$r = \sqrt{I/A} = \sqrt{3.73/2.88} = 1.138\ IN : \frac{Le}{r} = \frac{158}{1.138} = 139 - LONG$$

$$P_a = \frac{P_{CR}}{N} = \frac{\pi^2(29\times10^6)(2.88)}{(3)(139)^2} = \frac{42730 LB}{3} = 14243\ LB$$

11-29 AISI 1020 HR: $S_y = 331\ MPa$: $C_c = 105$: $r = \frac{40}{\sqrt{12}} = 11.55\ mm$

FIXED-FIXED: $L_e = 0.65(750) = 488\ mm$: $\frac{L_e}{r} = \frac{488}{11.55} = 42.2$ – SHORT

$$P_{CR} = A S_y\left[1 - \frac{S_y(L_e/r)^2}{4\pi^2 E}\right] = (2400\ mm^2)(331\ N/mm^2)\left[1 - \frac{331\ MPa\,(42.2)^2}{4\pi^2(207\times10^3\ MPa)}\right] = 737\ kN$$

LET $N = 3$: $P_a = \frac{P_{CR}}{N} = \frac{737\ kN}{3} = \underline{245\ kN}$

11-30 6061-T4: $S_y = 145\ MPa$ – $C_c = 98$: C102×2.568, $r_y = 16.26\ mm$

$\frac{L_e}{r} = \frac{4250\ mm}{16.26\ mm} = 261$ – LONG: $A = 954\ mm^2$ (APP. A-10 (SI))

$$P_a = \frac{P_{CR}}{N} = \frac{\pi^2 E A}{N(L_e/r)^2} = \frac{\pi^2(69\times10^3\ N/mm^2)(954\ mm^2)}{4(261)^2} = \underline{2376\ N}$$

11-31 NO IMPROVEMENT BECAUSE COLUMN IS STILL LONG AND BUCKLING LOAD FROM EULER FORMULA IS INDEPENDENT OF STRENGTH. E IS THE SAME.

11-32 W12×65, $A = 19.1\ IN^2$, $I_y = 174\ IN^4$

$r_{MIN} = r_y = \sqrt{I_y/A} = \sqrt{174\ IN^4/19.1\ IN^2} = 3.02\ IN$

ASTM A992: $S_y = 50\,000\ psi$, $E = 29\times10^6\ psi$

$SR = \frac{L_e}{r} = \frac{(0.8)(22.5\ FT)(12\ IN/FT)}{3.02} = 71.5$

$SR_t = 4.71\sqrt{E/S_y} = 4.71\sqrt{29\times10^6/5000} = 113$

$SR < SR_t$ USE EQ 11-10

$S_{CR} = \left[(0.658)^d\right] S_y \qquad d = S_y/S_e$

$S_e = \pi^2 E/SR^2 = \frac{\pi^2(29\times10^6)}{(71.5)^2} = 55987\ psi$

$d = 50000/55987 = 0.893$

$S_{CR} = \left[(0.658)^{0.893}\right] 50000 = 34406\ psi$

$P_m = S_{CR} A = 34406\ LB/IN^2\ (19.1\ IN^2) = 657{,}153\ LB$

$P_a = \frac{P_m}{1.67} = \frac{657153}{1.67} = \underline{393\,505\ LB}$

COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 11-33 TO 11-43

Prob. No.	L	K	L_e	s_y	E	C_c	A	r	SR	N	P_{cr}	Eqn.	Pa
11-33	163.2 in	0.65	106.2 in	36 ksi	29E06 psi	126	1.97 in^2	0.751 in	141	3	28260	Euler	9420 lb
11-34	83.2 in	0.80	66.6 in	36 ksi	29E06 psi	126	1.97 in^2	0.751 in	89	3	53403	Johnson	17801 lb
11-35	163.2 in	0.80	130.6 in	36 ksi	29E06 psi	126	1.97 in^2	0.751 in	174	3	18656	Euler	6219 lb
11-36	163.2 in	0.65	106.1 in	36 ksi	29E06 psi	126	2.44 in^2	1.11 in	96	3	62613	Johnson	20871 lb
11-37	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1570 mm^2	19.8 mm	134	2.27	173	Euler	75 kN
11-38	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1570 mm^2	28.2 mm	94	3	281	Johnson	93.8 kN

|--> HSS 76x76x6.4 tube (3x3x1/4)

Problem 11-39 Truss Analysis + Design of Compression Members
Example data only. Many possible designs — Listing of compression members only
Design factor on load = 3.0

Member	Load	L	Shape	Size	Material	P_a
AC	1925 lb	40 in	Circular	15/16 in	ASTM A36	2253 lb
CD	750	25 in	Circular	9/16 in	ASTM A36	750 lb
CE	650	40 in	Circular	11/16 in	ASTM A36	654 lb

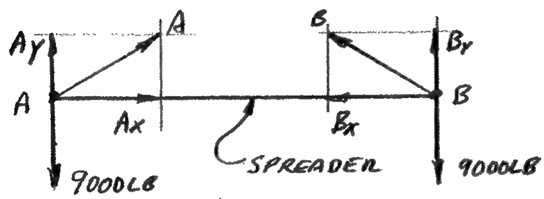
Problem 11-40 Truss Analysis + Design of Compression Members
Example data only. Many possible designs — Listing of compression members only
Design factor on load = 2.5

Member	Load	L	Shape	Size	Material	P_a
AC	4609 lb	120 in	Steel tube	2x2x1/4	ASTM A501	5950 lb
CG	3625 lb	120 in	Steel tube	2x2x1/4	ASTM A501	5950 lb
EH	1101 lb	154 in	Steel tube	2x2x1/4	ASTM A501	3613 lb
EG	3625 lb	120 in	Steel tube	2x2x1/4	ASTM A501	5950 lb
BE	4141 lb	120 in	Steel tube	2x2x1/4	ASTM A501	5950 lb

Problem 11-41 Truss Analysis + Design of Compression Members
Example data only. Many possible designs — Listing of compression members only
Design factor on load = 2.5

Member	Load	L	Shape	Size	Material	P_a
DE	2300 N	.25 m	Square	9 mm	Alum. 6061-T6	2383 N
BD	2597 N	.297 m	Square	11 mm	Alum. 6061-T6	3767 N
EF	2300 N	.20 m	Square	8 mm	Alum. 6061-T6	2324 N
FG	550 N	.15 m	Square	5 mm	Alum. 6061-T6	630 N
CF	800 N	.16 m	Square	6 mm	Alum. 6061-T6	1149 N

11-42 The support cables for the sling act at 30° to the horizontal and exert a direct axial compressive force on the spreader as shown below. Assume central loading of a straight column. The horizontal (axial) component of the cable force is 15 588 lb.



$A_y = B_y = 9000\,\text{LB}$

$A = B = 9000\,\text{LB}/\sin 30° = 18000\,\text{LB}$

$A_x = B_x = 18000\,\text{LB}(\cos 30°) = 15588\,\text{LB}$

Design decision: Use a hollow steel tube made from ASTM A501 structural steel. The column buckling analysis spreadsheet (Figure 14-9) was used to determine that the lightest size with adequate capacity is a 3x3x1/4 hollow steel tube. Other results are summarized below.

$L = L_e$ = 96 in; r = 1.11 in; SR = 86.5; A = 2.44 in^2; s_y = 36 000 psi; E = 29x10^6 psi; C_c = 126; Use N = 2.5 (design decision); Column is short; Use Johnson formula; P_{cr} = 67180 lb; P_a = 26872 lb.

11-43 The analysis is similar to Problem 11-42. With the angle of 15°, the axial force on the tube is 33 588 lb. The spreader now must be a HSS 4x4x1/4 steel tube with A = 3.37 in^2; r = 1.52 in; SR = 63.2; Short column; From the Johnson formula, P_{cr} = 106 103 lb; P_a = 42441 lb.

Crooked Columns

For Problems 11-44 to 11-49 loading data were taken from earlier problems as listed in the problem statements. The amount of initial crookedness is given. The Crooked Column Analysis spreadsheet (Figure 11-15) was used to determine the critical buckling load and the allowable load for a design factor of 3.0. The spreadsheet solves Equation 11-19. Results are summarized in the table on the following page.

Eccentrically-Loaded Columns

For Problems 11-50 to 11-58, data from the problem statements were entered into the Eccentric Column Analysis spreadsheet (Figure 11-16). Where the problem asks for the maximum stress and deflection, the design factor N = 1.0 was entered at the lower left column. For design problems, the requested design factor (typically N = 3.0) was entered. Results are summarized in the table on the following page.

Problem 11-59

Straight and crooked column analysis required for the 2-in schedule 40 steel pipe, 156 in long. The spreadsheets in Figures 11-9 (Straight columns) and 11-15 (crooked columns) were used to determine the following results.

a) Straight pipe: SR = 198; C_c = 126; Long column; P_{cr} = 7831 lb; P_a = 2610 lb.
b) Crooked pipe: a = 1.25 in; C_1 in Eqn. 11-19 = -21 766; C_2 = 3.36x10^7; Euler buckling load = 7831 lb; P_a = 1676 lb.

CROOKED COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 11-44 to 11-49

												Equation 11-19			
Prob.	**a**	***L***	***K***	***L_e***	***s_y***	***E***	***C_c***	***A***	***r***	***SR***	***N***	***P_{cr}***	***C_1***	***C_2***	***P_a***
11-44	4.00 mm	800 mm	1.00	800 mm	331 MPa	207 GPa	111	314 mm^2	5.00 mm	160	3	25.06 kN	-56362	2.89E08	5.71 kN
11-45	1.60 mm	210 mm	1.00	210 mm	469 MPa	207 GPa	93.3	300 mm^2	3.46 mm	60.6	3	111 kN	-146962	2.61E09	20.6 kN
11-46	14.0 mm	2800 mm	1.00	2800 mm	276 MPa	69 GPa	70.2	4743 mm^2	36.1 mm	77.6	3	537 kN	-761829	7.81E10	122 kN
11-47	0.75 in	150 in	0.80	120 in	50 ksi	29E06 psi	107	2.96 in^2	0.841in	143	3	41612 lb	-92180	6.84E+08	8143 lb
11-48	1.25 in	163.2 in	0.65	106.1 in	36 ksi	29E06 psi	126	1.97 in^2	0.751 in	141	3	28260 lb	-53938	2.23E+08	4505 lb
11-49	32.0 mm	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1570 mm^2	19.8 mm	134	1.068	173 kN	-862416	5.91E+10	75.0 kN

—> Iterated to find N for P_a = 75 kN

ECCENTRICALLY LOADED COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 11-50 to 11-58

Value of N in Eqn. 11-21 set equal to 1.0 to find maximum stress in column.

												Value of secant for:			
Prob.	***e***	***L***	***K***	***L_e***	***s_y***	***E***	***C_c***	***A***	***r***	***SR***	***N***	***Stress***	***Defl.***	***Stress***	***y_{max}***
11-50	0.60 in	42.0 in	1.00	42.0 in	21.0 ksi	10E06 psi	97	1.563 in^2	0.361 in	116	1	1.1527	1.1527	3456 psi	0.092 in
11-51	150 mm	3200 mm	1.00	3200 mm	331 MPa	207 GPa	111	1438 mm^2	29.55 mm	108	1	1.1716	1.1716	211 MPa	25.8 mm
11-52	0.30 in	14.75 in	1.00	14.75 in	40.0 ksi	28E06 psi	118	0.063 in^2	0.072 in	204	1	1.1511	1.1511	6687 psi	0.045 in

Problems 11-53 to 11-58 Eccentrically Loaded Column Analysis

Value of N = 3 used to evaluate safety

												Value of secant for:			
Prob.	***e***	***L***	***K***	***L_e***	***s_y***	***E***	***C_c***	***A***	***r***	***SR***	***N***	***Stress***	***Defl.***	***Reqd s_y***	***y_{max}***
11-53	0.50 in	40.0 in	1.00	40.0 in	50.0 ksi	29E06 psi	107	3.37 in^2	1.51 in	26.3	3	1.239	1.0703	103 ksi	0.035 in
	Design is not safe because reqd s_y is greater than given s_y. Prop is redesigned to find lightest steel tube that is safe.														
11-53a	0.50 in	40.0 in	1.00	40.0 in	50.0 ksi	29E06 psi	107	5.24 in^2	2.85 in	14.0	3	1.038	1.0123	48.4 ksi	0.006 in
	Square steel tube 8x4x1/4. Assumed pinned ends. Would be safer if ends are flat and parallel to the press ram.														
11-54a	0.90 in	72 in	1.00	72.0 in	90.0 ksi	30E06 psi	81.1	1.28 in^2	0.231 in	312	1	1.4289	1.4289	8319 psi	0.386 in
	This is the maximum stress in the column for an assumed design factor of N = 1.0.														
11-54b	0.9	72 in	1.00	72.0 in	90.0 ksi	30E06 psi	81.1	1.28 in^2	0.231 in	312	3	5.2212	1.4289	84.9 ksi	0.386 in
	This gives the required yield strength (84.9 ksi) of the material for N = 3. Specify AISI 1040 WQT 900 steel; s_y = 90.0 ksi.														

Problems 11-53 to 11-58 **Eccentrically Loaded Column Analysis (Continued)**
Value of N = 3 used to evaluate safety

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	Value of secant for: Stress	Defl.	Reqd s_y	y_{max}
11-55	0.467 in	112 in	1.00	112 in	36.0 ksi	29E06 psi	126	2.64 in^2	0.486 in	229	3	7.821	1.4864	104 ksi	0.2272 in

The design is not safe using a desired value of N = 3. Required yield strength is over 2.5 times higher that given yield strength.
The eccentricity is the distance from the centoidal axis and the middle of the flange width. C5x9 Steel channel; c = 1.412 in

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	Stress	Defl.	Reqd s_y	y_{max}
11-56	3.00 in	126 in	0.80	100.8	46.0 ksi	29E06 psi	112	6.02 in^2	1.41 in	71.5	3	1.2576	1.0749	46.0 ksi	0.225 in

The load for these data is* 19,263 *lb, found by iterating Equation 14-20 using the spreadsheet until the required yield strength became less than the given 46,000 psi. HSS 4x4x1/2 Steel tube; c = 2.00 in

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	Stress	Defl.	Reqd s_y	y_{max}
11-57a	1.75 in	40.0 in	1.00	40.0 in	40.0 ksi	10E06 psi	70.2	0.600 in^2	0.433 in	92.4	3	1.5127	1.1333	39.1 ksi	0.233 in

The load for these data is 675 lb, found by iterating Equation* 11-21 *using the spreadsheet until the required yield strength became less than the given 40,000 psi.
The analysis was done assuming that the loading in the plane of the drawing was critical. IT IS NOT! See the analysis below.
11-57b ***Buckling about the thickness of the bar is now checked assuming that the load is centrally applied.***
Column analysis spreadsheet is used to determine allowable load for N = 3.

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	Stress	Defl.	Reqd s_y	y_{max}
11-57b	0	40.0 in	1.00	40.0 in	40.0 ksi	10E06 psi	70.2	0.60 in^2	0.1154 in	347	3	-	-	-	

The allowable load for buckling about this axis is only 164 lb. This is the limiting load.

Prob.	e	L	K	L_e	s_y	E	C_c	A	r	SR	N	Stress	Defl.	Reqd s_y	y_{max}
11-58	20 mm	750 mm	1.00	750 mm	931 MPa	200 GPa	65.1	314 mm^2	7.29 mm	103	3	1.4512	1.1205	389 MPa	2.41 mm

The design is safe because the required yield strength is less than the actual yield strength of the given material.

Problem 11-59 **has two-parts. The first analysis is for a straight pipe. The second analysis is for the crooked pipe.**

Prob.	a	L	K	L_e	s_y	E	C_c	A	r	SR	N	P_{cr}	C_1	C_2	P_a
11-59a	0	156 in	1.00	156 in	36.0 ksi	29E06 psi	126	1.075 in^2	0.787 in	198	3	7831	-	-	2610 lb
11-59b	1.25 in	156 in	1.00	156 in	36.0 ksi	29E06 psi	126	1.075 in^2	0.787 in	198	3	7831	-21766	3.37E07	1676 lb

The allowable load decreases from 2610 lb to 1676 lb when the pipe is crooked.

Equation 11-4 used for Problem 11-59(a)
Equation 11-19 used for Problem 11-59(b)

12-1

$t = (200-184)/2 = 8.0\ mm$

LET D = MEAN DIA. $= (D_o + D_i)/2 = (200+184)/2 = 192\ mm$

$D/t = 192/8.0 = 24$ — THIN-WALLED SPHERE

$$\sigma = \frac{pD}{4t} = \frac{(19.2\ N/mm^2)(192\ mm)}{4(8.0\ mm)} = \underline{115\ MPa}$$

12-2

$D_m = D_o - t = 10\,500 - 12 = 10\,488\ mm$

$\frac{D_m}{t} = \frac{10488\ mm}{12\ mm} = 874$ - VERY THIN WALL ; AISI 1040 HR

$\sigma_d = \frac{S_y}{4} = \frac{414\ MPa}{4} = 103.5\ MPa$: $\sigma_{MAX} = \frac{pD}{4t}$

$$p = \frac{4t\sigma_d}{D_m} = \frac{4(12\ mm)(103.5\ MPa)}{10488\ mm} = \underline{0.474\ MPa} = \underline{474\ kPa}$$

12-3

$D_o = 1200\ mm$ - ASSUME THIN WALL ; FOR Ti-6Al-4V, $S_y = 1070\ MPa$

$\sigma_d = \frac{S_y}{4} = \frac{1070\ MPa}{4} = 267\ MPa$; $\sigma = \frac{pD_m}{4t} = \frac{p(D_o - t)}{4t} = \frac{pD_o - pt}{4t}$

$t = \frac{pD_o}{4\sigma_d + p} = \frac{(4.20\ MPa)(1200\ mm)}{4(267\ MPa) + 4.2\ MPa} = \underline{4.70\ mm}$ USE $t = 5.00\ mm$ CONVENIENT SIZE

$D_m/t \approx \frac{1195}{5} = 239$ - VERY THIN WALL

12-4

$\sigma_d = \frac{S_y}{4} = \frac{414\ MPa}{4} = 103.5\ MPa$

$t = \frac{pD}{4\sigma_d} = \frac{(4.20\ MPa)(1200\ mm)}{4(103.5\ MPa)} = \underline{12.2\ mm}$; $D/t = 1200/12.2 = 98$ - THIN

MASS = DENSITY × VOLUME OF SPHERE

VOLUME = $0.5236(D^3 - d^3)$; $D = 1200\ mm$; $d = D - 2t$

Ti : $d = 1200 - 2(4.71\ mm) = 1190\ mm$

$V = 0.5236(1200^3 - 1190^3) = 2.114 \times 10^7\ mm^3$

ALUM: $d = 1200 - (12.2)(2) = 1176\ mm$

$V = 0.5236(1200^3 - 1176^3) = 5.39 \times 10^7\ mm^3$

MASS OF Ti $= 4430\frac{kg}{m^3} \times 2.114 \times 10^7\ mm^3 \times \frac{1\ m^3}{(10^3\ mm)^3} = \underline{93.6\ kg}$ LIGHTER

MASS OF ALUM: $2770\frac{kg}{m^3} \times 5.39 \times 10^7\ mm^3 \times \frac{1\ m^3}{(10^3\ mm)^3} = 149\ kg$

12-5

$D_o = 10.750\ in$; $D_i = 10.020\ in$; $D_m = (D_o + D_i)/2 = 10.385\ in$

$t = 0.365\ in$; $D_m/t = 10.385/0.365 = 28.5$ (THIN)

$$\sigma = \frac{pD}{2t} = \frac{(50\ psi)(10.385\ in)}{2(0.365\ in)} = \underline{2134\ psi}$$

12-6 $D_m = D_i + t = 80 + 3.5 = 83.5\ MM$; $D_m/t = 83.5/3.5 = 23.9$ THIN

$\sigma = \frac{pD_m}{2t} = \frac{(1.85\ MPa)(83.5\ mm)}{2(3.5\ mm)} = \underline{34.0\ MPa}$

12-7 ASSUME THIN WALL: $\sigma_d = \frac{s_y}{4} = \frac{565\ MPa}{4} = 141.3\ MPa = \frac{pD}{2t}$

$t = \frac{pD}{2\sigma_d} = \frac{(1.7\ MPa)(300\ mm)}{2(141.3\ MPa)} = \underline{1.80\ mm}$

$D_m = D_o - t = 300 - 1.81 = 298.2$: $D_m/t = 298.2/1.80 = 164$ - VERY THIN

12-8 $t = \frac{pD}{2\sigma_d} = \frac{(15.2\ MPa)(250\ mm)}{2(141.3\ MPa)} = \underline{13.45\ mm}$

$D_m = D_o - t = 250 - 13.45 = 236.5\ mm$: $D_m/t = 236.5/13.45 = 17.6$ - THICK WALL

USE EQ. FOR σ_1 FROM TABLE 12-1. TRY $t = 14.0\ mm$

$D_i = D_o - 2t = 250 - 2(14) = 222\ mm$; $a = 111\ mm$, $b = 125\ mm$.

$\sigma_1 = \frac{p(b^2+a^2)}{b^2-a^2} = \frac{(15.2\ MPa)(125^2+111^2)}{(125^2-111^2)} = 128.6\ MPa$

OK FOR $\sigma_d = 141.3\ MPa$ BUT SOMEWHAT LOW.

FOR $t = 13.0\ mm$; $D_i = 224\ mm$; $a = 112\ mm$, $b = 125\ mm$

$\sigma = \underline{139.0\ MPa}$ $\underline{OK}$ USE $t = 13.0\ mm$

12-9 $D_m = D_o - t = 450 - 2.20 = 447.8\ mm$: $D_m/t = 447.8/2.20 = 204$ - THIN

$\sigma_{MAX} = \frac{pD_m}{2t} = \frac{(750 \times 10^3\ Pa)(447.8\ mm)}{2(2.20\ mm)} = \underline{76.3\ MPa}$

$N = \frac{s_y}{\sigma} = \frac{290\ MPa}{76.3\ MPa} = \underline{3.80}$

12-10 ASSUME THIN WALL: $\sigma_d = s_y/4 = 414/4 = 103\ MPa = \frac{pD}{2t}$

$t = \frac{pD}{2\sigma_d} = \frac{(750 \times 10^3\ Pa)(1800\ mm)}{2(103 \times 10^6\ Pa)} = \underline{6.55\ mm}$ LET $t = 7.0\ mm$

$D_m = D_o - t = 1800 - 7.0 = 1793\ mm$: $D_m/t = 1793/7.0 = 256$ VERY THIN

12-11 $D_m = D_o - t = 250 - 18 = 232\ mm$: $D_m/t = 232/18 = 12.9 < 20$ - THICK WALL

$b = 250/2 = 125\ mm$: $a = b - t = 125 - 18 = 107\ mm$

$\sigma_{1\,MAX} = \frac{p(b^3+2a^3)}{2(b^3-a^3)} = \frac{(70.0\ MPa)(125^3+2(107)^3)}{2(125^3-107^3)} = \underline{212\ MPa}$ TANGENTIAL

$\sigma_{3\,MAX} = -p = \underline{-70.0\ MPa}$ RADIAL

12-12

$D_o = 0.840\ in$; $D_i = 0.622$; $D_m = (D_o + D_i)/2 = 0.731$; $D_m/t = \frac{0.731}{0.109} = 6.71$ (THICK)

$b = D_o/2 = 0.420\ in$; $a = D_i/2 = 0.311\ in$

$\sigma_2 = \frac{p\,a^2}{b^2-a^2} = \frac{(250\ psi)(0.311)^2}{(0.420)^2-(0.311)^2} = \underline{303\ psi}$ LONGITUDINAL

$\sigma_1 = \frac{p(b^2+a^2)}{b^2-a^2} = \frac{(250\ psi)(0.420^2+0.311^2)}{0.420^2-0.311^2} = \underline{857\ psi}$ HOOP

$\sigma_3 = -p = \underline{-250\ psi}$ RADIAL

12-13

$$\frac{D_m}{t} = \frac{(300+220)/2}{(300-220)/2} = 6.5 \quad \text{(THICK)} \quad a = 110\,mm\,;\ b = 150\,mm$$

$$\sigma_1 = \frac{p a^2 (b^2+r^2)}{r^2(b^2-a^2)} = \frac{b^2+r^2}{r^2} \times \frac{p a^2}{b^2-a^2} = \frac{b^2+r^2}{r^2} \times \frac{(50\,MPa)(110)^2}{150^2-110^2} = 58.17\left[\frac{b^2+r^2}{r^2}\right]$$

r	σ_1 HOOP STRESS
110 mm	166 MPa – INNER SURFACE – MAXIMUM
120	149 MPa
130	136 MPa
140	125 MPa
150	116 MPa – OUTER SURFACE

12-14

$D_m = (D_o + D_i)/2 = (1.900 + 1.610)/2 = 1.755 \quad D_m/t = 1.755/0.145 = 12.1$ (THICK)

USING THICK WALLED EQN. $b = 1.90/2 = 0.95\,;\ a = 1.61/2 = 0.805$

$$\sigma_{1_{max}} = \frac{p(b^2+a^2)}{b^2-a^2} = \frac{(10.0\,MPa)(.95^2+.805^2)}{(.95^2-.805^2)} = \underline{60.9\,MPa}$$

USING THIN-WALLED EQN.

$$\sigma_1 = \frac{pD}{2t} = \frac{(10.0\,MPa)(1.755\,mm)}{2(0.145\,mm)} = \underline{60.5\,MPa}$$

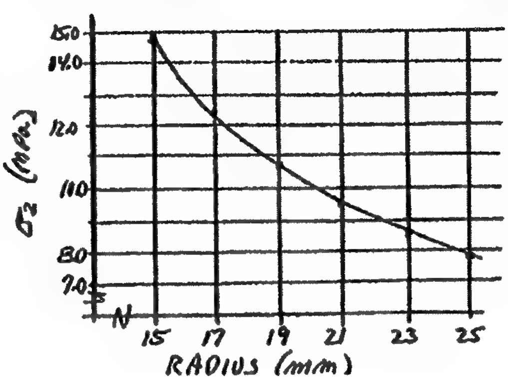
12-15 $D_m = (D_o + D_i)/2 = (50+30)/2 = 40\,mm\ :\ t = (D_o - D_i)/2 = (50-30)/2 = 10\,mm$

$D_m/t = 40/10 = 4.0$ – THICK : $b = D_o/2 = 25\,mm\ :\ a = D_i/2 = 30/2 = 15\,mm$

$$\sigma_{1_{max}} = \frac{p(b^2+a^2)}{b^2-a^2} = \frac{(8.0\,MPa)(25^2+15^2)}{(25^2-15^2)} = \underline{14.88\,MPa} \quad \text{TANGENTIAL}$$

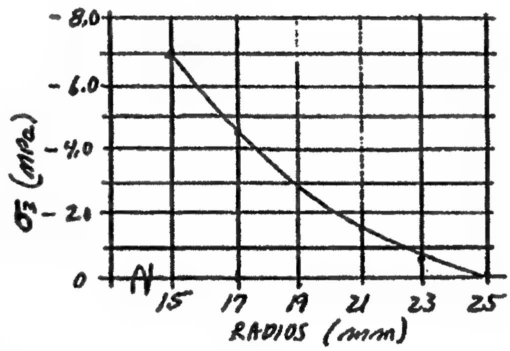
12-16 $$\sigma_1 = \frac{p a^2 (b^2+r^2)}{r^2(b^2-a^2)} = \frac{p a^2}{b^2-a^2} \times \frac{(b^2+r^2)}{r^2} = \frac{7.0(15)^2}{(25^2-15^2)} \times \frac{(b^2+r^2)}{r^2} = 3.94 \times \frac{(b^2+r^2)}{r^2}$$

r	σ_1 (MPa)
15	14.88 INNER SURFACE
17	12.45
19	10.75
21	9.52
23	8.59
25	7.88 OUTER SURFACE



12-17 $\sigma_3 = \frac{-p\,a^2(b^2-r^2)}{r^2(b^2-a^2)} = \frac{-pa^2}{b^2-a^2}\times\frac{b^2-r^2}{r^2} = \frac{-(7.0)(15^2)}{(25^2-15^2)}\times\frac{b^2-r^2}{r^2} = 3.94\,\frac{b^2-r^2}{r^2}$

r	σ_3 (MPa) RADIAL
15	−7.00 INNER SURFACE
17	−4.58
19	−2.88
21	−1.64
23	−0.71
25	0 OUTER SURFACE



12-18 ASSUMING THIN WALL THEORY

$\sigma_1 = \frac{pD_m}{2t} = \frac{(7.0\,MPa)(40\,mm)}{2(10\,mm)} = \underline{14.0\,MPa}$ (5.9% LOW)

FROM PROB. 12-15: ACTUAL $\sigma_{1\,MAX} = 14.88$ MPa

12-19 $D_m = D_o - t = 500 - 40 = 460$ mm ; $D_m/t = 460/40 = 11.5 < 20$ - THICK

$\sigma_d = \frac{S_y}{4} = \frac{931\,MPa}{4} = 232$ MPa AISI 501 OQT 1000 | $b = D_o/2 = 250$ mm ; $a = D_i/2 = 210$ mm

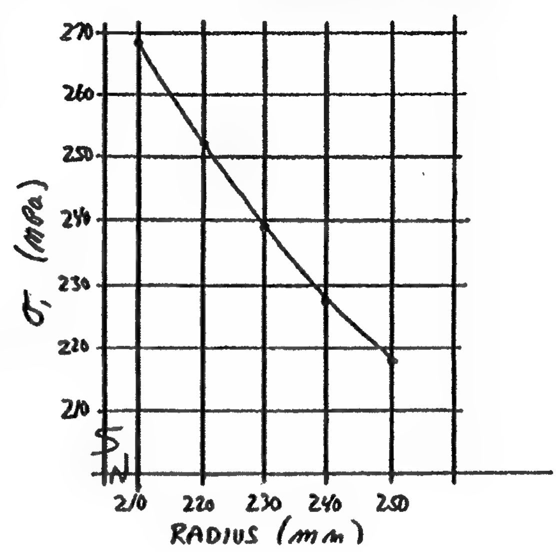
$\sigma_{MAX} = \sigma_{1\,MAX} = \frac{p(b^3+2a^3)}{2(b^3-a^3)}$; $p = \frac{2\sigma_d(b^3-a^3)}{(b^3+2a^3)} = \frac{2(232\,MPa)(250^3-210^3)}{[250^3+2(210^3)]}$

$\underline{p_{MAX} = 86.8\,MPa}$

12-20 FROM PROB. 12-19: THICK WALL: $b = 250$ mm, $a = 210$ mm

$\sigma_1 = \sigma_2 = \frac{pa^3(b^3+2r^3)}{2r^3(b^3-a^3)} = \frac{pa^3}{2(b^3-a^3)}\times\frac{(b^3+2r^3)}{r^3} = \frac{100\,MPa(210)^3}{2(250^3-210^3)}\times\frac{(b^3+2r^3)}{r^3}$

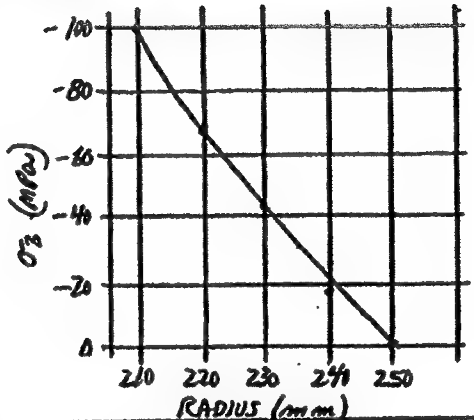
r	σ_1 (MPa) TANGENTIAL
210	268 INNER SURFACE
215	260
220	252
225	245
230	239
235	233
240	228
245	223
250	218 OUTER SURFACE



12-21 FROM PROB. 12-19: THICK WALL SPHERE: $b = 250\ mm$, $a = 210\ mm$

$$\sigma_3 = \frac{-pa^3(b^3 - r^3)}{r^3(b^3 - a^3)} = \frac{-pa^3}{(b^3 - a^3)} \times \frac{(b^3 - r^3)}{r^3} = \frac{-(100\ MPa)(210^3)}{(250^3 - 210^3)} \times \frac{(250^3 - r^3)}{r^3}$$

r	σ_3 (MPa) RADIAL	
210	-100	INNER SURFACE
215	-83.3	
220	-68.0	
225	-54.1	
230	-41.4	
235	-29.7	
240	-19.0	
245	-9.1	
250	0.0	OUTER SURFACE

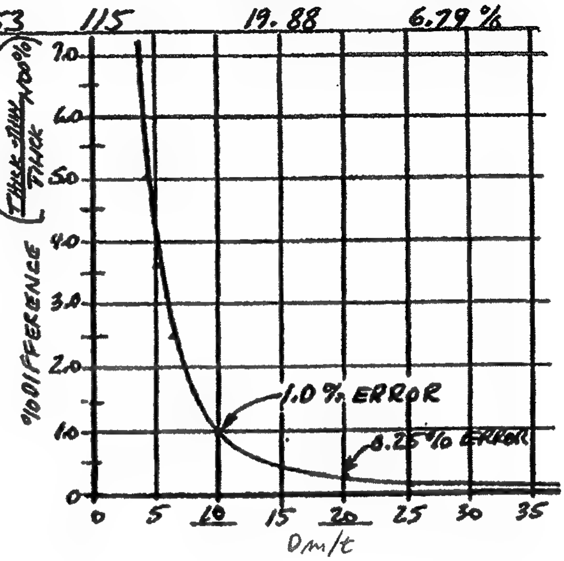


12-22 $D_o = 400\ mm$

t (mm)	$D_m = D_o - t$ mm	D_m/t	THIN $\sigma_1 = \frac{pD_m}{2t}$	$a = \frac{D_o - 2t}{2}$	THICK $\sigma_1 = \frac{p(b^2 + a^2)}{(b^2 - a^2)}$	% = $\frac{THICK - THIN}{THICK} \times 100\%$
5	395	79	395 MPa	195	395.06 MPa	0.015%
15	385	25.67	128.3	185	128.53	0.179%
19.05	380.95	20.0	100.0	180.95	100.25	0.25%
25	375	15.0	75.0	175	75.33	0.44%
35	365	10.43	52.14	165	52.62	0.91%
36.36	363.63	10.0	50.0	163.63	50.50	0.99%
45	355	7.89	39.44	155	40.08	1.60%
55	345	6.27	31.36	145	32.16	2.49%
65	335	5.15	25.77	135	26.74	3.63%
75	325	4.33	21.67	125	22.82	5.04%
85	315	3.71	18.53	115	19.88	6.79%

NOTES:

- $D_m/t = 20.0$ ADDED TO SHOW THAT ARBITRARY DIVISION BETWEEN THICK AND THIN WALL CYLINDERS RESULTS IN LESS THAN 0.25% ERROR FOR THIN-WALLED THEORY.
- FOR $D_m/t > 10$, ERROR IS LESS THAN 1.0%
- ERROR INCREASES RAPIDLY FOR $D_m/t < 10$.



12-23 $D_o = 400\ mm$: $D_i = 325\ mm$: $D_m = 362.5\ mm$: $D_m/t = 9.67$ - THICK

$t = 37.5\ mm$: $b = D_o/2 = 200\ mm$: $a = D_i/2 = 162.5\ mm$

$$\sigma_1 = \frac{pa^3(b^3 + 2r^3)}{2r^3(b^3 - a^3)} \quad : p = 10.0\ MPa$$

r (mm)	σ_1 (MPa)	σ_3 (MPa) PROB 15-24
162.5 = a	22.35	10.0
170.0	20.99	7.27
177.5	19.84	4.98
185.0	18.88	3.05
192.5	18.06	1.41
200.0 = b	17.35	0.00

12-24 SAME DATA AS 12-23: $\sigma_3 = \dfrac{-pa^3(b^3 - r^3)}{r^3(b^3 - a^3)}$

12-25 SCHEDULE 40 PIPE: APP. A-12: $D_m = (D_o + D_i)/2$

NOM. SIZE	D_m (IN)	t (IN)	D_m/t	
1/8	.337	.068	4.96	THICK
1/4	.452	.088	5.14	
3/8	.584	.091	6.42	
1/2	.731	.109	6.71	
3/4	.937	.113	8.29	
1	1.182	.133	8.89	
1 1/4	1.520	.140	10.86	
1 1/2	1.755	.145	12.10	
2	2.221	.154	14.42	
2 1/2	2.672	.203	13.16	

NOM SIZE	D_m (IN)	t (IN)	D_m/t	
3	3.284	0.216	15.20	THICK
3 1/2	3.774	.226	16.70	
4	4.263	.237	17.99	
5	5.305	.258	20.56	THIN
6	6.345	.280	22.66	
8	8.303	.322	25.79	
10	10.385	.365	28.45	
12	12.344	.406	30.40	
16	15.50	.500	31.00	
18	17.438	.562	31.03	

PROBLEMS 12-26 TO 12-35:

These problems are design problems so there may be more than one possible acceptable result. The approach taken in the following spreadsheets is to maintain the inner radius, *a*, at the stated minimum in the problem and then to adjust the outside radius, *b*, so that the maximum stress achieves the desired design factor. Attention was paid to whether the resulting design produced a thin-walled or thick-walled vessel. The spreadsheets are similar to that shown in Figure 12-7 in the text, augmented to enable the computation of the design stress and the volume of material in the cylinder or sphere. Both cylinders and spheres can be analyzed with the spreadsheet and the unused part of the sheet has been crossed out, leaving only the desired data.

Problems 12-26 through 12-30 call for a design stress of $s_u/8$ as the primary parameter. Secondarily, they call for computing the design factor based on yield strength if the maximum pressure was double the original design pressure. This approach reflects that the design pressure may be experienced many thousands of times in the life of the vessel and that fatigue of the material may be a failure mode. The higher pressure is considered a burst pressure test pressure that will be experienced only once or a few times in the life of the vessel.

PROBLEMS 12-26 TO 12-28 and 12-30:

These problems have the same design objectives with regard to the operating pressure and the length and inside diameter of a cylindrical pressure vessel. The material for the vessel is different for each problem. Following the given solution for Problem 12-30, a summary of the results for all four problems is given, comparing the wall thickness, volume of material in the cylinder, and the weight or of the cylindrical portion, not counting any end pieces or closures. This should give the student and the reader a feel for how material selection affects the final product design.

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:		Problem Number:	12-26		
Pressure = p =	450 psi	Wall thickness = t =	0.25 in		
Inside radius = a = R_i =	3.000 in	Mean diameter = D_m =	6.25 in		
Outside radius = b = R_o =	3.250 in	Ratio: D_m/t =	25.0	Thin	
		If Ratio < 20, Vessel is thick			
~~*Analysis of a Sphere*~~				~~Thin-wall sphere~~	
~~Max Tangential Stress =~~	~~2712 psi~~			~~2812.5 psi~~	
~~Max Radial Stress =~~	~~-450 psi~~				
Analysis of a Cylinder				Thin-wall cylinder	
Max Tangential Stress =	5634 psi	Volume of cylinder =	73.63 in^3	5625	psi
Max Longitudinal Stress =	2592 psi	Weight of cylinder =	7.363 lb	2812.5	psi
Max Radial Stress =	-450 psi	[Alum: 0.10 lb/in^3]			
Ultimate strength =	45000 psi		Thin-walled		
Su/8 =	5625 psi	Actual N for su =	8.00		
Yield strength =	40000 psi				
Sy/4 =	10000 psi	Actual N for sy =	7.11		
		N for p = 900 psi =	3.556		

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:		Problem Number:	12-27		
Pressure = p =	450 psi	Wall thickness = t =	0.065 in		
Inside radius = a = R_i =	3.000 in	Mean diameter = D_m =	6.065 in		
Outside radius = b = R_o =	3.065 in	Ratio: D_m/t =	93.3	Thin	
		If Ratio < 20, Vessel is thick			
~~*Analysis of a Sphere*~~				~~Thin-wall sphere~~	
~~Max Tangential Stress =~~	~~10388 psi~~			~~10497 psi~~	
~~Max Radial Stress =~~	~~-450 psi~~				
Analysis of a Cylinder				Thin-wall cylinder	
Max Tangential Stress =	20997 psi	Volume of cylinder =	18.58 in^3	20994	psi
Max Longitudinal Stress =	10273 psi	Weight of cylinder =	2.972 lb	10497	psi
Max Radial Stress =	-450 psi	[Titanium: 0.16 lb/in^3]			
Ultimate strength =	170000 psi		Thin-walled		
Su/8 =	21250 psi	Actual N for su =	8.10		
Yield strength =	155000 psi				
Sy/4 =	38750 psi	Actual N for sy =	7.38		
		N for p = 900 psi =	3.691		

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:		Problem Number: 12-28		
Pressure = p =	450 psi	Wall thickness = t =	0.052 in	
Inside radius = a = R_i =	3.000 in	Mean diameter = D_m =	6.052 in	
Outside radius = b = R_o =	3.052 in	Ratio: D_m/t =	116.4	Thin
		If Ratio < 20, Vessel is thick		

~~*Analysis of a Sphere*~~		~~Thin-wall sphere~~
~~Max Tangential Stress =~~	~~12983 psi~~	~~13093 psi~~
~~Max Radial Stress =~~	~~450 psi~~	

Analysis of a Cylinder				Thin-wall cylinder
Max Tangential Stress =	26188 psi	Volume of cylinder =	14.83 in^3	26187 psi
Max Longitudinal Stress =	12869 psi	Weight of cylinder =	4.167 lb	13093 psi
Max Radial Stress =	-450 psi	[St. Stl.: 0.281 lb/in^3]		
Ultimate strength =	210000 psi		Thin-walled	
Su/8 =	26250 psi	Actual N for su =	8.02	
Yield strength =	185000 psi			
Sy/4 =	46250 psi	Actual N for sy =	7.06	
		N for p = 900 psi =	3.532	

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:		Problem Number: 12-30		
Pressure = p =	450 psi	Wall thickness = t =	0.039 in	
Inside radius = a = R_i =	3.000 in	Mean diameter = D_m =	6.039 in	
Outside radius = b = R_o =	3.0391 in	Ratio: D_m/t =	154.5	Thin
		If Ratio < 20, Vessel is thick		

~~*Analysis of a Sphere*~~		~~Thin-wall sphere~~
~~Max Tangential Stress =~~	~~17265 psi~~	~~17376 psi~~
~~Max Radial Stress =~~	~~450 psi~~	

Analysis of a Cylinder		Graphite/Epoxy composite		Thin-wall cylinder
Max Tangential Stress =	34753 psi	Volume of cylinder =	11.13 in^3	34752 psi
Max Longitudinal Stress =	17152 psi	Weight of cylinder =	0.634 lb	17376 psi
Max Radial Stress =	-450 psi	[Composite: 0.057 lb/in^3]		
Ultimate strength =	278000 psi		Thin-walled	
Su/8 =	34750 psi	Actual N for su =	8.00	
Yield strength =	185000 psi			
Sy/4 =	46250 psi	Actual N for su =	8.00	
		N for p = 900 psi =	4	

SUMMARY OF RESULTS OF PROBLEMS 12-26 - 12-28 AND 12-30

Prob. No.	Material	Wall thickness	Weight	Ratio of weights based on composite design
12-26	Aluminum 6061-T6	0.250 in	7.363 lb	11.61
12-27	Titanium Ti-6Al-4V	0.065 in	2.972 lb	4.69
12-28	Stainless steel 17-4PH H900	0.052 in	4.167 lb	6.57
12-30	Graphite/epoxy composite	0.039 in	0.634 lb	1.00

PROBLEMS 12-31 TO 12-33:

These problems have the same design objectives with regard to the operating pressure, design factor, and inside diameter of a spherical pressure vessel. The material for the vessel is different for each problem. Following the given solution for Problem 12-33, a summary of the results for all three problems is given, comparing the wall thickness, volume of material in the sphere, and the weight or of the sphere. This should give the student and the reader a feel for how material selection affects the final product design.

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:	Problem Number: 12-31	
Pressure = p = 3000 psi	Wall thickness = t = 0.475 in	
Inside radius = a = R_i = 9.000 in	Mean diameter = D_m = 18.48 in	
Outside radius = b = R_o = 9.4750 in	Ratio: D_m/t = 38.9 Thin	
	If Ratio < 20, Vessel is thick	
Analysis of a Sphere	AISI 501 OQT 1000 St. Steel	Thin-wall sphere
Max Tangential Stress = 28472 psi	Volume of sphere = 509.5 in^3	29171 psi
Max Radial Stress = -3000 psi	Weight of sphere = 142.6 lb	
	St. Steel density = 0.280 lb/in^3	
~~*Analysis of a Cylinder*~~		~~Thin-wall cylinder~~
~~Max Tangential Stress = 58381 psi~~	~~Volume of cylinder = 413.5 in^3~~	~~58342 psi~~
~~Max Longitudinal Stress = 27690 psi~~	~~Weight of cylinder = 115.8 lb~~	~~29171 psi~~
~~Max Radial Stress = -3000 psi~~	~~[St. Steel: 0.280 lb/in^3]~~	
Ultimate strength = 175000 psi	Thin-walled	
Su/6 = 29167 psi	Actual N for su = 6.00	
Yield strength = 135000 psi		
Sy/4 = 33750 psi	Actual N for sy = 4.63	
	N for p = 6000 psi = 2.314	

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:	Problem Number: 12-32	
Pressure = p = 3000 psi	Wall thickness = t = 0.984 in	
Inside radius = a = R_i = 9.000 in	Mean diameter = D_m = 18.98 in	
Outside radius = b = R_o = 9.9840 in	Ratio: D_m/t = 19.3 Thick	
	If Ratio < 20, Vessel is thick	
Analysis of a Sphere	Aluminum 7075-T6	Thin-wall sphere
Max Tangential Stress = 13823 psi	Volume of sphere = 1115 in^3	14470 psi
Max Radial Stress = -3000 psi	Weight of sphere = 111.5 lb	
	Aluminum density = 0.100 lb/in^3	
~~*Analysis of a Cylinder*~~		~~Thin-wall cylinder~~
~~Max Tangential Stress = 29017 psi~~	~~Volume of cylinder = 880.3 in^3~~	~~28939 psi~~
~~Max Longitudinal Stress = 13008 psi~~	~~Weight of cylinder = 246.5 lb~~	~~14470 psi~~
~~Max Radial Stress = -3000 psi~~		
Ultimate strength = 83000 psi	Thick-walled	
Su/6 = 13833 psi	Actual N for su = 6.00	
Yield strength = 73000 psi		
Sy/4 = 18250 psi	Actual N for sy = 5.28	
	N for p = 6000 psi = 2.641	

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES						
Data Required:		Problem Number: 12-33				
Pressure = p =	3000 psi	Wall thickness = t =	0.49 in			
Inside radius = a = R_i =	9.000 in	Mean diameter = D_m =	18.49 in			
Outside radius = b = R_o =	9.4900 in	Ratio: D_m/t =	37.7	Thin		
		If Ratio < 20, Vessel is thick				
Analysis of a Sphere		Ti-6Al-4V Titanium			Thin-wall sphere	
Max Tangential Stress =	27604 psi	Volume of sphere =	526.4 in^3		28301 psi	
Max Radial Stress =	-3000 psi	Weight of sphere =	84.23 lb			
		Titanium density =	0.160 lb/in^3			
~~*Analysis of a Cylinder*~~					~~Thin-wall cylinder~~	
~~Max Tangential Stress =~~	~~56642 psi~~	~~Volume of cylinder =~~	~~426.9 in^3~~		~~56602 psi~~	
~~Max Longitudinal Stress =~~	~~26821 psi~~	~~Weight of cylinder =~~	~~68.31 lb~~		~~28301 psi~~	
~~Max Radial Stress =~~	~~-3000 psi~~	~~Titanium density =~~	~~0.160 lb/in^3~~			
Ultimate strength =	170000 psi		Thin-walled			
Su/6 =	28333 psi	Actual N for su =	6.01			
Yield strength =	155000 psi					
Sy/4 =	38750 psi	Actual N for sy =	5.48			
		N for p = 6000 psi =	2.738			

PROBLEMS 12-26 TO 12-33: Summary of Results

SUMMARY OF RESULTS OF PROBLEMS 12-31 - 12-33				
Prob. No.	**Material**	**Wall thickness**	**Weight**	**Ratio of weights based on titanium design**
12-31	AISI 501 OQT 1000 St. Steel	0.475 in	142.6 lb	1.69
12-32	Aluminum 7075-T6	0.984 in	111.5 lb	1.32
12-33	Titanium Ti-6Al-4V	0.490 in	84.23	1.00

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:		Problem Number: 12-34		
Pressure = p =	4200 kPa	Wall thickness = t =	25.9 mm	
Inside radius = $a = R_i$ =	225 mm	Mean diameter = D_m =	475.9 mm	
Outside radius = $b = R_o$ =	250.9 mm	Ratio: D_m/t =	18.4	Thick
		If Ratio < 20, Vessel is thick		
~~***Analysis of a Sphere***~~		Aluminum 6061-T6		~~Thin-wall sphere~~
~~Max Tangential Stress =~~	~~18.40 MPa~~	~~Volume of sphere =~~	~~0.018446 m^3~~	~~19.29 MPa~~
~~Max Radial Stress =~~	~~-4.20 MPa~~	~~Mass of sphere =~~	~~51.10 kg~~	
		Aluminum density =	2770 kg/m^3	
Analysis of a Cylinder		Length of cylinder =	600 mm	Thin-wall cylinder
Max Tangential Stress =	38.70 MPa	Volume of cylinder =	0.023234 m^3	38.59 MPa
Max Longitudinal Stress =	17.25 MPa	Weight of cylinder =	64.36 kg	19.29 MPa
Max Radial Stress =	-4.20 MPa	Aluminum density =	2770 kg/m^3	
Ultimate strength =	310 MPa		Thick	
Su/8 =	38.75 MPa	Actual N for su =	8.01	
Yield strength =	276 MPa			
Sy/4 =	69.00 MPa	Actual N for sy =	7.13	
		N for p = 8400 MPa =	3.57	

STRESSES IN THICK-WALLED CYLINDERS AND SPHERES

Data Required:		Problem Number: 12-35		
Pressure = p =	300 psi	Wall thickness = t =	0.301 in	
Inside radius = $a = R_i$ =	12.000 in	Mean diameter = D_m =	24.3 in	
Outside radius = $b = R_o$ =	12.301 in	Ratio: D_m/t =	80.7	Thin
		If Ratio < 20, Vessel is thick		
~~***Analysis of a Sphere***~~		AISI 1040 CD Steel		~~Thin-wall sphere~~
~~Max Tangential Stress =~~	~~5983 psi~~	~~Volume of sphere =~~	~~558.5 in^3~~	~~6055.1 psi~~
~~Max Radial Stress =~~	~~300 psi~~	~~Weight of sphere =~~	~~158 lb~~	
		Steel density =	0.283 lb/in^3	
Analysis of a Cylinder		Length of cylinder =	30 in3	Thin-wall cylinder
Max Tangential Stress =	12112 psi	Volume of cylinder =	689.4 in^3	12110 psi
Max Longitudinal Stress =	5906 psi	Weight of cylinder =	195.1 lb	6055.1 psi
Max Radial Stress =	-300 psi	Steel density =	0.283 lb/in^3	
Ultimate strength =	97000 psi		Thin	
Su/8 =	12125 psi	Actual N for su =	8.01	
Yield strength =	82000 psi			
Sy/4 =	20500 psi	Actual N for sy =	6.77	
		N for p = 600 psi =	3.386	

13-1(a) FIG. P13-1(a); 2 1/4-IN CARBON STEEL RIVETS; A36 STEEL: $S_y = 36$ KSI, $S_u = 58$ KSI

RIVET CAPACITY: 700 LB/RIVET × 2 RIVETS = 1400 LB (SHEAR)

BEARING ON A36: $\sigma_{ba} = 1.2 S_u = 1.2(58) = 69.6$ KSI

$F_b = \sigma_{ba} \cdot A_b = (69\,600\ \text{LB/IN}^2)(2)Dt = (69\,600)(2)(0.25)(0.375)$ LB

$F_b = 13\,050$ LB

TENSION: $F_{ta} = \sigma_{ta} A_t$

$\sigma_{tA} = 0.60 S_y = 0.60(36\,000) = 21\,600$ PSI

$A_t = [w - 2(D)]t = [3.00 - 2(0.25)]0.375 = 0.9375\ \text{IN}^2$

$F_{ta} = (21\,600\ \text{LB/IN}^2)(0.9375\ \text{IN}^2) = 20\,250$ LB

LIMITING LOAD = F_S = 1400 LB

13-1(b) FIG. 13-1(b): 3 3/16-IN CARBON STEEL RIVETS

FROM 13-1(a): $\sigma_{ba} = 69.6$ KSI, $\sigma_{ta} = 21\,600$ PSI (A36 STEEL)

SHEAR: $F_S = (540\ \text{LB/IN})(3\ \text{RIVETS}) = 1620$ LB LIMIT

BEARING: $F_b = \sigma_{ba} A_b = (69\,600\ \text{LB/IN}^2)(3)(0.1875)(0.375)\ \text{IN}^2$

$F_b = 14\,681$ LB

TENSION: $F_t = \sigma_{ta} A_t = (21\,600\ \text{LB/IN}^2)(3 - 3(0.1875))(0.375)\ \text{IN}^2$

$F_t = 19\,744$ LB

13-1(c) FIG. 13-1(c): 2 3/16-IN CARBON STEEL RIVETS; A36 STEEL PLATES

FROM 13-1(a): $\sigma_{ba} = 69.6$ KSI, $\sigma_{ba} = 21.6$ KSI; 3/8 PLATE IS CRITICAL.

SHEAR: $F_S = (540\ \text{LB/RIVET})(2\ \text{RIVETS})(2) = 2160$ LB (DOUBLE SHEAR) LIMIT

BEARING: $F_b = \sigma_{ba} A_b = (69\,600\ \text{LB/IN}^2)(2)(0.1875)(0.375)\ \text{IN}^2$

$F_b = 9788$ LB

TENSION: $F_{ta} = \sigma_{ta} A_t = (21\,600\ \text{LB/IN}^2)(3 - 2(0.1875))(0.375)\ \text{IN}^2$

$F_{ta} = 21\,263$ LB

13-1(d) SAME AS 13-1(C): F_S = 2160 LB LIMIT

13-2 (a) FIG. 13-2 (a); 4 3/16-IN RIVETS - STAINLESS ST., 950 LB/RIVET

AISI 430 STAINLESS ST. PLATES: $S_y = 80\,ksi$, $S_u = 90\,ksi$; FULL HARD

SHEAR: $F_s = (950\,LB/RIVET)(4) = \underline{3800\,LB}$ LIMIT

BEARING: $F_b = \sigma_{ba} A_b$

$\sigma_{ba} = 1.2 S_u = 1.2(90\,ksi) = 108\,ksi = 108\,000\,psi$

$A_b = (4)(D)(t) = (4)(0.1875)(0.50)\,IN^2 = 0.375\,IN^2$

$F_b = (108\,000\,LB/IN^2)(0.375\,IN^2) = \underline{40\,500\,LB}$

TENSION: $F_t = \sigma_{ta} \cdot A_t$

$\sigma_{ta} = 0.6 S_y = (0.6)(80000\,PSI) = 48\,000\,PSI$

$A_t = [4.0 - 2(0.1875)](0.50)\,IN^2 = 1.8125\,IN^2$

$F_t = (48\,000\,LB/IN^2)(1.8125\,IN^2) = \underline{87\,000\,LB}$

13-2 (b) FIG. 13-2(b): 6 5/32-IN RIVETS - STAINLESS STEEL, 650 LB/RIVET

AISI 430 PLATES: $\sigma_{ba} = 108000\,psi$, $\sigma_{ta} = 48000\,psi$ (FROM 13-2(a))

SHEAR: $F_s = (650\,LB/RIVET)(6\,RIVETS)\underbrace{(2)}_{\text{DOUBLE SHEAR}} = \underline{7800\,LB}$ LIMIT

BEARING: $F_b = \sigma_{ba} \cdot A_b = (108\,000\,LB/IN^2)(6)(0.156)(0.5)\,IN^2$

$F_b = \underline{50625\,LB}$

TENSION: $F_t = \sigma_{ta} \cdot A_t = (48000\,LB/IN^2)(4 - 3(0.156))(0.50)\,IN^2$

$F_t = \underline{84750\,LB}$

13-2(C) FIG. 13-2(c): 4 3/16-IN RIVETS - STAINLESS STEEL, 950 LB/RIVET

AISI 430 PLATES: $\sigma_{ba} = 108000\,psi$; $\sigma_{ta} = 48000\,psi$ (FROM 13-2(a))

SHEAR: $F_s = (950\,LB/RIVET)(4\,RIVETS)(2) = \underline{7600\,LB}$ (DOUBLE SHEAR)

BEARING: $F_b = \sigma_{ba} A_b = (108000\,LB/IN^2)(4)(0.1875)(0.5) = \underline{40500\,LB}$

TENSION: $F_t = \sigma_{ta} \cdot A_t = (48000\,LB/IN^2)(4 - 2(.1875))(0.5)\,IN^2 = \underline{87000\,LB}$

$\underline{LIMIT = F_s = 7600\,LB}$

13-2(d) FIG. 13-2(d): 2 1/4-IN RIVETS - STAINLESS STEEL, 1700 LB/RIVET

AISI 430 PLATES: $\sigma_{ba} = 108000\,LB/IN^2$; $\sigma_{ta} = 48000\,LB/IN^2$; FROM 13-2(a)

SHEAR: $F_s = (1700\,LB/RIVET)(2\,RIVETS)(2) = \underline{6800\,LB}$ (DOUBLE SHEAR)

BEARING: $F_b = \sigma_{ba} \cdot A_b = (108000\,LB/IN^2)(4 - 2(0.25))(0.5)\,IN^2 = \underline{189000\,LB}$

$\underline{LIMIT = 6800\,LB = F_s}$

13-3(a) FIG. 13-1(a); ASTM A307 STEEL BOLTS – 2 – 1/4 IN DIA.
PLATES: ASTM A242 HSLA; $S_y = 50$ KSI, $S_u = 70$ KSI
$\sigma_{ba} = 1.2 S_u = 1.2(70\text{ KSI}) = 84$ KSI
$\sigma_{ta} = 0.6 S_y = 0.6(50\text{ KSI}) = 30$ KSI
BOLTS: $\tau_a = 12$ KSI (NO THREADS IN SHEAR PLANE)
SHEAR: $F_s = \tau_a \cdot A_s = (12000\text{ LB/IN}^2)(2)(\pi)(0.25\text{ IN})^2/4 = \underline{1178\text{ LB}}$
BEARING: $F_b = \sigma_{ba} \cdot A_b = (84000\text{ LB/IN}^2)(2)(0.25)(0.375)\text{ IN}^2 = \underline{15750\text{ LB}}$
TENSION: $F_t = \sigma_{ta} \cdot A_t = (30000\text{ LB/IN}^2)(3.0 - 2(0.25 + 0.063))(0.375)\text{ IN}^2$
$F_t = \underline{26708\text{ LB}}$ [HOLE DIA. = D + 1/16 IN]
LIMIT = F_s = $\underline{1178\text{ LB}}$

13-3(b) FIG. 13-1(b); 3 3/16 IN BOLTS – SEE PROB. 13-3(a) FOR DATA.
SHEAR: $F_s = \tau_a \cdot A_s = (12000\text{ LB/IN}^2)(3)(\pi(0.1875)^2/4)\text{ IN}^2 = \underline{994\text{ LB}}$
BEARING: $F_b = \sigma_{ba} \cdot A_b = (84000\text{ LB/IN}^2)(3)(0.1875)(0.375)\text{ IN}^2 = \underline{17719\text{ LB}}$
TENSION: $F_t = \sigma_{ta} \cdot A_t = (30000\text{ LB/IN}^2)(3.0 - 3(0.1875 + 0.063))(0.375)\text{ IN}^2$
$F_t = \underline{25296\text{ LB}}$
LIMIT = $F_s = \underline{994\text{ LB}}$

13-3(c) FIG. 13-1(c): 2 3/16 IN BOLTS – DOUBLE SHEAR – SEE PROB. 13-3(a)
SHEAR: $F_s = \tau_a \cdot A_s = (12000\text{ LB/IN}^2)(2)(2)(\pi(0.1875)^2/4)\text{ IN}^2 = \underline{1325\text{ LB}}$
BEARING: $F_b = \sigma_{ba} \cdot A_b = (84000\text{ LB/IN}^2)(2)(0.1875)(0.375)\text{ IN}^2 = \underline{11813\text{ LB}}$
TENSION: $F_t = \sigma_{ta} \cdot A_s = (30000\text{ LB/IN}^2)(3.0 - 2(0.1875 + 0.063))(0.375)\text{ IN}^2$
$F_t = \underline{28114\text{ LB}}$
LIMIT = $F_s = \underline{1325\text{ LB}}$

13-3(d) FIG. 13-3(d): SAME AS 13-3(c), $F_s = 1325$ LB

13-4(a) FIG. 13-2(a): 4 3/16-IN BOLTS, ASTM A325 STEEL, $\tau_a = 30$ KSI
ASTM A514 ALLOY STEEL: $S_y = 100$ KSI, $S_u = 110$ KSI
$\sigma_{ba} = 1.2 S_u = 1.2(110) = 132$ KSI
$\sigma_{ta} = 0.6 S_y = 0.6(100\text{ KSI}) = 60$ KSI
SHEAR: $F_s = \tau_a \cdot A_s = (30000\text{ LB/IN}^2)(4)(\pi(0.1875)^2/4)\text{ IN}^2 = \underline{3313\text{ LB}}$
BEARING: $F_b = \sigma_{ba} \cdot A_b = (132000\text{ LB/IN}^2)(4)(0.1875)(0.5)\text{ IN}^2 = \underline{49500\text{ LB}}$
TENSION: $F_t = \sigma_{ta} \cdot A_t = (60000\text{ LB/IN}^2)(4.0 - 2(0.1875 + 0.063))(0.5)\text{ IN}^2$
$F_t = \underline{112485\text{ LB}}$
LIMIT = $F_s = \underline{3313\text{ LB}}$

13-4(b) FIG. 13-2(b): 6 5/32 BOLTS, DOUBLE SHEAR, SEE PROB 13-4(a)

SHEAR: $F_s = \tau_a \cdot A_s = (30\,000\ LB/IN^2)(6)(2)(\pi(0.156)^2/4)IN^2$

$F_s = 6903\ LB$

BEARING: $F_b = \sigma_{ba} \cdot A_b = (132\,000\ LB/IN^2)(6)(0.156)(0.5)IN^2 = 61875\ LB$

TENSION: $F_s = \sigma_{ta} \cdot A_t = (60\,000\ LB/IN^2)(4.0-3(0.156+0.063))(0.5)IN^2$

$F_s = 100\,268\ LB$

LIMIT $= F_s = 6903\ LB$

13-4(c) FIG. 13-2(c): 4 3/16-IN BOLTS, DOUBLE SHEAR, SEE PROB. 13-4(a)

SHEAR: $F_s = \tau_a \cdot A_s = (30\,000\ LB/IN^2)(4)(2)(\pi(0.1875)^2/4)IN^2 = 6627\ LB$

BEARING: $F_b = \sigma_{ba} \cdot A_b = (132\,000\ LB/IN^2)(4)(0.1875)(0.5)IN^2 = 49500\ LB$

TENSION: $F_t = \sigma_{ta} \cdot A_t = (60\,000\ LB/IN^2)(4.0-2(0.1875+0.063))(0.5)IN^2 =$

$F_t = 104970\ LB$

LIMIT $= F_s = 6627\ LB$

13-4(d) FIG. 13-2(d): 2 1/4-IN BOLTS, DOUBLE SHEAR, SEE PROB. 13-4(a)

SHEAR: $F_s = \tau_a \cdot A_s = (30\,000\ LB/IN^2)(2)(2)(\pi(0.25)^2/4)IN^2 = 5890\ LB$

BEARING: $F_b = \sigma_{ba} \cdot A_b = (132\,000\ LB/IN^2)(2)(0.25)(0.50)IN^2 = 33000\ LB$

TENSION: $F_t = \sigma_{ta} \cdot A_t = (60\,000\ LB/IN^2)(4.0-2(0.25+0.063))(0.5)IN^2 =$

$F_t = 101220\ LB$

LIMIT $= F_s = 5890\ LB$

13-5 FIG. P13-5 $P = 6500\ LB$, $M = P \cdot 48 = (6500\ LB)(48\ IN) = 312\,000\ LB \cdot IN$

LOADS SHARED BY CONNECTIONS ON TWO SIDES OF COLUMNS

EACH SIDE: $P = 3250\ LB$, $M = 156000\ LB \cdot IN$ - SIX BOLTS ASTM A325

$R_P = 3250\ LB/6 = 542\ LB/BOLT \downarrow$ $\tau_a = 30\ KSI$

$\Sigma(x^2+y^2) = 6(2.0)^2 + 4(2.5)^2 = 49\ IN^2$

FOR UPPER RIGHT BOLT:

$$R_{1x} = \frac{M\,y_1}{\Sigma(x^2+y^2)} = \frac{(156000\ LB \cdot IN)(2.5\ IN)}{49\ IN^2} = 7959\ LB$$

$$R_{1y} = \frac{M\,x_1}{\Sigma(x^2+y^2)} = \frac{(156000)(2.0)}{49} = 6367\ LB\downarrow$$

$$R_{R1} = \sqrt{(7959^2 + (542+6367)^2)} = 10206\ LB$$

REQ'D. $A = R_{R1}/\tau_a = 10206\ LB / 30\,000\ LB/IN^2 = 0.340\ IN^2 = \pi D^2/4$

$$D_{MIN} = \sqrt{4A/\pi} = \sqrt{\frac{4(0.340\ IN^2)}{\pi}} = 0.658\ IN$$

SPECIFY 3/4 IN = 0.750 IN BOLTS

CHECK BEARING ON 3/8 IN A36 STEEL PLATE: $S_u = 58\ KSI$

$\sigma_{ba} = 1.2 S_u = 1.2(58000) = 69\,600\ LB/IN^2$

$$\sigma_b = \frac{F_b}{A_b} = \frac{10206\ LB}{(0.75)(0.375)IN^2} = 36\,288\ LB < \sigma_{ba}\ \text{OK}$$

13-6 FIG. P13-6

POSSIBLE SOLUTION –

$P_X = P\sin 30° = 13.0\,kN$

$P_Y = P\cos 30° = 22.5\,kN$

$M = (22.5)(850) = 19125\,kN\cdot mm$

12-ASTM A325 BOLTS –

$\tau_a = 30.0\,ksi = 207\,MPa$

ON BOLT ①

$$\frac{P_X}{12} = \frac{13.0\,kN}{12} = 1.083\,kN \leftarrow$$

$$\frac{P_Y}{12} = \frac{22.5\,kN}{12} = 1.875\,kN \uparrow$$

FORCES DUE TO MOMENT:

$$\Sigma(X^2+Y^2) = 6(50)^2 + 6(100)^2 + 8(100)^2 = 155000\,mm^2$$

$$R_{1X} = \frac{M\,y_i}{\Sigma(X^2+y^2)} = \frac{19125\,kN\cdot mm(100\,mm)}{155000\,mm^2} = 12.34\,kN \leftarrow$$

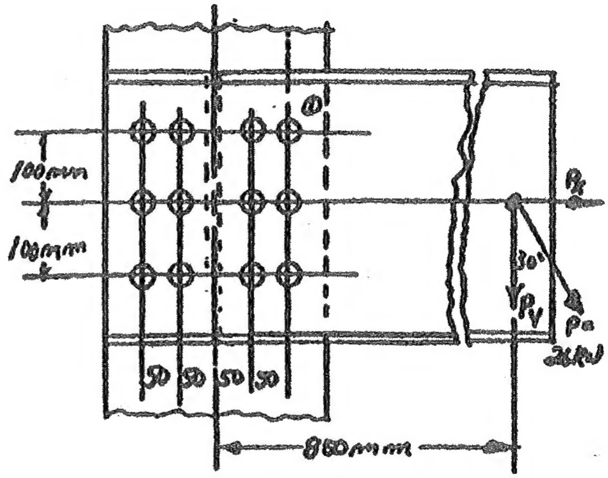
$$R_{1Y} = \frac{M\,X_i}{\Sigma(X^2+y^2)} = \frac{19125\,kN\cdot mm(100\,mm)}{155000\,mm^2} = 12.34\,kN \uparrow$$

$$R_1 = \left[(1.083+12.34)^2 + (1.875+12.34)^2\right]^{1/2} = 19.55\,kN$$

$$REQ'D.\ A = \frac{R_1}{\tau_a} = \frac{19550\,N}{207\,N/mm^2} = 94.4\,mm^2 = \pi D^2/4$$

$$D_{MIN} = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(94.4\,mm^2)}{\pi}} = 10.96\,mm$$

SPECIFY M12 BOLT, D = 12.0 mm



13-7 FIG. P13-1 (a)

$P = \tau_a L t = (18000\ lb/in^2)(6\ in)(0.707)(0.3125\ in) = \underline{23860\ lb\ on\ welds}$

ON STRAPS: $P = \sigma_{ba}(3)(3/8) = (0.6)(36000\ lb/in^2)(3\ in)(0.375\ in) = 24\,300\ lb$

WELDS GOVERN JOINT STRENGTH

13-8 FIG. P13-2 (c)

$P = \tau_a L t = (21000\ lb/in^2)(8\ in)(0.707)(0.250\ in) = \underline{29700\ lb\ on\ welds.}$

ON STRAPS: $P = (0.6)(50000\ lb/in^2)(0.5\ in)(4\ in) = 60\,000\ lb$

THE SOLUTIONS SHOWN BELOW FOR PROBLEMS 13-9, 13-10, AND 13-11 ARE JUST SAMPLES OF MANY POSSIBLE SOLUTIONS. THE GENERAL CONFIGURATION AND NUMBER OF FASTENERS SHOWN IN FIG. 13-1 ARE USED, BUT OTHERS COULD BE USED.

13-9 TOTAL LOAD $= 15.0\ Mg = (15.0\times10^3\ kg)(9.81\ m/s^2) = 147\,150\ N$

4 SUPPORTS: LOAD/SUPPORT $= 147150/4 = 36788\ N$ DOUBLE SHEAR

USE 6 RIVETS AS SHOWN. $F_s = 36788\ N/(2)(6) = 3066\ N$

SPECIFY 6.35 mm (1/4 IN) CARBON STEEL RIVETS, 3114 N CAP'Y.

CHECK BEARING ON WEB OF TEE, $t = 10.6\ mm$, A36 STEEL

$S_y = 248\ MPa$, $S_u = 400\ MPa$; $\sigma_{ba} = 1.2 S_u = 1.2(400) = 480\ MPa$

$$\sigma_b = \frac{F}{A_b} = \frac{2(3066\ N)}{(6.35)(10.6)\ mm^2} = 91.1\ N/mm^2 = 91.1\ MPa < 480\ MPa\ \ \underline{OK}$$

DETERMINE THICKNESS OF STRAPS (2)

TENSION: $\sigma_{ta} = 0.6 S_y = 0.6(248\ MPa) = 148.8\ MPa$

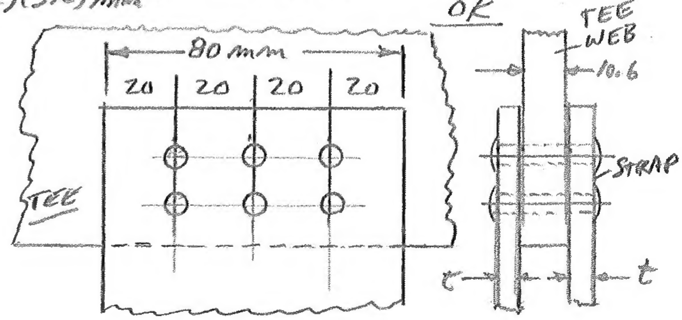
$F_t = \sigma_{ta} A_t$; $A_t = (w - 3D)2t = [80 - 3(6.35)](2)t = 121.9t$

$36788\ N = (148.8\ N/mm^2)(121.9t) = 18139t$

$t_{MIN} = 36788\ N/18139 = 2.03\ mm$ SPECIFY $t = 3.0\ mm$

CHECK BEARING ON STRAPS

$$\sigma_b = \frac{F}{A_b} = \frac{36788\ N}{(6)(6.35)(2)(3.0)\ mm^2} = 161\ N/mm^2 = 161\ MPa < 480\ MPa\ \ \underline{OK}$$



13-10 SEE FIG. 13-1. FORCE ON JOINT = 36788 N (PROB 13-9)

USE 2 BOLTS, DOUBLE SHEAR, ASTM A325, $\tau_a = 207\,MPa$

$$\tau = F_S/A_S \;\therefore\; A_S = \frac{F_S}{\tau_a} = \frac{36788\,N}{207\,N/mm^2} = 177.7\,mm^2 \text{ TOTAL}$$

$$A_{BOLT} = \frac{A_S}{\underbrace{(2\,BOLTS)(2)}_{\text{DOUBLE SHEAR}}} = 44.43\,mm^2 = \pi D^2/4$$

$$D_{MIN} = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(44.43)}{\pi}} = 7.52\,mm \text{ ; SPECIFY M8 BOLTS } D = 8.0\,mm$$

CHECK TENSION ON STRAPS $\sigma_{ta} = 148.8\,MPa$ (PROB 13-9)

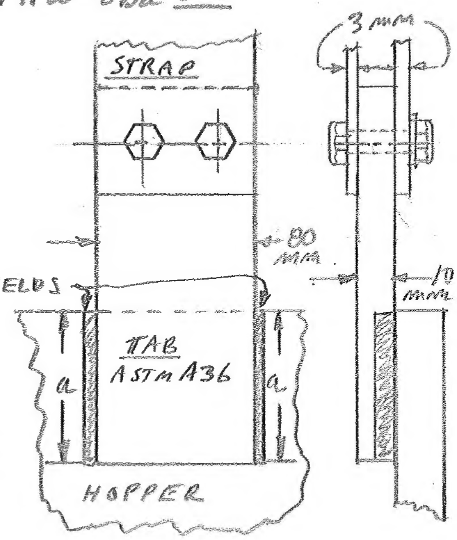
$$\sigma_t = F/A_t \;,\; A_t = (80 - 2(8))(2)(3) = (w - 2D)(2)(t) = 384\,mm^2$$

$$\sigma_t = \frac{36788\,N}{384\,mm^2} = 95.8\,MPa < 148.8\,MPa \;\underline{=OK}$$

CHECK BEARING ON STRAPS $\sigma_{ba} = 480\,MPa$ (PROB 13-9)

$$\sigma_b = \frac{F}{A_b} = \frac{36788\,N}{(2)(8)(2)(3)\,mm^2} = 383\,MPa < \sigma_{ba} \;\underline{-OK}$$

(2) – 2 BOLTS; (8) – D_{BOLT}; (2) – 2 STRAPS; (3) – t OF STRAP



13-11 SEE FIG. 13-1. WELDED JOINT

FORCE ON JOINT = 36788 N (PROB 13-9)

ASTM A36 STEEL – E60 ELECTRODE

$\tau_a = 124\,MPa$

$$\tau = \frac{F}{A_w} = \frac{36788\,N}{L \cdot t}$$

$L = 2a$; $t = 0.707\,w$

w = WELD LEG SIZE = 5.0 mm (SPECIFIED)

$t = 0.707(5) = 3.535\,mm$

$$\text{REQ'D } L = F/\tau_a t = \frac{36788\,N}{(124\,N/mm^2)(3.535\,mm)}$$

$L_{MIN} = 83.93\,mm = 2a$

$a_{MIN} = 83.93/2 = 41.96\,mm$

SPECIFY $a = 45\,mm$